O'GORMAN'S

INTUITIVE CALCULATIONS

EDITED BY

PROFESSOR YOUNG

INTUITIVE CALCULATIONS;

OR,

EASY AND COMPENDIOUS METHODS

OF PERFORMING THE VARIOUS ARITHMETICAL OPERATIONS REQUIRED IN

COMMERCIAL AND BUSINESS TRANSACTIONS;

TOGETHER WITH

FULL EXPLANATIONS OF DECIMALS AND DUODECIMALS, SEVERAL USEFUL TABLES, AND AN EXAMINATION AND DISCUSSION OF THE BEST SCHEMES FOR

A DECIMAL COINAGE.

BY

DANIEL O'GORMAN.

The Twenty-Fourth Edition, Corrected and Enlarged by

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PREFACE.

I HAVE undertaken the editorship of this twenty-fourth edition of the INTUITIVE CALCULATIONS for two reasons. One is that I was personally acquainted with the author:—he was a student in one of my Mathematical Classes in the College of Belfast; and this circumstance alone made me feel a more than ordinary interest in his book. The other reason is that I considered the book itself, upon the whole, to be an ingenious, an instructive, and, in some respects, a remarkable performance.

It seems to have been the author's design to produce a work that should form a practically useful supplement to the ordinary treatises on common arithmetic; and disregarding the hackneyed precepts laid down in those treatises for executing, by a *general* Rule, every example coming under a distinct subdivision of the subject, to devise *special* Rules for *special* cases, and thus to economize figure-work as much as possible.

Those who are familiar with school-arithmetic, and whose professional avocations, or whose private pursuits apart from these, may demand an occasional exercise of its principles, seldom feel any difficulty in recalling to memory the general precepts here adverted to, or in applying them successfully in practice. The Rules themselves are comparatively few in number, and are therefore usually remembered without much effort. Each general Rule is applicable to a large class of particular examples; and special direc-

tions are but seldom given for simplifying the calculation in individual instances, however these may be distinguished, by favourable peculiarities, from the general mass.

The present work is constructed upon an entirely different principle. *Here*, the main object is to give special Rules for special business-callings; and to introduce *general* Rules only when there is no unnecessary expenditure of figures in any of the individual instances coming under them.

As a consequence of these more minute subdivisions of the subject, the Rules in this book are more numerous than those given in the school treatises. It is not, however, designed to be a school manual of arithmetic; but rather to be a depository of easy and expeditious methods of calculation for the guidance and use of those whose business occupations require them to be more especially expert in some particular department of the general subject: it is, in fact, in so far as its scope extends, to be regarded more as a sort of Arithmetical Dictionary, or Book of Reference, for the use of such commercial men, traders, artificers, &c., as may have to do with those arithmetical calculations *only* which are exclusively connected with their own respective callings. As these callings are *special* so are the Rules.

But although not intended for ordinary school-purposes, yet it is a book which, I think, every teacher of arithmetic should possess. It will show to him, as it has shown to me, how much the exercise of common sense, and a little ingenuity, may sometimes do in the way of shortening and simplifying numerical computations; and a judicious teacher may, from time to time, avail himself of much that this work will supply,—to the advantage of his pupils, in the form of oral instruction. It is likely that many of these compendious methods, *after* they are pointed out, may seem to spontaneously suggest themselves, so to speak, from the very conditions of the inquiry in hand. Such is really the case; and it was no doubt this fact that induced the author

to adopt the title he has given to his book. And I think it all the more meritorious in Mr. O'Gorman that he should have *observed*, and have turned to profitable account, what so many others had merely *seen*,—and disregarded.

But to what extent the author of the Intuitive Calculations may have been indebted to preceding writers for the numerous compendious Rules of operation he has introduced into his book, and how many of these Rules are due entirely to his own sagacity, I am unable to say. I may, however, affirm with confidence that there does not exist any work on arithmetic in which so many ingenious expedients are devised for abridging labour and saving time, and so much judicious advantage is taken of the resources of common arithmetic, as in the present volume; in fact, I consider the book to be *unique*.

Of course I could not speak in these laudatory terms of any work of my own; and even in reference to the Intuitive Calculations I cannot but admit that, notwithstanding its many excellencies, there were blemishes, and even inaccuracies of no trifling kind. The mere blemishes were principally those observable in want of system in the arrangement of the several parts of the performance, want of clearness and of completeness in certain of the explanations and proofs of Rules, and also in occasional departures from good taste in the way in which the author has descanted upon the merits of his own work: the inaccuracies were for the most part those necessarily involved in the so-called doctrine of multiplying money by money, and in the further attempt to multiply together concrete quantities generally.

In the present edition, these, as also some minor defects, are removed; and the work has been considerably enlarged.

When a revision of the book was first proposed to me, I thought that nothing *more* than a revision would be necessary; but, upon carefully examining it throughout, I came to the conclusion that it would be an easier task to

re-write the whole, preserving, however, in substance, all that I regarded as really creditable to the author and useful to the reader; and making such additions as any one instructed, as I felt myself to have been, by Mr. O'Gorman's ingenious arithmetical expedients, might easily supply.

This twenty-fourth edition, therefore, is printed from *manuscript;* the whole book has been re-written, with the exception of perhaps two, or at most three pages, which, a few lines, transcribed here and there, may amount to. And I venture to hope, as I have exerted my best endeavours to do every justice to Mr. O'Gorman and his book, that this edition will be found to be an improvement upon the editions which have preceded it. I have handled the book with unreserved freedom, because I felt satisfied that, if the author himself could have been consulted, he would have given me full liberty to do so.

But whatever there may be in the present impression which may be thought to appear in favourable contrast, or comparison, with what the original author may himself have delivered under the same head (should any such grounds for preference really exist), I beg that it may be distinctly understood that I consider the credit—such as it is—to be due entirely to O'Gorman. I have but "ploughed with his heifer," and if I have turned up an additional furrow or two, I have done so only upon his own ground. In some things I was his teacher; but there is not a single item in his book for which he is indebted to me.

With the view, however, of making the present edition as comprehensively useful as I could, within the limits assigned to it, I have introduced some topics not included in the former impressions, and have considerably enlarged upon others. The compact letter-press has enabled me thus to amplify the matter, without greatly increasing the bulk or price of the volume. I need specify here only the articles on what is called ALLIGATION, which concerns the mixing

together of different ingredients, so as to form a desired compound; and I have taken care to supply proofs (usually omitted) of the several Rules of operation. I have also extended the articles on ARTIFICERS' WORK; and, by repeated cautionary observations, have endeavoured to preclude the possibility of erroneous conceptions, by the reader, of the duodecimal notation, and of what the several denominations of superficial and cubic measures really are, when these measures are expressed in that notation; for it is a subject upon which misconception prevails to some extent among workmen, and even in books. The articles on Interest too, and on the collateral topics—Commission, Brokerage, Purchase of Stocks, &c., have likewise been extended. But the annexed Table of Contents gives a sufficiently detailed enumeration of the multifarious particulars discussed in the book; and the reader has only to refer, at random, to a few of the *special* Rules there pointed to, to satisfy himself that, under the guidance of those Rules, he would be conducted, in each case, to the desired end, by a shorter and an easier path, than by following the stereotyped directions of any treatise on Arithmetic with which he may be acquainted.

I shall only add here that two distinct articles "ON THE DECIMAL COINAGE" will be found at the end of the work; and that the second of these articles is but a slight modification of what has appeared in the former editions. This is a topic in reference to which public interest has considerably subsided of late years: it may possibly be revived. But the compendious methods of calculation taught in this book, will show that in a great variety of ordinary business transactions, in which numerical computations are necessary, the calculator may perform his work very expeditiously—even with our present systems of money, weights, and measures. The advantages to him, which would accrue from any change in these, cannot be *fairly* estimated unless it be first seen that arithmetic has done its best with things as they are. In

whatever proportion monetary and commercial calculations can be facilitated, without any alteration of our existing systems at all—in that proportion will the desire for alteration become less urgent. And I cannot but think that the general circulation of the easy Rules here delivered would do much to moderate the demand for a DECIMAL COINAGE.

I feel reluctant to conclude this preface without adding a few remarks personal to the author. I have adverted above to Mr. O'Gorman's manifest propensity to extol the merits of his own performance: this is observable in his "INTRODUCTION," and also, here and there, in the body of his book; and, from this peculiarity, some of his readers may have inferred that O'Gorman was an ostentatious and self-sufficient person. He was the very reverse: his manners were gentle, modest, and retiring; and while a student at Belfast, his unobtrusive demeanour, his genial disposition, and his uniform propriety of conduct, secured to him the general esteem of his class-fellows, and made him an especial favourite. My appreciation of his amiable qualities, and the melancholy recollection of his sad end, have caused me to feel a deep and peculiar interest in this undertaking. The circumstances of his death may be recorded in a few words. Intending to proceed to Australia, he embarked as passenger for Melbourne, in THE LONDON; and he perished in that ill-fated vessel, January 11th, 1866, when only five days' sail from Plymouth harbour. It seemed to me that an improved edition of the INTUITIVE CALCULATIONS was the most befitting tribute which it was in my power to offer to the memory of my lamented pupil.

Tyrrel Road, East Dulwich. J. R. YOUNG.

*** *The editor will feel thankful for any hints or suggestions for the future improvement of this work. Communications to be addressed to the Publishers, Messrs. Lockwood and Co., 7, Stationers' Hall Court, London, E.C.*

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Errata.—Page 14, last line, for 108 read 182. Page 229, line 3, for "take half the number," &c., *read* "regard the unit's figure as a *decimal*, and divide by 2"."

7

INTUITIVE CALCULATIONS.

The present work is not intended to teach children the first principles of common Arithmetic. These first principles are here presumed to have been previously acquired from one or other of the numerous manuals for the use of schoolboys

ERRATA.

Page 148, lines 9 and 11, *for* $1\frac{1}{2}$, $\frac{3}{4}$, *read* $\frac{2}{3}$, $\frac{1}{3}$.
„ 206, line 3, *for* 5750 $\frac{250}{493}$, *read* 575050 $\frac{350}{493}$.

Intuitive Calculations.

calculation, but further to prove to him that what he is thus told inculcates no violation of the arithmetical principles he may have learnt at school, but only a judicious economizing of time and figures.

It should here be remarked, however, that what are sometimes declared to be short methods of calculation, and which actually appear to be such to the eye, are deceptive. There may be fewer figures put down upon the paper, and yet more time and thought be expended than the operation with twice that number of figures would demand; and in this kind of brevity, the risk of error, and the trouble of revising the process, are sure to be increased. In what follows, all such merely apparent abbreviations of work will be excluded: the saving of figures *only* is of comparatively but little moment.

That cannot surely be called a *short* path to any terminus in which the *time* of passage is lengthened, and the risk of a false step increased.

In each of the various commercial callings of civilized life, the calculations in request are almost always of a specific and limited kind. In some, the *weights* only of commodities are matters of consideration; in others, *measures;* in others, *measurements*,—linear, superficial, or solid; and in others again (purely monetary transactions), not one of these is a subject of concernment; though, in certain business occupations, all may enter into combination in the estimate of the work performed, or the service rendered, and the pecuniary accommodation (or credit) granted.

It is the design of this book to teach, under distinct heads, the most expeditious methods of executing the different commercial calculations here alluded to; but such of these calculations as do not admit of being reduced to a simpler or more convenient form, by any modification of the rules in common use, will be retained in their proper places here, without alteration. As to the simple and fundamental rules for Addition, Subtraction, Multiplication, and Division, we shall omit them altogether, except certain cases of the two latter, in which a departure from the common practice will be seen to be attended with advantage.

We shall merely add, in concluding these introductory remarks, that although this book is not prepared for schoolboy use, yet that the judicious schoolmaster may perhaps consult its pages with profit; and in the exercise of his arduous profession may occasionally make selections from them for the special benefit of such of his pupils as he may know to be destined for specified commercial callings.

SIMPLE MULTIPLICATION.

Case 1.

When the multiplier is a number between 10 *and* 20.

Rule.—Multiply each figure of the multiplicand, in succession, by the units-figure of the multiplier, as in the

common method; but here, to each product add, not only what is carried, but also the next right-hand figure of the multiplicand as well: to the last figure of the multiplicand add what is carried only.

Thus, in the first of the examples following, 3 times 5 are 15, and 1 is carried; 3 times 8, with this 1, are 25, which, by including the back-figure, 5, makes 30, the 3 being carried; then 3 times 3, with this carried figure, makes 12, and adding in the back-figure, 8, we get 20; and the 2, now carried to the last figure, 3, of the multiplicand, gives 5; and the work is finished.

EXAMPLES.

Multiply	385	679	4873	6958	7956	7685
by	13	16	18	17	15	19
Product	5005	10864	87714	118286	119340	146015

No explanation of this method can be needed: it is obvious that the several back-figures, added in at once with the carryings, are the figures actually written down, in the common method, and added vertically with those carryings.

If the first figure of the multiplier, instead of being unit, is two units, then each back-figure is to be doubled, the rule being this.

CASE 2.

When the multiplier is a number between 20 *and* 30.

RULE.—Multiply as in last case, taking in the *double* of the back-figure; and add what is carried, from the last multiplication, to the last figure of the multiplicand thus *doubled*.

EXAMPLES.

Multiply	798	567	395	395	487	6784	123
by	22	23	23	27	27	28	29
Product	17556	13041	9085	10665	13149	189952	3567

CASE 3.

When the multiplier is 111, *or* 112, *or* 113, *&c., up to* 119.

RULE.—Multiply by the first figure on the right-hand, as in the first case; but instead of *one* back-figure, add the sum

of the *two* back-figures, so soon as there are two to add; that is, when the third figure of the multiplicand is reached. Add what is carried from the *final* product to the sum of the last two figures, and if anything is carried from the result, add it to the *last* figure itself; but if nothing is carried, merely bring down this last figure.

Examples.

Multiply	2183	4296	5589	6273	7182	83716
by	111	112	113	114	115	116
Product	242313	481152	631557	715122	825930	9711056

To illustrate these operations, let us take the third example. Here 3 times 9 are 27, carry 2; 3 times 8 are 24, which, with the 2 carried, gives 26, and the back-figure 9 being taken in, we have 35, carry 3. Then 3 times 5 are 15, and 3 are 18, and 8 are 26, and 9 are 35 (8 and 9 being the two back-figures), carry 3. Then 3 times 5 are 15, and 3 are 18, and 5 are 23, and 8 are 31 (5 and 8 being the two back-figures), carry 3. Again, add the 3, thus carried from the final product (31), to the sum of 5 and 5, the result is 13, carry 1, this added to the last figure (5) gives 6.

In each of the first two examples, nothing is carried to the last figure, which is therefore merely brought down.

It is readily seen that the back-figures, added in this method, are the very figures actually written down, and added vertically in the common operation.

The following general rule includes the cases 1 and 3 above, and renders special directions for *them* unnecessary.

General Rule.

When the multiplier is a single figure preceded by any number of ones.

Rule.—1. Prefix to the multiplicand as many *ciphers* as there are prefixed *ones* in the multiplier.

2. Multiply by the figure to which the ones are prefixed, adding in, as we proceed, first the single back-figure, then the two back-figures, then the three back-figures, and so on, till the back-figures thus added in are just as many in

number as there are prefixed ciphers: they are never to be more in number.

3. These multiplications and additions are to be continued up to the last of the prefixed ciphers inclusive; and when this is reached the work terminates.

We shall take the fourth example in each of the two cases alluded to, and give two additional examples in which three ones are prefixed, and a final example in which four are prefixed.

Multiply	06958	006273	0006958	0006273	0000435216
by	17	114	1117	1114	11115
Product	118286	715122	7772086	6988122	4837425840

If the reader will only take the trouble to work the last of these examples in the ordinary way, he will see that the saving is considerable, as well in time as in figures; and he will at the same time perceive that the back-figures, added in, in this method, are the same as the figures actually written down, and added vertically, in the old method.

To Multiply by any Number of Nines.

Rule.—Add as many ciphers to the right-hand of the multiplicand as there are nines in the multiplier, and from the result subtract the original multiplicand, the remainder will be the product.

Examples.

1.—Multiply 2368 by 999.

	2368000
	2368
Product	2365632

2.—Multiply 37568 by 999999.

	37568000000
	37568
Product	37567962432

Instead of the proposed multiplier, if we were to take 1, followed by as many ciphers as there are nines in the true multiplier, it is obvious that the product would be *once* the multiplicand too great. But by annexing the ciphers, as above, we *do*, in effect, multiply by this additional *once;* so that once the multiplicand has to be subtracted in order that the true product may be obtained.

LONG DIVISION.

THE operation for *short* division, that is, when the divisor is only a single figure or digit, is too simple, by the common method, to be susceptible of, or to need, any abbreviation; but when the divisor consists of two or more digits much greater compactness may be given to the work by conducting it in accordance with the following rule.

RULE.—1. Place the divisor to the left of the dividend, as in the ordinary arrangement, and draw a horizontal at some little distance below the dividend.

2. As in the common method, find how often the divisor is contained in the number given by the first few figures of the dividend,—two, three, or four, &c., as may be found necessary; and write the corresponding quotient-figure below this horizontal line, and directly under the last of the dividend-figures used.

3. Multiply the divisor by this quotient-figure, subtract the result from the used figures of the dividend, and place the figures of the remainder, one below another, as they arise, vertically under the next unused figure of the dividend.

4. The figures, thus in vertical column (above the horizontal line), when read *upwards*, give the next number to which the division is to be applied; and, as before, the new quotient-figure is to be written below the horizontal line, beside the first; and, also as before, the figures of the remainder, one after another, are to be written vertically under the *next* figure of the dividend; and so on, as in the following examples.

EXAMPLES.

1.—786547632 ÷ 14

14)	78	6	5	4	7	6	3	2	
		8	2	1	2	3	0	5	
		.	.	1	.	1	1	.	
Quotient	5	6	1	8	1	9	7	3	10 rem.

2.—237869547 ÷ 17

17)	23	7	8	6	9	5	4	7	
		6	6	5	3	5	4	0	
		.	1	1	.	.	.	1	
Quot.	1	3	9	9	2	3	2	6	5 rem.

In each of these examples, the two leading figures form a number which suffices for the first partial dividend. Taking the first of these examples, this number is 78, which contains 14, 5 times, leaving the remainder 8, which is placed vertically under the 6, the next figure of the dividend. The

partial dividend is now 86, which contains 14, 6 times, leaving the remainder 2. We now have 25, which contains 14 once, leaving the remainder 1 from the 5 and 1 from the 2. The next partial dividend is therefore 114, which contains 14, 8 times, giving 2 for remainder; and 27 contains 14 once, giving a remainder 3 from the 7 and 1 from the 2. The dividend now is 136, containing 14, 9 times, leaving for remainder 0 from the 6 and 1 from the 3. The dividend, 103, gives 7 for quotient and 5 for remainder; and, lastly, 52 gives 3 for quotient and 10 for remainder.

Of course the vertical lines, drawn above, would not be introduced in actual practice: they are marked here only the more clearly to show what figures are to be kept in column.

3. 18) 395816432
3767403
1111

Quot. 21989801, 14 rem.

4. 19) 437652789
5068603
11

Quot. 23034357, 6 rem.

5. 28) 3147869754
36160557
1 12 2

Quot. 112423919, 22 rem.

6. 32) 476987546
5810792
12 21

Quot. 14905860, 26 rem.

This method is given here chiefly on account of its conciseness: whether or not the computer can save time by employing it, will depend upon his degree of expertness in performing the two processes of multiplication and subtraction at once. A different arrangement, involving the same double process, is that exhibited in the second working of the example below, in which, however, more figures have to be written down.

7. 234) 78678543654
45472169
84577215
1 2

Quot. 336233092, 126 rem.

234) 78678543654 (336233092
847
1458
545
774
723
2165
594
126 rem.

Here it will be seen that each partial quotient, formed in the second operation from the remainder and new figure brought down, is written horizontally, while in the first operation the same partial quotient is written vertically, and the bringing down of the successive figures of the dividend is thus dispensed with. A sufficient reason for the first operation is seen by comparing it with the second.

NOTE.—Instead of writing the several quotient-figures *below* a horizontal line, drawn at some distance beneath the dividend, we may write them immediately under the dividend; placing the figures of the remainder, at each step, vertically *over* the next unused figure of the dividend; as in the example here annexed. In this arrangement, we dispense with the necessity of estimating the distance below the dividend at which the horizontal line should be drawn; and, moreover, the divisor, the dividend, and the quotient, have the same relative positions as they have in short division. The learner may work the preceding examples in this manner, and judge for himself as to which is the more convenient arrangement.

$$\begin{array}{rl} & 1\quad 121 \\ & 4\ 81404 \\ 21) & 567854635 \\ \hline & 27040696,\ 19 \text{ rem.} \end{array}$$

REDUCTION.

MONEY, WEIGHTS, AND MEASURES.

REDUCTION is the name given to those arithmetical operations by which a quantity of one denomination is converted into another quantity of different denomination, but of the same value: the operation, for instance, by which pounds, in money, are converted into their equivalent in shillings, or pence, or farthings: those by which hundred-weights or tons are converted into pounds, ounces, &c.; or lengths, such as miles or furlongs, into yards, feet, and inches. This reducing of higher denominations into lower is sometimes called *Reduction descending;* while the contrary operations, for converting the lower denominations into higher, as pence into pounds, ounces into hundred-weights, feet into miles, &c., are called *Reduction ascending.*

In money calculations it is necessary that the computer should have what is called the *Pence Table* at his fingers' ends.

Pence Table.

			s.	d.				s.	d.				s.	d.
12	*pence*	are	1	0	60	*pence*	are	5	0	108	*pence*	are	9	0
20	,,		1	8	70	,,		5	10	110	,,		9	2
24	,,		2	0	72	,,		6	0	120	,,		10	0
30	,,		2	6	80	,,		6	8	130	,,		10	10
36	,,		3	0	84	,,		7	0	132	,,		11	0
40	,,		3	4	90	,,		7	6	140	,,		11	8
48	,,		4	0	96	,,		8	0	144	,,		12	0
50	,,		4	2	100	,,		8	4	150	,,		12	6

It should also be kept in remembrance that

960 *farthings* = 240 *pence* = *one pound*, or 20*s*.

Reduction of Money.

General Rule.—All higher denominations are reduced to lower by multiplication; and all lower to higher by division. The pounds, multiplied by 20, are reduced to shillings; the shillings, multiplied by 12, to pence; and these are reduced to farthings by multiplying them by 4. On the other hand, farthings are reduced to pence by dividing by 4; pence to shillings by dividing by 12; and shillings to pounds by dividing by 20. In these operations the final denomination sought is reached by passing regularly through all the intermediate denominations in succession, but in certain cases of frequent occurrence the final denomination may be arrived at at a single step, as in the examples, after the 7th, in the following collection.

Examples.

1. Reduce £247 to shillings.

```
 20
-----
4940s.
```

2. Reduce 468s. to pence.

```
 12
-----
5616d.
```

3. Reduce 273d. to farthings.

```
  4
-----
1092f.
```

4. Reduce 7656s. to pounds.

```
2,0) 765,6
     -----
  £382 16s.
```

EXAMPLES—*continued.*

5. In 89594*d.* how many shillings?

12) 89594

Ans. 7466*s.* 2*d.*

6. Reduce £55 19*s.* 7*d.* to pence.

20

1119
12

13435*d.*

7. Reduce £754 17*s.* 9¾*d.* to farthings

20

15097
12

181173
4

724695 *f.*

Thus far all the operations are those of the ordinary school-books. In the examples which follow, shorter methods of working are employed.

8. In £478 how many pence?
240 (No. of pence in £1)

Ans. 114720*d.* (See page 3.)

9. In £478 how many farthings?
960 (No. of farth. in £1.)

28680
4302

Ans. 458880*f.*

Or shorter thus:

478000
19120

458880*f.*

Here we have multiplied the 478 by 1000, which exceeds 960 by 40; and have then corrected the product by subtracting 40 times the 478.

10. In 114720*d.* how many pounds? (See page 6.)

240) 114720
72
89
11

Ans. £478

11. In 7376640*f.* how many pounds?

960) 7376640
664
508
683

Ans. £7684

NOTE.—In *general*, however, it is better to work up to the higher denomination through the lower denominations, conformably to the Rule above, since some or all of these may enter the final result, as in the converse of ex. 7.

REDUCTION OF WEIGHTS.

To reduce a weight in one denomination to its equivalent in another denomination, whether higher or lower, certain

Tables are necessary, which are different for different classes of commodities weighed: they will be inserted here under their respective heads.

TROY WEIGHT,—APOTHECARIES' WEIGHT.

24 Grains (gr.) make 1 Pennyweight = 24 grains.
20 Pennyweights (dwt.) 1 Ounce (oz.) = 480 grains.
12 Ounces (oz.) 1 Pound (lb.) = 5760 grains.

As used by Apothecaries, in compounding liquid medicines, the Troy ounce is divided into 8 *drams*, and the dram into 3 *scruples:* the weight of the dram is therefore 60 grains, and the weight of the scruple 20 grains.

The following examples sufficiently show how this Table is to be used, without any formal *Rule.*

EXAMPLES.

1. How many grains are there in 24 lb.?

```
 24 lb.   Or, (see page 3.)
 12            5760 gr. in 1 lb.
 ——              24
288 oz.       ———————
 20           138240 gr. in 24 lb.
 ———
5760 dwt.
  24
———————
138240 gr.
```

2. In 14 lb. 11 oz. 19 dwt. 16 gr., how many gr.?

Here the weight is 15 lb. all but 8 gr., so that $5760 \times 15 - 8 = 86400 - 8 = 86392$ gr.

3. How many spoons, each weighing 4 oz. 10 dwt., can be made out of 2 lb. 4 oz. 6 dwt. of silver?

As the weight of the silver is 28 oz. 6 dwt., it is plain that not more than six spoons can be made, the weight of which is 27 oz.: hence there are 1 oz. 6 dwt. to spare.

4. In 92517 gr. how many lb. troy?

```
  3|92517
  8|30839
2,0|385 ,4 ,7 × 3 = 21 gr.
 12|192 14 dwt.
    16 lb. 0 oz. 14 dwt. 21 gr.
```

Here we divide by 3 and by 8, instead of by 24, for convenience. The remainder 7, from the division by 8, is not seven *grains*, but 7 of the units in 30839, *each* of which is 3 grains, or three of the units in 92517, as is obvious: hence the 7 denotes 7×3 grains.

If we had divided by 8 first, and then by 3, we should have got from the first operation the remainder 5; these are of course 5 *grains:* the remainder from the second division would have been 2, each unit of *it* being *eight* grains: the complete remainder is therefore twice 8 grains and 5 grains, that is, 21 grains.

5. How many pounds troy are there in 138240 grains? *Ans.* 24 lb.
6. In 14 lb. 11 oz. 1 dwt. 16 gr., how many grains? *Ans.* 85960 gr.
7. In 75 lb. 11 oz. 19 dwt. 23 gr., how many grains? *Ans.* 437759 gr.
8. In 16 lb. 0 oz. 14 dwt. 21 gr., how many grains? *Ans.* 92517 gr.
9. How many pounds troy are there in 176360 grains? *Ans.* 30 lb. 7 oz. 8 dwt. 8 gr.
10. How many pounds troy are there in 85963 grains? *Ans.* 14 lb. 11 oz. 1 dwt. 19 gr.
11. How many grains are there in eight silver teapots, each weighing 3 lb. 9 oz. 18 dwt. 13 gr.? *Ans.* 17360 gr.
12. In 7 oz. 5 drs. 3 scr., how many scruples? *Ans.* 186 scr.
13. How many pounds are there in 4896 scruples? *Ans.* 17 lb.
14. A patient takes 2 drs. 2 scr. of bark daily: how long will 7 lb. last him? *Ans.* 252 days.
15. What weight of gold will be required to make twelve ornaments, each weighing 1 oz. 18 dwt. 12 gr. *Ans.* 23 oz. 2 dwt.

NOTE.—In this last example, we see that the weight of each ornament is 2 oz. less $1\frac{1}{2}$ dwt.; so that the weight of the twelve ornaments must be 24 oz. diminished by 12 times $1\frac{1}{2}$ dwt.; that is, by 18 dwt., hence the weight is 23 oz. 2 dwt., which is thus easily determined without putting pen to paper.

Gold and Silver Coins.

The coins, gold and silver, in present use in the United Kingdom, are the following.

GOLD COINS.

Sovereign,	in value	20*s.*,	in weight	5 dwt. 3·274 gr.
Half-sovereign,	,,	10*s.*,	,,	2 dwt. 13·637 gr.

The above decimals are respectively the 274 *thousandths*, and the 637 *thousandths* of a grain. The amount of *pure* gold in a sovereign is (or should be) 113 grains and one-thousandth of a grain; but the whole weight of a new sovereign, expressed in grains, is 123 grains and the 274 *thousandths* of a grain.

The coins are not entirely of pure gold, a metal which is too soft, and therefore too easily bruised and battered, to be well adapted for use, in its pure state, as money. It is sufficiently hardened by being mixed with an *alloy*, to the amount of one-twelfth ($\frac{1}{12}$) of the whole weight, of copper; so that the gold coin has only $\frac{11}{12}$ of its weight pure gold. A mass of this mixed metal is called Mint Gold or *Standard* Gold, and is said to be 22 *carats fine;* which means that if the

mass, of whatever weight, be divided into 24 equal parts, or *carats*, 22 of those parts only will express the weight of pure or fine gold, and the other two parts the weight of copper, in the mass; so that into every pound troy of Standard Gold there enters one ounce of alloy. The Mint price of this standard gold is £3 17*s.* $10\frac{1}{2}$*d.* per ounce, or £46 14*s.* 6*d.* per pound; a pound of it is therefore coined into 46 $\frac{29}{40}$* sovereigns, or 40lb. into 1869 sovereigns.

*** The legal standard for gold watch-cases is 18 carats fine. What is called the *Hall mark* (a crown and the figures 18), stamped by the authority of the Goldsmiths' Company on the case, is a warrant for this degree of purity, namely, that one-fourth part only is alloy.

Silver Coins.

Crown-piece,	in value 5*s.*,	in weight, 18 dwt.	4·3636 gr.	or 18 dwt.	$4\frac{4}{11}$ gr.
Half-crown,	,, 2*s.* 6*d.*	,, 9 ,,	2·1818	,, 9 ,,	$2\frac{2}{11}$,,
Florin,	,, 2*s.* 0*d.*	,, 7 ,,	6·5454	,, 7 ,,	$6\frac{6}{11}$,,
Shilling,	,, 12*d.*	,, 3 ,,	15·2727	,, 3 ,,	$15\frac{3}{11}$,,
Sixpence,	,, 6*d.*	,, 1 ,,	19·6363	,, 1 ,,	$19\frac{7}{11}$,,
Four-penny-piece	,, 4*d.*	,, 1 ,,	5·0909	,, 1 ,,	$5\frac{1}{11}$,,
Three-penny-piece	,, 3*d.*	,, 0 ,,	21·8181	,, 0 ,,	$21\frac{9}{11}$,,

In standard silver for coinage there are $\frac{37}{40}$ths of the whole of pure silver, and the remaining $\frac{3}{40}$ths of alloy; so that a pound of standard silver contains 11 oz. 2 dwt. of fine silver, and 18 dwt. of alloy; that is, 2 dwt. less than an ounce: it is coined into 66 shillings, its Mint price being at the rate of 66*d.*, or 5*s.* 6*d.* an ounce.

NOTE.—As already observed above, the word *carat*, when used in reference to the purity of the precious metals, denotes merely the twenty-fourth part of the entire mass; but the same term, when employed in reference to the weight of diamonds, stands for $3\frac{1}{5}$ grains. A diamond of the first *water*, that is, of the first quality, when without flaw and properly cut, is worth £8 if it weigh 1 carat; it is worth four times as much, or £32, if it weigh 2 carats; nine times as much, or £72, if it weigh three carats; sixteen times as much, or £128, if it weigh 4 carats; and so on, the worth being estimated at £8, multiplied by the *square* of the number of carats.

---

* 14*s.* 6*d.*, expressed as a fraction of £1, is $£\frac{14\frac{1}{2}}{20} = £\frac{29}{40}$.

By Act of Parliament "all articles sold by weight shall be by Avoirdupois Weight, except, gold, silver, platina, diamonds, and other precious stones, and drugs when sold by retail, and that such excepted articles, and none others, may be sold by troy weight."

Apothecaries, though always compounding their medicines by troy weight, yet buy and sell the ingredients by avoirdupois.

Avoirdupois Weight.

16 drams	make	1 ounce = 437½ grains.
16 ounces	,,	1 pound = 7000 ,,
14 pounds	,,	1 stone.
28 pounds	,,	1 quarter of a cwt. (hundred-weight.)
4 quarters	,,	1 cwt. = 112 lb. = 8 stone.
20 cwt.	,,	1 ton = 2240 lb.

Note.—1 pound Avoirdupois is equal to 14 oz. 11 dwt. 15½ gr. Troy; so that if a person were to get a *pound* of any commodity, weighed by the troy-pound-weight, he would get *less* than if the article were weighed by the avoirdupois-pound-weight: but if he were to receive an *ounce*, weighed by the troy-pound-weight, he would get *more* than if it were weighed by the avoirdupois-pound-weight; for an avoirdupois-ounce is only 437½ grains, whilst a troy-ounce is 480 grains, the *grain* being the same weight in both cases.

Formerly the stone varied in different parts of the kingdom, from 8 lb. to 16 lb.; but by an act of Parliament, passed in 1834, the legal stone was fixed at 14 lb.: nevertheless, butchers, in London and the suburbs, use a stone of 8lb. for meat.

Besides the above denominations used in avoirdupois weight, there are several others peculiar to the particular class of commodities weighed: thus the following denominations are employed for Wool.

Wool Weight.

	Av. lb.		Av. lb.
1 Clove = ½ a stone =	7	1 Sack = 2 Weys =	364
1 Tod = 2 stone =	28	1 Last = 12 sacks =	4368
1 Wey = 6½ tod =	108	= 39 cwt.	

And for Hay and Straw, the additional terms *truss* and *load* are used thus—

Hay and Straw.

	Av. lb.		cwt.	Av. lb.
1 Truss of Straw	= 36	1 Load of Old Hay	= 18	= 2016
1 ,, Old Hay	= 56	1 ,, New Hay	= 19 32 lb	= 2160
1 ,, New Hay	= 60	1 ,, Straw	= 11 64lb	= 1296

It will be seen from this Table that 36 Trusses go to a Load, whether they be of Straw or Hay, new or old; so that the term *load* here does not imply a fixed weight or number of pounds, but only a fixed number of trusses, namely, 36.

There are several other articles of merchandize, certain weights of which carry particular names: the following are the principal of these.

Miscellaneous Articles.

	Av. lb.		Av. lb.
A Firkin of Butter	= 56	A Puncheon of Prunes	= 1120
A ,, Soap	= 64	A Bushel of Flour	= 56
A barrel of ,,	= 256	A Sack of Flour	= 280
A ,, Anchovies	= 30	A Fother of Lead, 19 cwt. 2 qr.	= 2184

A Sack of Coal weighs 2 cwt., or 224 lb.; so that 10 sacks make a ton.

The following are compendious methods of reducing hundred-weights, quarters, and pounds, to pounds.

RULE I.—Multiply the cwts. by 12, adding in the overplus weight reduced to *pounds*. Write the result under the cwts., so that the place of hundreds may be directly under the place of units of the cwts., and then add. Or

RULE II.—Repeat the number of cwts. under itself; repeat again, this time pushing the figures one place in advance to the left; repeat still again, pushing the figures one place more to the left; then add the four rows up, taking in the odd *pounds* that are over and above the cwts.

The following examples will practically illustrate both these short and convenient rules.

EXAMPLES.

1. How many pounds are there in 123 cwt. 3 qr. 10 lb?

By the first rule, we have to multiply the 123 by 12, taking in the

3 qr. 10 lb. in *pounds*, namely, 94 lb., placing the result, 1570, as directed, thus—

	cwt.	lb.		cwt.	lb.		cwt.	lb.
By Rule I.	123	94	By Rule II.	123	94	or,	123	94
	1570			123			123	
Ans.	13870 lb.			123			1234	
				12394			1239	
			Ans.	13870 lb.		*Ans.*	13870 lb.	

This last operation differs from that immediately preceding it only as to the manner of disposing of the odd 94 lb. It is mere matter of taste which arrangement be adopted.

2. How many pounds are there in 26 cwt. 1 qr. 13 lb.?

	cwt.	lb.		cwt.	lb.		cwt.	lb.
By Rule I.	26	41	By Rule II.	26	41	or,	26	41
	353			26			26	
Ans.	2953 lb.			26			261	
				2641			264	
			Ans.	2953 lb.		*Ans.*	2953 lb.	

3. In 14 tons 17 cwt. 0 qr. 3 lb., how many pounds?

As 20 times 14 are 280, the weight in cwts. and lbs. is therefore 297 cwt. 3 lb.

	cwt.	lb.		cwt.	lb.
By Rule I.	297	3	By Rule II.	297	3
	3567			297	
Ans.	33267 lb.			2973	
				297	
			Ans.	33267 lb.	

The explanation of these rules is as follows: take the last example. Multiplying 297 by 112 is evidently the same as multiplying it first by 100, and then adding 12 times 297 to the result. But this result is 29700, and 12 times 297 is 3564; and these are the two numbers which, with the 3 lb., are added together above, conformably to the first rule.

Again. In multiplying 297 by 112, in the usual way, we take 297 *twice*, thus getting 594, under which 297 is afterwards twice written down, but the figures are pushed each time a place to the left, exactly as above.

The first of these rules involves multiplication by 12, the

second requires addition only. We shall add two examples of the reverse operation, the bringing of lbs. into cwts.

4. In 2953 lb. how many cwt.? [See page 6 for rule for Division.]

```
112) 2953
        1
        7
     ----
     26 cwt. 41 lb.
   = 26 cwt. 1 qr. 13 lb.
```

5. In 13870 lb. how many cwt.?

```
112) 13870
        63
        24
      ----
     123 cwt. 94 lb.
   = 123 cwt. 3 qr. 10 lb.
```

6. How many lbs. are there in 75 cwt. 3 qr. 14 lb.? *Ans.* 8498 lb.
7. How many lbs. are there in 976 cwt. 3 qr. 27 lb? *Ans.* 109423 lb.

This example will of course be computed for 977 cwt., and then 1 lb. deducted from the result. In like manner the weight in the preceding example may be regarded as 76 cwt., and 14 lb. be deducted afterwards.

8. In 3 cwt. 2 qr. 14 lb. of sugar, how many half-pound parcels are there? *Ans.* 812.
9. In 264 cwt. 3 qr. 12 lb. 11 oz. how many oz? *Ans.* 474635 oz.
10. In 249901 oz. how many cwt.? *Ans.* 139 cwt. 1 qr. 22 lb. 13 oz.
11. How many pounds are there in 24 bags of flour, each weighing 2 cwt. 2 qr. 13 lb.? *Ans.* 7032 lb.

Some articles are sold wholesale (by *tale*) by what is called the *Long Hundred;* that is, by the six score or 120, instead of by the five score or 100. The following is a general rule for converting Hundreds of the one kind into Hundreds of the other kind.

To reduce common hundreds to long hundreds, and the contrary.

Rule.—From the number of Common Hundreds subtract the *sixth* part of that number: the remainder will be the number of Long Hundreds.

To the number of Long Hundreds add the *fifth* part of that number: the sum will be the number of Common Hundreds: thus—

1.
```
6) 468 Common Hundreds.
    78
   ---
   390 Long Hundreds.
```

2.
```
5) 390 Long Hundreds.
    78
   ---
   468 Common Hundreds.
```

The reason is pretty obvious: there will be only $\frac{5}{6}$ as many Long as there are Common Hundreds, seeing that $\frac{5}{6}$ of 1 Long Hundred = 1 Common Hundred; and $\frac{5}{6}$ of any number is that number *minus* $\frac{1}{6}$ of it. Also there must be $1\frac{1}{5}$ as many Common Hundreds as there are Long Hundreds, since $1\frac{1}{5}$ of the former make but 1 of the latter; and $1\frac{1}{5}$ of any number is that number *plus* $\frac{1}{5}$ of it. The examples following will suffice for exercise in this rule.

3. How many Long Hundreds are there in 320 Common Hundreds? *Ans.* $266\frac{2}{3}$.
4. How many Common Hundreds are there in 256 Long Hundreds? *Ans.* $307\frac{1}{5}$.
5. How many Long Hundreds are there in 24000? *Ans.* 200.
6. How many Common Hundreds are there in 173 Long Hundreds? *Ans.* $138\frac{2}{5}$.

The answer to example 3 implies that there will be 266 Long Hundreds, and 80 individual articles besides; the answer to the 4th example shows that there will be 307 Common Hundreds, and 20 articles over; and the answer to example 6, shows that there will be 138 Common Hundreds, and 40 articles besides.

[A similar rule is sometimes given for converting what is assumed to be *Long weight* (120 lb. = 1 cwt.) into Common weight (112 lb. = 1 cwt.), but it is fallacious: it gives, for instance,—

347 cwt. 1 qr. 16 lb. Common weight = 324 cwt. 0 qr. $26\frac{2}{15}$ lb. Long wt.; and 176 cwt. 3 qr. 19 lb. Long wt. = 189 cwt. 2 qr. $6\frac{5}{14}$ lb. Com. wt.; results which are obviously erroneous, since the weight on the left of the sign = is in each case an exact number of lbs., without any fraction of a lb. Such a rule, however, though accurately given, would be useless.]

Reduction of Measures.

Measures, like weights, are reduced to equivalent measures, having other denominations, by the aid of one or other of the following tables.

I. Measures of Length, or Long Measure.

The principal measures of length used in this kingdom are these :—

12	inches	make	1 foot.
3	feet	,,	1 yard.
$5\frac{1}{2}$	yards	,,	1 rod, pole, or perch.
40	poles	,,	1 furlong = 220 yards.
8	furlongs	,,	1 mile = 1760 yards = 5280 feet

The old Scotch and Irish miles are respectively $1\frac{1}{8}$ and $1\frac{3}{11}$ English, so that 8 Scotch miles are equal to 9 English, and 11 Irish to 14 English.

Surveyors measure land by the *chain*, consisting of 100 links. The length of the chain is 4 poles, or 22 yards* = 66 feet, so that the length of a single link is the 100th part of 66 feet, that is, $7\frac{92}{100} = 7{\cdot}92$ inches. Distances both at land and sea are sometimes measured in *leagues*, the league being a length of 3 miles. It should be mentioned, however, that the land mile and league are not the same as the *nautical* mile and league; the nautical mile (the 60th part of a degree of the equator) exceeds the land mile: it is 6086 feet; and the nautical league is three of *these* miles. Sea-depths or *soundings* are measured in *fathoms*, the fathom being 6 feet.

For the height of horses, the unit of measure is the *hand* = 4 inches; so that a horse fifteen hands high is 5 feet in height.

Drapers and mercers use the measures *ell*, *yard*, *inch*, and *nail*, but not the *foot*. The measures are as follow :—

Cloth Measure.

$2\frac{1}{4}$	inches	make	1 nail.
4	nails	,,	1 quarter (of yd.)
4	quarters	,,	1 yard.
5	quarters	,,	1 English ell, and 6 qrs. 1 French ell.

Note. In early times the inch was estimated as the length furnished by putting together three grains of barley, end to end; and that "Three Barley-corns make one Inch," is a statement still retained in many tables of linear measure.

---

* This length is chosen for the surveyor's chain, for convenience in the calculation of acres of surface : an acre being 4840 square yards, it is equal to 10 times a square chain, that is, to 10 square chains.

To reduce miles to yards, and the contrary.

Rule I.—Multiply 1760 by the number of miles; but if this number consist of three or more figures, it will usually be found more convenient to work by one or other of the two rules following.

Rule II.—Multiply the number of miles by 44, then the result by 4, and annex a 0 to the final product. Or:

Rule III.—Multiply the number of miles by 16 *in one line*, repeat this line of figures underneath, commencing one place back to the left, and then add up the two rows of figures, annexing 0 to the result.

By either of these rules the miles will be reduced to yards. To convert yards into miles we must divide by 1760.

Examples.

1. How many yards are there in 374 miles?

By Rule I.	By Rule II.	By Rule III.
374	374	374
1760	44	16
22440	1496	5984
6358	1496	5984
Ans. 658240 yds.	16456	*Ans.* 658240 yds.
	4*	
	Ans. 658240 yds.	

2. How many yards are there in 2683 miles?

By Rule I.	By Rule II.	By Rule III.
2683	2683	2683
1760	44	16
160980	10732	42928
45611	10732	42928
Ans. 4722080 yds.	118052	*Ans.* 4722080 yds.
	4*	
	Ans. 4722080 yds.	

The operation by Rule I. requires no explanation: the

* This multiplier may of course be used first, instead of last; that is, we may multiply four times the number of miles by 44.

multiplication by the 17 is performed in one line. According to Rule II., the multiplication is by 44, 4, and 10; and $44 \times 4 \times 10 = 1760$. By Rule III., the number 176 is split into the two numbers 16 and 160: the first row of figures in the work arises from multiplying by the 16, and the second row being 10 times the first, arises from the multiplication by the 160: the sum of the two rows is therefore the product arising from the multiplier 176, and 0 annexed to this gives the product resulting from the multiplier 1760.

Although in multiplying by so small a number as 1760, the trouble and risk of error by the ordinary method are but little, yet by either of the methods II., III., both are still less. It may be worthy of notice too that the three 4s, employed in the second rule, may serve to recall the number of yards in a mile, should it escape the memory. The third mode of proceeding is always applicable to any multiplier, of three significant figures, in which the first figure is unit, and the third a unit less than the second; as, for instance, to the multipliers 132, 143, 154, 165, &c. We have only to take the first and third figures as one number, and to proceed as above, annexing 0s to the result only when 0s are annexed to the multiplier. Thus, taking the multipliers 143, 187, and the mutiplicand 4681572, we work as follows.

$$\begin{array}{r} 4681572 \\ 13 \\ \hline 60860436 \\ 60860436 \\ \hline 669464796 \\ \hline \end{array} \qquad \begin{array}{r} 4681572 \\ 17 \\ \hline 79586724 \\ 79586724 \\ \hline 875453964 \\ \hline \end{array}$$

*** The multipliers 143 and 187 are here virtually employed in the forms

$$\left\{ \begin{array}{r} 13 \\ 13 \end{array} \right\} \quad \text{and} \quad \left\{ \begin{array}{r} 17 \\ 17 \end{array} \right\}$$

If ciphers had been annexed to either of the multipliers, we should of course have annexed as many to the product.

It may be added, that however numerous the figures of any multiplier may be, yet that to every set of three, coming under the above conditions, the method here proposed may be applied. A similar principle may be employed whenever the

second of the three figures is double of the first; thus the multipliers 245, 367, 483, may be replaced by

$$\begin{Bmatrix} 25 \\ 22 \end{Bmatrix}, \begin{Bmatrix} 37 \\ 33 \end{Bmatrix}, \begin{Bmatrix} 43 \\ 44 \end{Bmatrix}$$

3. How many miles are there in 658247 yards, and in 4722084 yards, respectively? [See Rule for Division, p. 6.]

```
176,0) 65824,7          176,0) 472208,4
        00 0                     062 0
        37 0                     245 0
        1                        11
        ——                       ——
Ans. 374 miles 7 yds.   Ans. 2683 miles 4 yds.
```

4. How many yards are there in 3057 miles? *Ans.* 5380320 yds.
5. How many miles are there in 1793440 yards? *Ans.* 1019 miles.
6. How many feet are there in the earth's diameter at the equator, its measure in miles being 7925·648 miles? *Ans.* 41847421·44 feet.

In this example we are required to reduce miles to feet. By the usual method of operating, we should multiply the number of miles by 5280, the number of feet in a mile. But the trouble will be somewhat lessened by employing the principle of Rule III. (Page 20.) The number $528 = 44 \times 3 \times 4$; and $44 \times 3 = 132$; so that, by the principle referred to, we may use the multipliers $\begin{Bmatrix} 12 \\ 12 \end{Bmatrix}$ and 4: we shall here exhibit the operation in juxtaposition with that by the common rule.

```
   7925·648                   7925·648
       5280                         12
  ─────────                  ─────────
  634051840                   95107776
 15851296                    95107776
39628240                     ──────────
 ─────────                  1046185536
Ans. 41847421·440 feet               4
 ─────────                  ──────────
                       Ans. 41847421·440 feet.
```

As the terminating figures here are *decimals*, the final cipher is of course superfluous. It is inserted merely that in the second method of working the principle explained above may be strictly conformed to.

There is the same amount of figure-work in the second as there is in the first of these operations; but there is unquestionably some saving of head-work in the second mode of proceeding, and it is *this* kind of saving which is the main object of consideration in forming an estimate of a *short method.* [For an explanation of *Decimals* see APPENDIX.]

II. SUPERFICIAL OR SQUARE MEASURE.

Table I., at page 19, is applied exclusively to the measure of *lengths:* this table is applied exclusively to the measure of *surfaces.*

144 square inches make 1 square foot.
9 square feet make 1 square yard.
100 square feet make 1 square of flooring.
272¼ square feet (30¼ sq. yds.) make 1 square rod, pole, or perch.
40 square rods, or poles, make 1 rood = 1210 sq. yds. = 2½ sq. chains.
4 roods (4840 sq. yds.) make 1 acre = 10 square chains.
640 acres make 1 square mile = 6400 sq. chains.

Hence a piece of land which is equivalent in surface to a square each side of which is 10 chains, measures an acre; and a piece equivalent in surface to a square of which each side is 80 chains, measures a square mile, since $80 \times 80 = 6400$.

III. CUBIC OR SOLID MEASURE.

This measure is used whenever three dimensions, length, breadth, and thickness (or depth) are all taken into consideration. A cube is a figure in the form of the gambler's die, the three dimensions being all equal. When each dimension is an inch, the matter or space is a cubic inch of that matter or space; when each dimension is a foot, it is a cubic foot, and so on.

1728 cubic inches	make	1 cubic foot.
27 cubic feet	,,	1 cubic yard.

A cubic yard of earth is reckoned to be 1 *Load.*

⁂ These tables will come into application hereafter, in the calculations of artificers' work.

MEASURES OF CAPACITY.

I. Dry Measure.

(Chiefly for corn, meal, flour, peas, beans, &c.)

2 pints	make	1 quart.	4 bush.	make	1 coomb.
4 quarts	,,	1 gallon.	2 coombs	,,	1 quar. = 8 bush.
2 gallons	,,	1 peck.	5 quarters	,,	1 wey or load.
4 pecks	,,	1 bushel.	2 weys	,,	1 last, or 10 qrs.
2 bushels	,,	1 strike.	A pottle is half a gallon.		

*** The gallon measure contains 277·274 cubic inches.

II. Wine and Spirit Measure.

4 gills (or qtns)	make	1 pint.	54 gallons	make	1 hogshead.
2 pints	,,	1 quart.	2 hogsheads	,,	1 pipe or butt.
4 quarts	,,	1 gallon.	4 hhds. (2 pipes)		1 tun.

Note.—The last three terms in this table—the terms, Hogshead, Pipe, and Tun, are names which are used more to designate the *kind* of casks than to denote definite measures of capacity. The measure of a Pipe, namely, 108 gallons, given above, applies exclusively to Sherry wine; for Port, Claret, Madeira, and other wines, the number of gallons to the pipe is different. But it is the practice to *gauge* the casks which bear the before-mentioned names, and thus to ascertain the number of gallons contained in them by direct measurement.

III. Ale and Beer Measure.

2 pints	make	1 quart.
4 quarts	,,	1 gallon.
9 gallons	,,	1 firkin.
2 firkins (18 gal.)	,,	1 kilderkin.
2 kilderkins	,,	1 barrel = 36 gallons.
3 kilderkins	,,	1 hogshead = 54 gallons.
2 hogsheads	,,	1 butt = 108 gallons.
2 butts	,,	1 tun = 216 gallons.

Note.—Till the year 1826, the gallon was a measure of varying capacity, its cubic contents being different for different dry or liquid commodities measured by it: the wine gallon, the ale and beer gallon, and the corn gallon, were all of different cubic contents. The imperial gallon is now the only *legal* gallon, and it alone is to be universally employed throughout the British dominions as the measure bearing that name, without any distinction as to the articles measured. It is enacted that the gallon shall contain 277·274 cubic inches, and as this number of inches is the cubic measure of 10 lbs. avoirdupois of distilled

water (when the temperature of the air is at 62° Fahrenheit, and the barometer stands at 30 in.), we learn that

"A pint of pure water
Weighs a pound and a quarter."

The weight, in grains, of a gallon of pure water, at the above temperature and pressure, is 70000 grains; the avoirdupois pound weighing 7000 grains, as stated at page 14.

IV. Divisions of Time.

60 seconds	make	1 minute
60 minutes	,,	1 hour.
24 hours	,,	1 day.
28, 29, 30, or 31 days	,,	1 calendar month.
12 calendar months	,,	1 year.
365 days	,,	1 common year.
365¼ days	,,	1 Julian year.
365 days 5 hours 48 minutes 48 sec.	=	1 Solar Year

The solar year is the time in which the earth makes one revolution in its orbit round the sun, or the time which elapses between the departure of the sun from the vernal equinox (or where its path crosses the equinoctial) till its return to the vernal equinox again. As the equinoctial points shift a little, it is the interval of time, on the average—the Mean Solar Year, which 365 days 5 hrs. 48 min. 48 sec. measures. This period of time is only 11 min. 12 sec., or 11⅕ min., short of 365¼ days; and since, from neglecting the fraction of a day beyond the 365 days, the Roman calendar retrograded more and more from the true period of the year which it nominally indicated, Julius Cæsar caused it to be readjusted, and enacted that every fourth year afterwards should be a year of 366 days; hence our *bissextile*, or *leap-year*, in which an additional day is added to the month of February every fourth year, the other years counting as 365 days only. But as the *Julian Year* is 11 min. 12 sec. in excess, the error would be *one day* in excess in 129 years, another adjustment was, therefore, afterwards seen to be necessary; and this important "*Reformation of the Calendar*," as it is called, was accomplished by direction of the enlightened Pontiff Gregory XIII., in the year 1582. As the error, in excess, of the Julian reckoning amounted to about three days in 400 years, it was enacted that the additional

day added to February, in ordinary leap-years, should be omitted in the years which *completed centuries*, unless when these centenary years were multiples of the number 400. The error by this adjustment becomes so small that it amounts to only a day in about 3600 years. To compensate for the accumulated errors of the Julian reckoning, the Reformed Calendar commenced with the suppression of ten of the days of the Julian Calendar, it being ordered that the 5th of October in that year (1582) should be called the 15th.

The annexed table, showing the number of days from one date to another, will occasionally be found useful; it will be sufficiently understood from an example or two of its application.

Examples.

1. Required the number of days from the 9th of May to the 17th of September in the same year. Opposite May, on the right, will be found 120, the number of days from the 1st day of January, inclusive, to the last day of April, inclusive; hence, adding the 9 days in May, we have 129 days, including the 9th of May itself. Again, opposite September, on the right, is found 243, the number of days from the 1st of January, inclusive, to the last day of August, inclusive. Adding therefore the 17 days in September, we have 260 days from January 1 to September 17, both inclusive; then subtracting the 129 from the 260, there remains 131, the number of days which follow the first of the given dates up to the second, including that second date.

Days.		Months.	Days.	
31	334	January .	00	31
59	306	February	31	28
90	275	March . .	59	31
120	245	April . .	90	30
151	214	May. . .	120	31
181	184	June . .	151	30
212	153	July. . .	181	31
243	122	August .	212	31
273	92	September	243	30
304	61	October .	273	31
334	31	November	304	30
365	00	December	334	31

Of course, if the end of a leap-year February be included in the interval, another day must be added.

2. How many days are there from the 5th of November, 1868, to the 16th of May (inclusive), 1869?

Here some of the days are in one year, and the remaining days in the next year; and we proceed thus:—

Subducting the 5 days of November, already elapsed, from the 30 days of November, there remain 25 days: add these to the 31 days, opposite November, on the left; to the sum (56) add the 120 opposite May, thus bringing the days to the end of April; then taking account of the 16 days *following* the end of April, we have 192 for the total number of days from the first given date to the second, including that second date.

As before, if the end of a leap-year February occur in the interval, another day must be added.

It will be observed that in the column immediately *following* the column of MONTHS, the number against any month expresses the number of days of the year which have elapsed *before* the 1st of that month. And in the column immediately *preceding* the column of MONTHS, the number against any month expresses the number of days requisite, *after* the last day of that month, to complete the year. The first column in the table merely gives the number of days in the first month of the year, in the first *two* months, the first *three* months, and so on, up to the whole twelve months; while the last column expresses the number of days in each individual month.

[For the purpose of calling these days to mind nothing can be more suitable than the well-known doggerel:—

Thirty days hath September,
April, June, and dull November,
February hath twenty-eight alone,
And all the others thirty-one,
Except leap-year, and that's the sign
That February has twenty-nine.

or

Leap-year coming once in four,
February has one day more.]

3. How many days are there from July 18 to December 27, inclusive, in the same year? *Ans.* 162 days.
4. How many days are there from November 17, 1868, to September 12, 1869? *Ans.* 299 days.
5. How many days are there from November 17, 1867, to September 12, 1868? *Ans.* 300 days.

DIVISIONS OF THE CIRCLE.

For certain purposes of calculation, where arcs of circles are concerned, it has been found convenient to regard the entire circumference of a circle (of whatever magnitude) to be divided into 360 equal parts, and to call each of these parts a *degree;* the degree is further subdivided into 60 equal parts called *minutes*, and each of these again into 60 equal parts called *seconds;* a portion of arc still smaller than a second is expressed as a fraction or decimal of a second. An arc of, say, 23 degrees 47 minutes 13 seconds, is denoted thus, $23^\circ\ 47'\ 13''$, and similarly in other cases. An arc of 90° is called a *quadrant.* It may be well to mention that,

from this mode of measurement, no estimate can be formed of the actual *length* of arc, unless that of the entire circumference to which it belongs be stated, since the term *degree* does not imply any definite length, any more than the word *circumference* does. As the earth performs a complete rotation in 24 hours, the equator, and every circle parallel to it (every parallel of latitude), turns through a 24th part of the entire circumference, or 360°, every hour; that is, it rotates at the rate of 15° an hour. We can thus readily find the *time* corresponding to any given arc of the earth's rotation, or the difference of time, at the same instant, between two places whose difference of longitude is given. The rule for doing this is as follows:—

To find the time corresponding to an assigned number of degrees, minutes, and seconds of longitude.

RULE.—1. Divide the number of degrees by 15; the quotient is the number of hours.

2. Multiply the remainder (if there be a remainder) by 4; the product is the number of minutes of time.

3. Divide the minutes of arc by 15; the quotient is the number of minutes of time.

4. Multiply the remainder, if any, by 4; the product is seconds of time.

5. Divide the seconds of arc by 15; the quotient is seconds of time. The sum of these results will be the hours, minutes, and seconds corresponding to the given arc.

EXAMPLES.

1. Required the time corresponding to 108° 24′ 22″.

			h.	m.	s.
Time for	108°	. . . .	7	12	0
,,	24′	. . . .		1	36
,,	22″	. . . .			1·47
Time for 108° 24′ 22″		. . . .	7	13	37·47

The decimal ·47 is written for the interminable decimal ·466 . . . .

That the foregoing operations must lead to the correct result is pretty obvious. The complete quotient of 108 divided by 15 is $7\frac{3}{15}$; so that $7\frac{3}{15}$ hours correspond to 108 degrees; but the fraction of an hour is converted into

minutes by multiplying it by 60; and 60 times 3 divided by 15 is 4 times 3. In like manner for the 24′; $24 \div 15 = 1\frac{9}{15}$; and $\frac{9}{15} \times 60 = 4$ times 9. As to the seconds of arc, $22 \div 15 = 1\frac{7}{15}$; and by proceeding in the same way with *this* fraction, we should get from it 4 times 7 *thirds;* but as *thirds* of time are not used, the fraction is expressed in decimals of *seconds*, namely, $\frac{7}{15}$ seconds = ·4666 . . . seconds.

2. Required the time corresponding to 84° 42′ 30″. *Ans.* 5 h. 38 m. 50 s.
3. Required the time corresponding to 93° 47′ 41″.
   Ans. 6 h. 14 m. 30·72 s.
4. Required the time corresponding to 230° 32′ 10″.
   Ans. 15 h. 22 m. 8·7 s.

To find the angular measure corresponding to an assigned time.

This problem is the converse of the preceding one: the rule is this:—

RULE.—1. Multiply the number of hours by 15; the product is so many degrees.

2. Divide the minutes and seconds of time by 4, and reckon every unit of remainder as 15′, if minutes be the dividend, and as 15″, if seconds be the dividend.

EXAMPLES.

1. What angular measure (or arc) corresponds to 3h. 14m. 23s.?

For 3 h. . . .	45°		
,, 14 m. . . .	3	30′	
,, 23 s. . . .		5	45″
Angular Measure	48°	35′	45″

2. Find the angular measure for 2 h. 18 m. 58·26 s.

For 2 h. . . .	30°		
,, 18 m. . . .	4	30′	
,, 58·26 s. . . .		14	33·9″
Angular Measure	34°	44′	33·9″

The remainder from the 58·26 seconds is 2·26; and 15 times this is 33·9.

3. Find the arc corresponding to 5 h. 19 m. 37 s. *Ans.* 79° 54′ 15″.
4. What angular measure corresponds to 2 h. 18 m. 58·27 s?
   Ans. 34° 44′ 34″.
5. Required the degrees, minutes, and seconds, corresponding to 7 h. 13 m. 37·4666 . . . s. *Ans.* 108° 24′ 22″.

COMPOUND MULTIPLICATION,

OR MULTIPLICATION OF QUANTITIES MADE UP OF DIFFERENT DENOMINATIONS.

WE commence the practical application of the particulars tabulated in the last few pages with COMPOUND MULTIPLICATION, because the easy operations of addition and subtraction, as taught in every schoolboy course of arithmetic, admit of no simplifications or abridgments; and because, moreover, these operations will actually be involved in the working of most of the examples which follow. Previously, however, to entering upon these, we shall give a table of the factors of *Composite Numbers*, as far, at least, as the number 10000.* A composite number is one that admits of decomposition into integral factors; that is, it is a number that may be produced by the multiplication together of *whole numbers* only. When the multiplier of a quantity made up of several denominations is a composite number of two or three figures, it is, in general, much more convenient to multiply by the factors of it, one after another, than to multiply by the higher number itself; and the Table will point out what these lower multipliers, in each case coming within its limits, are. And it will hereafter be seen that, even in the case of a non-composite, or *prime* number, as it is called, a reference to the table will always suggest a saving of trouble, and diminish the risk of error in the work.

A TABLE of those Factors of the Composite Numbers from 75 to 10000, which fall within the Limits of the Multiplication Table.

No.	Factors.	No.	Factors.	No.	Factors.
75	5 5 3	128	8 8 2	168	8 7 3
98	7 7 2	135	9 5 3	175	7 5 5
105	7 5 3	147	7 7 3	176	11 8 2
112	8 7 2	154	11 7 2	189	9 7 3
125	5 5 5	162	9 9 2	192	12 8 2
126	9 7 2	165	11 5 3	196	7 7 4

* Those composite numbers, the factors of which are given by the multiplication table itself, it would be superfluous to include in *this* table: they are therefore omitted.

TABLE, &c.—*continued.*

No.	Factors.	No.	Factors.	No.	Factors.
198	11 9 2	616	11 8 7	1375	11 5 5 5
216	12 9 2	625	5 5 5 5	1386	11 9 7 2
224	8 7 4	648	9 9 8	1408	11 8 8 2
225	9 5 5	672	12 8 7	1452	12 11 11
231	11 7 3	675	9 5 5 3	1458	9 9 9 2
242	11 11 2	686	7 7 7 2	1485	11 9 5 3
243	9 9 3	693	11 9 7	1512	9 8 7 3
245	7 7 5	704	11 8 8	1536	8 8 8 3
252	12 7 3	726	11 11 6	1568	8 7 7 4
256	8 8 4	729	9 9 9	1575	9 7 5 5
264	11 6 4	735	7 7 5 3	1584	12 12 11
275	11 5 5	756	12 9 7	1617	11 7 7 3
288	12 12 2	768	12 8 8	1694	11 11 7 2
294	7 7 6	784	8 7 7 2	1701	9 9 7 3
297	11 9 3	792	12 11 6	1715	7 7 7 5
308	11 7 4	825	11 5 5 3	1728	12 12 12
315	9 7 5	847	11 11 7	1764	9 7 7 4
324	9 9 4	864	12 9 8	1782	11 9 9 2
336	12 7 4	875	7 5 5 5	1792	8 8 7 4
343	7 7 7	882	9 7 7 2	1815	11 11 5 3
352	11 8 4	891	11 9 9	1848	11 8 7 3
363	11 11 3	896	8 8 7 2	1875	5 5 5 5 3
375	5 5 5 3	924	12 11 7	1925	11 7 5 5
378	9 7 6	945	9 7 5 3	1936	11 11 4 4
384	8 8 6	968	11 11 8	1944	9 9 8 3
385	11 7 5	972	12 9 9	2016	9 8 7 4
392	8 7 7	1008	12 12 7	2025	9 9 5 5
396	11 9 4	1024	8 8 8 2	2048	8 8 8 4
405	9 9 5	1029	7 7 7 3	2058	7 7 7 6
432	12 9 4	1056	12 11 8	2079	11 9 7 3
441	9 7 7	1078	11 7 7 2	2112	11 8 8 3
448	8 8 7	1089	11 11 9	2156	11 7 7 4
462	11 7 6	1125	9 5 5 5	2178	11 11 9 2
484	11 11 4	1134	9 9 7 2	2187	9 9 9 3
486	9 9 6	1152	12 12 8	2205	9 7 7 5
495	11 9 5	1155	11 7 5 3	2268	9 9 7 4
504	9 8 7	1176	8 7 7 3	2304	9 8 8 4
512	8 8 8	1188	12 11 9	2352	8 7 7 6
525	7 5 5 3	1215	9 9 5 3	2376	11 9 8 3
528	12 11 4	1225	7 7 5 5	2401	7 7 7 7
539	11 7 7	1232	11 8 7 2	2464	11 8 7 4
567	9 9 7	1296	12 12 9	2475	11 9 5 5
576	12 12 4	1323	9 7 7 3	2541	11 11 7 3
588	12 7 7	1331	11 11 11	2592	9 9 8 4
594	11 9 6	1344	8 8 7 3	2625	7 5 5 5 3
605	11 11 5	1372	7 7 7 4	2646	9 7 7 6

TABLE, &c.—*continued.*

No.	Factors.	No.	Factors.	No.	Factors.
2662	11 11 11 2	4356	11 11 9 4	6804	12 9 9 7
2673	11 9 9 3	4374	9 9 9 6	6860	10 7 7 7 2
2688	8 8 7 6	4375	7 5 5 5 5	6875	11 5 5 5 5
2695	11 7 7 5	4455	11 9 9 5	6912	12 9 8 8
2744	8 7 7 7	4536	9 9 8 7	7056	9 8 7 7 2
2772	11 9 7 4	4608	9 8 8 8	7128	11 9 9 8
2816	11 8 8 4	4704	12 8 7 7	7168	8 8 8 7 2
2835	9 9 7 5	4725	9 7 5 5 3	7203	7 7 7 7 3
2904	11 11 8 3	4752	11 9 8 6	7392	12 11 8 7
2916	9 9 9 4	4802	7 7 7 7 2	7425	11 9 5 5 3
3024	9 8 7 6	4851	11 9 7 7	7546	11 7 7 7 2
3025	11 11 5 5	4928	11 8 8 7	7560	12 10 9 7
3072	8 8 8 6	5082	11 11 7 6	7623	11 11 9 7
3087	9 7 7 7	5103	9 9 9 7	7744	11 11 8 8
3125	5 5 5 5 5	5145	7 7 7 5 3	7776	12 9 9 8
3136	8 8 7 7	5184	9 9 8 8	7875	9 7 5 5 5
3168	11 9 8 4	5292	12 9 7 7	7938	9 9 7 7 2
3234	11 7 7 6	5324	11 11 11 4	7986	11 11 11 6
3267	11 11 9 3	5346	11 9 9 6	8019	11 9 9 9
3375	9 5 5 5 3	5376	12 8 8 7	8064	9 8 8 7 2
3388	11 11 7 4	5445	11 11 9 5	8085	11 7 7 5 3
3402	9 9 7 6	5488	8 7 7 7 2	8192	8 8 8 4 4
3456	9 8 8 6	5544	11 9 8 7	8232	8 7 7 7 3
3465	11 9 7 5	5625	9 5 5 5 5	8316	12 11 9 7
3528	9 8 7 7	5632	11 8 8 8	8448	12 11 8 8
3564	11 9 9 4	5775	11 7 5 5 3	8505	9 9 7 5 3
3584	8 8 8 7	5808	11 11 8 6	8575	7 7 7 5 5
3645	9 9 9 5	5832	9 9 9 8	8624	11 8 7 7 2
3675	7 7 5 5 3	5929	11 11 7 7	8712	11 11 9 8
3696	11 8 7 6	6048	12 9 8 7	8748	12 9 9 9
3773	11 7 7 7	6075	9 9 5 5 3	9072	9 9 8 7 2
3872	11 11 8 4	6125	7 7 5 5 5	9075	11 11 5 5 3
3888	9 9 8 6	6144	12 8 8 8	9216	9 8 8 4 4
3969	9 9 7 7	6174	9 7 7 7 2	9261	9 7 7 7 3
3993	11 11 11 3	6237	11 9 9 7	9317	11 11 11 7
4032	9 8 8 7	6272	8 8 7 7 2	9375	5 5 5 5 5 3
4096	8 8 8 8	6336	11 9 8 8	9408	8 8 7 7 3
4116	12 7 7 7	6468	12 11 7 7	9504	12 11 9 8
4125	11 5 5 5 3	6534	11 11 9 6	9604	7 7 7 7 4
4158	11 9 7 6	6561	9 9 9 9	9625	11 7 5 5 5
4224	11 8 8 6	6615	9 7 7 5 3	9702	11 9 7 7 2
4235	11 11 7 5	6655	11 11 11 5	9801	11 11 9 9
4312	11 8 7 7	6776	11 11 8 7	9856	11 8 7 4 4

Case I.

When the multiplier is a composite number, in which no factor greater than 12 necessarily enters.

Rule.—Multiply the quantity by one of the *factors* (none of them exceeding 12) of the multiplier, the product by another, this result by another, and so on, till all the factors have been used.

Examples.

1. 72 cwt. at 6*s*. 7¾*d*. per cwt.
   72 = 12 × 6.

	s.	d.
	6	7¾
		12
	79	9
		6
2,0)	47,8	6
Ans.	£23 18*s*.	6*d*.

Since 12 times any number of *pence* is obviously that number of *shillings*, 12 times 7¾ pence is 7¾ shillings; that is to say, 7*s*. 9*d*.; hence the result of the first multiplication is 79*s*. 9*d*.

Again, 6 times 9*d*. are 9 sixpences = 4*s*. 6*d*.: hence the final result is 478*s*. 6*d*., or £23 18*s*. 6*d*.

2. 96 yards at 1*s*. 10¾*d*. per yard.
   96 = 12 × 8.

	s.	d.
	1	10¾
		12
	22	9
		8
2,0)	18,2	0
Ans.	£9 2*s*.	0*d*.

3. 42 cwt. at £4 10*s*. 7*d*. per cwt.
   42 = 7 × 6

£	s.	d.
4	10	7
		7
31	14	1
		6
Ans. £190	4*s*.	6*d*.

4. Multiply 8 lb. 5 oz. 17 dwt. 4 gr. troy by 28 = 7 × 4.

lb.	oz.	dwt.	gr.
8	5	17	4
			7
59	5	0	4
			4
Ans. 237 lb.	8 oz.	0 dwt.	16 gr.

5. Multiply 17 lb. 7 oz. 9 dr. avoirdupois, by 168.

By a reference to the foregoing table, we find that $168 = 8 \times 7 \times 3$: we may therefore multiply by these factors in succession.

lb.	oz.	dr.
17	7	9
		8
139	12	8
		7
978	7	8
		3

Ans. 2935 lb. 6 oz. 8 dr. = 2935 lb. $6\frac{1}{2}$ oz.

6. Multiply £19 13*s.* $5\frac{1}{4}$*d.* by 28. *Ans.* £550 16*s.* 3*d.*
7. Required the price of 16 cwt. of tallow at £1 18*s.* 8*d.* per cwt. *Ans.* £30 18*s.* 8*d.*
8. At 1*s.* $10\frac{3}{4}$*d.* per lb., what will 96 lb. cost? *Ans.* £9 2*s.*
9. Multiply 23 miles 1 furlong 31 perches 2 yards by 256. *Ans.* 6025 m. 6 fur. 4 per. 3 yd.
10. Multiply 7*s.* $10\frac{3}{4}$*d.*, by 7986. *Ans.* £3152 16*s.* $1\frac{1}{2}$*d.*

Case 2.

When the multiplier is not a composite number; or, being a composite number, when any of its factors are inconveniently large.

[The number 69, for instance, is a composite number, since it has integral factors, namely, 23 and 3; but 23 is a multiplier which is inconvenient for *compound* multiplication. The present case includes all numbers in each of which an integral factor higher than the number 12 *necessarily* enters; as also, of course, every number which has no integral factors at all, except the number itself and unit, that is, every prime number.]

Rule.—1. Refer to the table for the composite number which is the nearest in value to the given multiplier, and use, as in last case, the factors of it.

2. Multiply the quantity by the difference between the given number and that taken from the table; and if the tabular number be *less* than the given one, *add* the product; if it be *greater*, subtract the product.

EXAMPLES.

1. 79 yards at 7*s*. 10*d*. per yard.
Here the nearest composite number to 79 is $75 = 5 \times 5 \times 3$, so that we work thus:

	s.	*d.*	
	7	10 × 4	
		5	
1	19	2	
		5	
9	15	10	
		3	
29	7	6	
Add 1	11	4	for the 4 yds.
Ans. £30	18*s.*	10*d.*	

2. Multiply 13 acres 3 roods 17 poles, by 511.
The nearest composite number is $512 = 8 \times 8 \times 8$.

acres.	roods.	poles.
13	3	17 × 1
		8
110	3	16
		8
886	3	8
		8
7094	1	24
Sub. 13	3	17
Ans. 7080 ac.	2 rds.	7 po.

3. What is the price of 114 stone of meat at 15*s*. $3\frac{3}{4}$*d*. per stone? *Ans.* £87 5*s*. $7\frac{1}{2}$*d*.
4. What sum of money must be divided among 108 men, so that each may receive £14 6*s*. $8\frac{1}{3}$*d*.? *Ans.* £1548 4*s*. 6*d*.
5. Multiply 2 sq. yds. 8 ft. 123 in. by 563. *Ans.* 1679 sq. yds. 7 ft. 129 in.

When the given multiplier has $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ connected with it, disregard the fraction, and proceed as above; then for $\frac{1}{4}$ add a fourth part of the quantity multiplied to the result; for $\frac{1}{2}$ add half that quantity; and for $\frac{3}{4}$ add half and the half of that half; or if the multiplier actually used *exceed* that given by a fraction, *subtract*. (See Example 8.)

6. $117\frac{1}{4}$ gallons at 12*s*. 6*d*. per gallon.

$$117 = 112 + 5, \text{ and } 112 = 8 \times 7 \times 2.$$

		s.	*d.*	
	4)	12	6 × 5	
			8	
	5	0	0	
			7	
	35	0	0	
			2	
	70	0	0	
	3	2	6	for the 5 gallons.
		3	$1\frac{1}{2}$	for the $\frac{1}{4}$.
Ans.	£73	5*s.*	$7\frac{1}{2}$*d.*	

7. 85¾ cwt. at £1 7*s*. 8*d*. per cwt.

$85 = 84 + 1$, and $84 = 12 \times 7$.

£.	s.	d.	
1	7	8 × 1	
		12	
16	12	0	
		7	
116	4	0	
1	7	8	for the 1 cwt.
	13	10	for ½ } = ¾
	6	11	for ¼ }

Ans. £118 12*s*. 5*d*.

8. What is the price of 87¾ bushels of wheat at 4*s*. 3*d*. per bushel?

$87 = 88 - 1$, and $88 = 11 \times 8$.

£	s.	d.	
0	4	3 × 1;	
		11	
2	6	9	
		8	
18	14	0	
Sub.	4	3	for 1 bushel.
18	9	9	
	2	1½	} for ¾ bush.
	1	0¾	}

Ans. £18 12*s*. 11¼*d*.

Or better thus: since 87¾ = 88 − ¼.

£	s.	d.	
0	4	3	
		11	
2	6	9	
		8	
18	14	0	
Sub.	1	0¾	for ¼ bush.

Ans. £18 12*s*. 11¼*d*.

9. Multiply 16° 51′ 43″ by 231¾. *Ans*. 3907° 45′ 20¼″.
10. Multiply 19 days 13 hours 27 minutes by 443¼. *Ans*. 8670 d. 3 h. 42 m. 45 s.
11. Multiply 9 oz. 17 dwt. 20 gr. by 616¼. *Ans*. 6095 oz. 14 dwt. 19 gr.
12. Multiply 14 tons 13 cwt. 2 qr. 11 lb. by 243¾. *Ans*. 3578 tons 4 cwt. 2 qr. 7½ lb.

Remarks on the real character of Multiplication, and on some erroneous opinions respecting it.

Multiplication, whether simple or compound, is a short way of finding the result of addition, and it is nothing more than this. The multiplier always expresses the *number* of

things, each equal to the multiplicand, that are to be incorporated into one sum; and this *sum*, by the operation of multiplication, is furnished by the *product*. The multiplier, therefore, can never be anything but an abstract number; it can never be a commodity, as a sum of money or a weight of goods; nor yet any measure of length, surface, or capacity; it simply denotes how many *repetitions*, or *times*, some other abstract number, or concrete quantity, is to be taken and exhibited in one whole.

There are people, however, who, strangely departing from this strictly correct and common-sense view of multiplication, speak of multiplying money by money, weights by weights, and measures by measures, and who actually set forth the calculations by which this impossible work is performed. They tell us that "2*s*. 6*d*. multiplied by 2*s*. 6*d*. is $3\frac{3}{4}$*d*.," and exhibit the figure-work by which the absurd result is obtained.* They forget that, if this were true, then $3\frac{3}{4}$*d*. divided by 2*s*. 6*d*. would be half-a-crown; a conclusion which, we presume, even these persons themselves would at once reject. As a test of the arithmetical skill of others, it is frequently proposed, by people who themselves are ignorant of the real nature and object of multiplication, to multiply £19 19*s*. $11\frac{3}{4}$*d*. by itself: they maintain that the product must be something short of £400, *because* £20 × £20 = £400: a statement which is inadmissible because it is absurd.

There is little doubt that these erroneous and ridiculous notions of the true purport of multiplication have arisen from the fact that workmen, in computing superficial and solid measurement, speak of multiplying feet and inches *by* feet and inches, and that books on mensuration tolerate and even adopt this inaccurate form of expression. But if it were *not* inaccurate, the precedent could furnish no justification for the so-called multiplication of money by money, or lbs. by lbs.; for the product in the former case is a quantity

* This figure-work is as follows, first by decimals and then by vulgar fractions.—

1. 2*s*. 6*d*., or $\frac{1}{8}$th of a pound, is ·125*l*.; and ·125*l*. × ·125*l*. = ·015625*l*. = $3\frac{3}{4}$*d*.

2. $\frac{1}{8}l. \times \frac{1}{8}l. = \frac{1}{64}l.$, which, in pence, is $\frac{1}{64} \times 240$ pence $= \frac{240}{64}d. = 3\frac{3}{4}d.$

altogether *different in kind* from the quantities multiplied together: thus, although we might say in reference to linear measure, that 20 ft. × 20ft. = 400 sq. ft., yet we could not, on that account, be permitted to say that 20*l.* × 20*l.* = 400*l.*, but rather, *on that account*, we should be *forbidden* to say so. In the case of the *feet*, the result is something wholly different from linear feet: it is *square* feet, not length, but surface; so in the case of the *pounds*, in money, the result, consistently with this, should not be pounds, but something different in kind; it should not be *money* at all, any more than surface is mere length.

These considerations alone are sufficient to show that there is no analogy at all between the multiplication of feet by feet, and the multiplication of money by money, even supposing "multiplication of feet by feet" to be an accurate form of expression for a certain operation; that is, even supposing that we really *do* multiply by the concrete quantity feet, and not by an abstract number merely. But there is no such thing as multiplication *except* by an abstract number: a multiplier always expresses *a number of times*, and never *concrete things;* the number of times, namely, that a proposed quantity, be it concrete or not, is to be taken, and all these repetitions of the same individual thing (or equivalents of that thing) incorporated in one whole; and this whole must, of course, *necessarily* be the same in kind as each of the individual quantities, themselves all of one kind, of which it is composed.

When the multiplier is a whole number and a fraction, it directs us to take the multiplicand that whole number of times, and *also* to add in the proposed fractional part of a time. It would be fastidious and hyper-critical to dispute about a fractional part of *once* the given quantity being called a *multiplication* of that quantity. The propriety of calling *once*, or *one time* itself, a multiplication might be objected to on similar grounds. The multiplying a quantity by a number, whole or mixed, or fractional, means the taking that quantity the proposed number of times, together with the proposed fractional part of it, and finding the whole sum without the trouble of writing down these items and then adding them all up.

As to the apparent practical departure from the above-

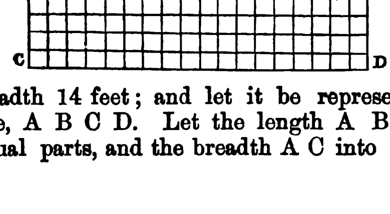
mentioned general principle in the particular instances of computing surfaces and solids from their given linear dimensions, we shall show, in the next article, that the departure is *only* apparent; and that the rules for such computations are worded as they are solely for brevity of expression, and as a sort of artificial aid to the memory of the computer, as to the successive steps of the mere figure-work.

Duodecimals, or Cross Multiplication.

At page 23 we have given a table of superficial or *square* measure, as it is sometimes called, the inches, feet, &c., in that table being exclusively *square* inches, feet, &c. By a square foot, or inch, is meant the surface, or *area*, of a square, each side of which is a linear foot, or a linear inch; so that when a surface is said to measure 10 square feet, the meaning is that it is equal in area to 10 times the surface of 1 square foot.

The simplest kind of plane figures is the *rectangle*, a figure of four sides, each pair of opposite sides being equal and parallel straight lines. The square is itself a rectangle; but in this particular kind of rectangle, not only are the *opposite* sides equal sides, but the sides are all four of them equal. A general rule for computing the area of a rectangle, from knowing the measures of the length and breadth of it, may be deduced thus:—

Suppose the length of the proposed rectangle to be 18 feet,



and its breadth 14 feet; and let it be represented by the above figure, A B C D. Let the length A B be divided into 18 equal parts, and the breadth A C into 14; then if

parallel lines be drawn from the points of division, as here represented, the rectangle will be cut up into *squares* of which each side is (or at least stands for) *one foot*. By counting these squares, we should find that the rectangle contains 252 square feet; but without taking this trouble, since we know that there are fourteen horizontal rows of squares between A B and C D, and that each row consists of eighteen squares, the exact number of squares will be found by simply multiplying 18 by 14; hence $18 \times 14 = 252 =$ the *number* of square feet in the surface. Here the 18, the 14, and the 252, are abstract numbers: we have multiplied the *number* of feet in the length by the *number* of feet in the breadth, and we know, from the above diagram, that the result is the *number* of square feet in the rectangular surface; and it is plain that the area would be found, in a similar manner, if the length and breadth had each been any other number of feet. And if a fraction of a foot had been added to this number of feet, the operation would still have been that of ordinary multiplication: thus, in the foregoing example, if the length had been $18\frac{1}{2}$ feet, then a vertical row of half-squares would have been added to the vertical row B D of fourteen whole squares, and each of the fourteen horizontal rows would have consisted of $18\frac{1}{2}$ square feet, so that the number of square feet in the rectangle would have been 14 times $18\frac{1}{2}$, or 259. If with this length, the breadth had been $14\frac{1}{4}$ feet, then a row of quarter-squares would have been added to the horizontal row C D of eighteen whole squares, *together* with a quarter of the half-square already added on at D, since a vertical column of half-squares has been added on at B D; hence the entire number of squares is 14 times $18\frac{1}{2}$ together with $\frac{1}{4}$ of $18\frac{1}{2}$; that is, the number of square feet is $18\frac{1}{2} \times 14\frac{1}{4} = 263\frac{5}{8}$, which is thus got, as before, by simply multiplying the *number* of feet in the length by the *number* of feet in the breadth.

From these particular illustrations it is obvious that, whatever be the length and breadth of a rectangle in feet and fractions of a foot, the number of square feet in the area of it is found by multiplying the number of linear feet in the one dimension by the number of linear feet in the other; and that the numbers thus multiplied together are *abstract*, not *concrete* numbers; that is, we do *not* multiply feet by feet.

When, however, the linear dimensions involve fractions of

a foot, it is usually the more convenient course to convert the fractions into decimals, and to work with *them;* thus, writing 18·5 for 18½, and 14·25 for 14¼, we should multiply as in the margin (*See* APPENDIX). The decimal ·625Ft. may be converted into inches by multiplying it by 144, the number of square inches in a square foot, or by multiplying by 12 and 12. The result is 90 square inches, so that the area of the rectangle is 263 Ft. 90 In. We write the initial letters of the feet and inches in capitals, in order to distinguish them from linear feet and inches; this form should, we think, be generally used, to save the trouble of always inserting the word "square."

$$\begin{array}{r} 14{\cdot}25 \\ 18{\cdot}5 \\ \hline 7125 \\ 25650 \\ \hline 263{\cdot}625 \text{ Ft.} \\ \hline \end{array}$$

But there is another way of calculating the area, the way, indeed, usually practised by workmen, when the linear dimensions are taken in feet and inches. This is by DUODECIMALS,—a subject now to be explained.

The *decimal notation* is so called because the value of every figure of a number expressed in that notation diminishes at a *tenfold* rate, as it is removed one place at a time towards the right: thus, if the figure 6, for instance, occur in any number, its local situation in that number may be such that, in that place, it stands for 600; if it be removed one place more to the right, it will stand for 60; removed one place still further to the right, it will stand for 6 only. Introducing a separating mark here (the decimal point) to distinguish integers from fractions, and still continuing to push back the figure, it will stand successively for 6 *tenths*, 6 *hundredths*, and so on. If, instead of this tenfold diminution, the diminution were at a *twelve-fold* rate, the notation would be the *duodecimal notation;* and in the measurement of *lengths*, the denominations feet and inches do actually descend in value at this rate, an inch being the twelfth part of a foot; and hence for the purpose of computing surfaces it is convenient to have a duodecimal arithmetic, at least for the operation of multiplication. Let us inquire how this new kind of multiplication ought to be performed.

Take the instance already considered, the length of the rectangle being 18 ft. 6 in., and its breadth 14 ft. 3 in., or, as it is usually expressed, the dimensions being 18 ft. 6 in., by 14 ft. 3 in. Now as in ordinary multiplication we carry the

number of *tens*, so here we must carry the number of *twelves*. In $6 \times 3 = 18$, there is 1 twelve, therefore, to be carried, and the overplus 6 to be put down; in 18×3, *plus* the 1 carried, that is, in 55, there are 4 twelves, and 7 over. The first line of the multiplication is therefore to be written as in the margin; it expresses $4 + \frac{7}{12} + \frac{6}{144}$, just as 4·76 expresses $4 + \frac{7}{10} + \frac{6}{100}$. The 7, in the above work, denotes seven twelfths of one of the units in the 4, and the 6 denotes six-twelfths of one of the units in the 7, or six 144ths of one of the units in the 4. For the second line of the work we have $6 \times 14 = 84 =$ seven twelves, and nothing over; and this seven added to 18×14 gives 259. The product is therefore 263 7 6, so that the area of the rectangle is 263 square feet, 7 twelfths of a square foot, and 6 square inches. The twelfth of a square foot is 12 inches, therefore the area is 263 Ft. 90 In., or, as at page 40, $263\frac{5}{8}$ Ft.

18	6	
14	3	
4	7	6
259	0	
263	7	6

It is customary to call the twelfth of a square foot a *Part*, certainly a very vague designation. Using this term, however, the foregoing result would be read, 263 Ft. 7 Pts. 6 In. It would be far better, however, to call these *Parts* twelfths, meaning twelfths of a square foot, each "Part" being a surface one foot in length and one inch in breadth: these Parts, or twelfths, are, however, usually turned into Inches, and the result expressed in Feet and Inches, and fractions of Inches only.

It should here be mentioned that in the numerical operation above, we have arranged the steps of the work just as they would be arranged if we had been dealing with ordinary decimals instead of with duodecimals, in order that the strict analogy between the two processes may be clearly seen. But in actual practice this conformity of mere arrangement is usually departed from, the second line of the work being placed first, agreeably to the following

RULE *for computing the surface of a rectangle.* Set down the two dimensions one under the other, feet under feet, and inches under inches, and work as follows:—

1. Commence with the *feet* in the multiplier, and multiply each term of the multiplicand, beginning with the inches (or lowest denomination), by that number. For every 12 in the product carry 1; and write the overplus, or remainder, under

that term of the multiplicand which has supplied the product.

2. In like manner multiply the multiplicand, beginning as before with the lowest denomination in it, by the number of *inches* in the multiplier, rejecting, as before, the twelves in the product (carrying 1 for each 12), and *here* write the remainder *one place further to the right of the term multiplied.* Add the two lines together, and the result will be the area in Feet, Parts, and Inches.

Should the given dimensions consist of more than two duodecimal denominations (feet and inches), and include twelfths of inches, and even twelfths of one of these twelfths, the same course is to be followed. At each multiplication the term first written down is to be pushed one place more to the right, since a unit of each multiplier (after the feet) is always one-twelfth of a unit of the immediately preceding multiplier, and the denominations, from left to right, uniformly descend by *twelfths.* See the third working of example 9, at page 45.

NOTE.—For the more easy recollection of the denominations to which each partial result in the several steps of the work refers, it is common to say (and for this purpose only, allowable to say) that feet multiplied by feet produce Feet; inches multiplied by inches produce Inches; and inches multiplied by *feet* produce Parts, or (which is a better term), *twelfths* of a Foot.

We shall now give a few examples, commencing with the example already worked, under the slightly different arrangement alluded to above.

EXAMPLES.

	feet.	inches.			feet.	inches.			feet.	inches.	
1.	18	6		2.	17	9		3.	22	5	
	14	3			20	6			16	11	
	259	0			355	0			358	8	
	4	7	6		8	10	6		20	6	7
	263 Ft.	7 Pts.	6 In.		363 Ft.	10 Pts.	6 In.		379 Ft.	2 Pts.	7 In.

And, turning the Parts into Inches, these several results are:

263 Ft. 90 In., 363 Ft. 126 In., and 379 Ft. 31 In.

Seeing that the successive products in operations of this kind are each divided by 12, and the remainders only put

down, it will conduce to facility to bring the common *pence table* to our aid, regarding, for the moment, every partial product as so many *pence* to be converted into *shillings*. Thus, in the first example, we may say, 6 times 14 are 84, 84 pence are 7 shillings, 0 and carry 7; then, for the next line, 3 times 6 are 18, one and sixpence, 6 and carry 1; 3 times 18, and the 1, make 55, 55 pence are four and seven pence. In the third example, we may say, 5 times 16 are 80, 80 pence are six and eight pence, 8 and carry 6; and, in the second line, 5 times 11 are 55, four and seven pence, 7 and carry 4; 11 times 22 are 242, and this, with the 4 carried, is 246 *pence* = 20*s*. and 6*d*; and so in all other instances.

4. Find the area of a rectangle, the dimensions being 8 ft. 5 in. by 4 ft. 7 in. *Ans.* 38 Ft. 83 In.
5. Required the area of a rectangle 9 ft. 8 in. by 7 ft. 6 in. *Ans.* 72 Ft. 72 In.
6. What is the area of a rectangle 75 ft. 7 in. by 9 ft. 8 in.? *Ans.* 730 Ft. 92 In.
7. Find the area when the dimensions are 75 ft. 9 in. by 17 ft. 7 in. *Ans.* 1331 Ft. 135 In.
8. Find the area when the dimensions are 179 ft. 3 in. by 38 ft. 10 in. *Ans.* 6960 Ft. 126 In.
9. Required the area of a plank 20¾ feet long and 12½ inches broad. *Ans.* 21 Ft. 88½ In.

As fractions occur in these dimensions, we shall give the work first by duodecimals, and then without them.

feet.	in.	
20	9	
1	0½	
20	9	
	10	4½
21 Ft.	7 Pts.	4½ In.
= 21 Ft.	88½ In.	

	feet.	in.	in.
	20	9	=249
			12½
			2988
			124½
12)			3112½ In.
12)			259 Pts. 4½ In.
			21 Ft. 7 Pts. 4½ In.

The first method is evidently the shorter and the more convenient; but the following mode of working, which is throughout by duodecimals (the ½ being replaced by 6

twelfths), is upon the whole to be preferred. We shall indicate the twelfth of a linear inch by a dash (′).

feet.	in.	′		
20	9	0		
1	0	6		
20	9	0		
	10	4	6	0

(See the RULE.)

21 Ft. 7 Pts. 4 In. 6‵ = 21 Ft. 88½ In.

Among mechanics, generally, linear measurements are not taken to a greater nicety than the 16th of an inch, their divisions of the inch being the half, the quarter, the eighth, and the sixteenth. In duodecimal workings these are all to be replaced by their equivalents in twelfths, and this is easily done, for

$$\frac{1}{2}=\frac{6}{12};\ \frac{1}{4}=\frac{3}{12};\ \frac{1}{8}=\frac{3}{24}=\frac{1\frac{1}{2}}{12};\ \frac{1}{16}=\frac{3}{48}=\frac{\frac{3}{4}}{12};$$

that is, using, as above, the simple dash to mark twelfths of a linear inch, the double dash to mark twelfths of these twelfths, and so on, we have

$$\overset{\text{in.}}{\frac{1}{2}}=6';\ \overset{\text{in.}}{\frac{1}{4}}=3';\ \overset{\text{in.}}{\frac{1}{8}}=1'\,6'';\ \overset{\text{in.}}{\frac{1}{16}}=\frac{3}{4}\text{ of }1'=\frac{3}{4}\text{ of }12''=9''.$$

10. Find by duodecimals the area of a rectangle measuring 7 ft. 5¾ in. by 3 ft. 5½ in. *Ans.* 25 Ft. 8 Pts. 6 In. 2‵ 3‶.

It may seem that we have worked this last example to an unnecessary degree of precision. A practical man would reject from his result the small portions of surface marked 2‵ and 3‶, the reversed dashes being here used to distinguish these portions of surface from the linear measures 2′ and 3″. Yet, that the result may be correct to the nearest square inch, he must first compute it to this extent, rejecting the insignificant parts of it afterwards. It is necessary that he should bear in mind the influence of the *carryings*, in the multiplications, and also remember that in the adding up of small quantities, the sum may become appreciable. Upon performing the calculation which leads to the result stated above, the computer will feel the advantage of availing him-

self of the *pence table*, as suggested at p. 44. Rejecting the $3''$, the answer in feet and inches is 25 Ft. 102$\frac{1}{8}$ In., or including the $3''$, it is 25 Ft. 102$\frac{3}{16}$ In.

Whenever the highest denomination in the linear measurement exceeds feet, as yards, poles, &c., then this higher denomination must be reduced to feet before duodecimals can be applied, seeing that a yard is not *twelve* times, but *three* times a foot. The result being obtained in square feet, &c., it may then be converted into square yards, &c., as in the following example.

11. How many square yards are there in a carpet 7 yds. 1 ft. 4 in. long and 5 yds. 2 ft. 3 in. wide?

	feet.	in.	
	22	4	
	17	3	
	379	8	
	5	7	0
9)	385 Ft. 3 Pts. = 42 Yds. 7 Ft. 36 In. = 42 Yds. 7$\frac{1}{4}$ Ft.		

12. What is the area of a rectangle 13 yds. 2 ft. 9 in. by 5 yds. 1 ft. 7 in.? *Ans.* 76 Yds. 8 Ft. 51 In.

COMPOUND DIVISION.

It has already been noticed that multiplication (by an integer) is nothing more than a short way of finding the result of addition when the quantities to be added, be they abstract numbers or concrete quantities, are *all equal:* there can be no multiplication operation by a whole number that cannot be replaced by addition. There are books which tell us that division, in like manner, is merely an abridgment of the process of repeated subtraction, but this is not true. Suppose, for instance, we are asked to divide a sum of money, say £34 16*s*. 8*d*., by 24: how is it possible that the result sought could be arrived at by subtraction? If we had to *multiply* by 24, instead of to divide by 24, we might write down the sum of money twenty-four times, and then, adding the whole up, we should get the same amount for the *sum* that, in the multiplication, we should get for the *product*. The sum to be repeatedly added, we *know:* it is £34 16*s*. 8*d*.; but where is

the sum to be repeatedly *subtracted?* It does not exist; it is in fact the very thing we are asked to discover.

There are two distinct aspects under which division should be viewed. When we are required to divide a concrete quantity by an abstract number, as, for instance, by 4, 9, 24, &c., the demand is to determine the 4th, 9th, 24th, &c., part of that quantity; but when we are required to divide one concrete quantity by another concrete quantity of the same kind, then the demand is to find how many *times* the divisor is contained in the dividend; and the quotient, or answer to the inquiry, is then always an abstract number. In the example proposed above of dividing £34 16*s.* 8*d.* by 24, the answer (to the nearest farthing) is £1 9*s.* 0¼*d.*, this being the 24th part of the given sum; but if we had been required to divide by 24*s.*, that is, to find how many sums, each equal to £1 4*s.*, are contained in £34 16*s.* 8*d.*, the answer would have been 29 of such sums with 8*d.* over; in other words, that £34 16*s.* 8*d.* contains 24*s.* 29 times with 8*d.* to spare.

In order that he may have correct notions of what he is about, the reader should be careful to discriminate between these two kinds of division; to notice that when he is required to divide a concrete quantity by a mere *number*, the quantity is supposed to be cut up into that number (the divisor) of equal parts, and his business is to find the value of *one* of those parts; and that when he is required to divide one concrete quantity by another, then he is to find how many *times* the one quantity (the divisor) is contained in the other: he does not *here* get a concrete quantity for his result, as in the former case, but an abstract number.

From these observations, the reader will at once see that however repugnant to common sense it may be (and pure nonsense it certainly is) to speak of *multiplying* money by money, yet we may propose to *divide* money by money, that is, to find how many times a smaller sum is contained in a greater, with strict propriety. If money be the multiplicand, the multiplier *must* be a mere number; if money be the dividend, the divisor may be either a mere number, or be itself money: in the former case the quotient will be money, in the latter it will be an abstract number.

We shall now give the necessary rules for these two cases, illustrating them by suitable examples.

Case I.

When the divisor is an abstract number.

Rule.—Apply the divisor to the leading, or highest, denomination in the compound quantity, and if the divisor do not exceed 12, put the quotient underneath; reduce the remainder to the next lower denomination, and carry it, thus reduced, to the term of that lower denomination in the dividend. Divide the result in the same way, reducing the remainder to the *next* lower denomination, and adding in the term of like denomination in the dividend, as before; and proceed in this way from term to term.

And if the divisor be greater than 12, but be composed of factors none of which exceed 12, we may still proceed by short division as above directed, with each of these factors in succession, instead of dividing at once by the composite number itself. Whether the proposed divisor can or cannot be decomposed into factors suitable for the operation of short division, we may discover at once by reference to the table at page 30. But when the divisor is a large number, the factors of which are unsuitable for these successive short division steps, reduce the compound quantity to the lowest denomination in it, and *then* divide. The quotient will be a quantity in that lowest denomination, which may be brought into the higher denomination by reduction.

Note.—When the number in the highest denomination actually *contains* the divisor, we may apply the divisor to *it* at once, and reduce what remains to the lowest denomination afterwards. (See Ex. 3.)

Examples.

1. Find the 24th part of £34 16*s*. 8*d*.

$24 = 8 \times 3$

	£	*s*.	*d*.
8)	34	16	8
3)	4	7	1
	£1	9*s*.	0¼*d*. + ⅓ *far*.

So that, in strict accuracy, if £34 16*s*. 8*d*. is to be equally divided among 24 persons, each person should get £1 9*s*. 0¼*d*., and ⅓ of a farthing besides; but as we have no coin so small as the third of a farthing, and since the overplus is twenty-four of these thirds, that is, two pence, the sum, to be practically divisible, with accuracy (if no *increase* of it be allowed), should have been £34 16*s*. 6*d*.

2. Divide £780 12*s*. 4*d*. by 168.

By the table (p. 30), $168 = 7 \times 6 \times 4$.

	£	s.	d.	
4)	780	12	4	
6)	195	3	1	
7)	32	10	6	. . 1
	£ 4	12*s*.	11*d*.	. 1

Now, as already explained (p. 11), the fraction of a penny belonging to this final quotient, is $1 \times 6 + 1$ divided by 7×6, or 42; that is, it is $\frac{7}{42}d.$, or one-sixth of a penny;* so that the sum, to be rendered accurately divisible by 168, should be diminished by 168 of these sixths, that is, since $\frac{168}{6}d. = 28d.$, it should be diminished by 2*s*. 4*d*.; or else it should be increased by 168 farthings *less* this sum; which increase is 1*s*. 2*d*. In the former case, the exact 168th part would be £4 12*s*. 11*d*., and in the latter, a farthing more than this. We shall here work the example under both these changes.

	£	s.	d.
4)	780	10	0
6)	195	2	6
7)	32	10	5
	£4	12*s*.	11*d*.

	£	s.	d.
4)	780	13	6
6)	195	3	4½
7)	32	10	6¾
	£4	12*s*.	11¼*d*.

This work merely verifies the above conclusion, reached at once with scarcely any work at all; and when any sum is proposed for division, and we find, upon trial, that accurate division, without fractions of a farthing, is impracticable, we thus see how readily we may find the *smallest* amount by which the proposed sum must be diminished or increased in order that the required part of it may be practically payable in existing coin.

* The *complete* quotient from the divisor 6 is £32 10*s*. 6*d*. + $\frac{1}{6}d.$; and the next complete quotient is therefore £4 12*s*. 11*d*. + $\frac{1}{7}d.$ + $\frac{1}{42}d.$; and the sum of these two fractions is $\frac{1 \times 6}{42} + \frac{1}{42} = \frac{7}{42}$. (See the article on *Fractions*, p. 53.)

3. £837 13*s.* 6*d.* is to be equally divided among a ship's company of 273 persons; what will be the share of each?

As we see, by a reference to the table at p. 30, that 273 is not decomposable into factors suitable for short division, we must proceed by long division, as follows:—

The lowest denomination being 6*d.*, we shall reduce the sum to sixpences.

```
        £   s.  d.                            £   s.  d.  £   s.  d.
       837  13  6   Or, better thus:   273) 837  13   6  (3   1   4¼
        20                                  819
      -----                                 ---
      16753                                  18
          2                                  20
      ----- sixpences.                      ---
273) 33507 (122 = 61s. = £3 1s.             373 (1s.
      273                                   273
      ---                                   ---
       620                                  100
       546                                   12
       ---                                  ----
        747                                 1206 (4d.
        546                                 1092
        ---                                 ----
        201                                  114
          6                                    4
       ----                                 ----
       1206 (4d.                             456 (1 far.
       1092                                  273
       ----                                  ---
        114                                  183
          4                                  ---
        ---
        456 (1 far.
        273
        ---
        183               Ans. £3 1s. 4¼d. + 183/273 f.
        ---
```

In order that the share of each man may be exactly £3 1*s.* 4¼*d.*, without any overplus fraction of a farthing, the proposed sum must be diminished by 183 farthings, that is, by 3*s.* 9¾*d.*, thus reducing the amount to be divided to £837 9*s.* 8¼*d.* But instead of diminishing the sum to be distributed by 3*s.* 9¾*d.*, an additional farthing will fall to the share of each person by increasing that sum by 273 farthings *less* these 183 farthings; that is, by 90*f.* = 1*s.* 10½*d.*, and thus making it £837 15*s.* 4½*d.*

4. Divide £64 19*s.* by 36. *Ans.* £1 16*s.* 1*d.*
5. Divide £46 14*s.* 6*d.* by 24. *Ans.* £1 18*s.* 11¼*d.*

6. Divide 315 days 17 hours 37 minutes by 112.
   Ans. 2 days 19 hours 39 minutes 26¼ seconds.
7. Divide 128° 45′ 52″ by 125. *Ans.* 1° 2′ 48″ + $\frac{52}{125}$″.
8. Divide 496 miles 5 furlongs 2 perches by 594.
   Ans. 6 furlongs 27 perches 3 yards.
9. Divide 7080 acres 2 roods 7 poles by 511. *Ans.* 13 ac. 3 rd. 17 po.
10. Divide 1679 sq. yds. 7 sq. ft. 129 sq. in. by 563.
    Ans. 2 Yds. 8 Ft. 123 In.

*** Why the initial letters in this last answer are capitals is explained at page 41.

Case II.

When the divisor is a concrete quantity.

Rule.—Reduce the two quantities to the *lowest* denomination that occurs in either of them, and then perform the division with *the results:* the quotient will express the number of times the smaller of the two concrete quantities is contained in the greater.

Examples.

1. How many times is £2 17*s.* 8*d.* contained in £18 3*s.*?

The lowest denomination here is *pence*; we have therefore to reduce both divisor and dividend to pence, and to perform the division with the results, thus:—

£ s. d.	£ s. d.
2 17 8	18 3 0
20	20
57	363
12	12
692	692) 4356 (6
	4152
	204 *rem.*

Hence the smaller sum is contained in the greater six times, with 204 pence to spare. If the £18 3*s.* be diminished by this, that is, by 17*s.*, it will then contain the £2 17*s.* 8*d.* *exactly* 6 times. As it is, however, the greater contains the less 6 times and the $\frac{204}{692}$ part of a time; or, which is the same thing, 6 times the less, and the $\frac{204}{692}$ of that less besides.

2. How many parcels, each weighing 9 oz. 17 dwt. 20 gr., may be made up out of 380 lb. 15 oz. 14 dwt. 19 gr.?

```
oz. dwt. gr.                                 lb.  oz. dwt. gr.
 9   17  20                                  380  15  14   19
     20 = number of dwts. in 1 oz.            16 = number of oz. in 1 lb.
 ---                                         ----
 197                                         6095
  24 = number of grs. in 1 dwt.                20
 ----                                       ------
  788                                       121914
 396                                            24
 ----                                       ------
 4748 grs. in each parcel.                  487665
 ----                                      243829
                                           -------
                                   4748) 2925955 (616
                                         28488
                                         -----
                                           7715
                                           4748
                                          -----
                                          29675
                                          28488
                                          -----
                                           1187
                                           ----
```

It thus appears that the number of parcels is 616, and that there will be 1187 grains to spare. Now it is easily seen that these overplus grains amount to just one-fourth of another parcel, for $\frac{1}{4}$ of 4748 *is* 1187: hence we should say that the greater weight contains exactly $616\frac{1}{4}$ times the smaller.

There are no means of abridging operations of this kind.

3. Divide £46 14*s.* 6*d.* by £3 17*s.* $10\frac{1}{2}$*d.* *Ans.* 12.
4. Divide 2 tons 13 cwt. 5 lb. by 3 qr. 17 lb. *Ans.* $58\frac{83}{101}$.
5. Divide 8631 days 48 minutes 45 seconds by 19 days 13 hours 27 minutes. *Ans.* $441\frac{1}{4}$.

Compendious Methods of Calculation in Special Cases

(With some Preliminary Remarks on Vulgar Fractions).

In the preceding pages we have given general rules of operation universally applicable to all inquiries coming under the several heads into which the subjects treated of have been divided; and, in a few instances, we have replaced hackneyed methods by shorter modes of proceeding. It is our purpose now to explain how, by the exercise of a little ingenuity, and by availing ourselves of certain obvious ex-

pedients, these *general* rules may, *in special cases*, be put aside altogether; and *particular* rules, expressly applicable to such cases, and involving less numerical work, be substituted for them.

In the various avocations of civilized life, those of them in which calculation is essential are usually such that each distinct calling brings into exercise one particular class of arithmetical operations much more frequently than any other class: that portion of arithmetic which might do very well for the draper and the mercer, would not suffice for the carpenter or the bricklayer; and what might fully answer the requirements of either of these would be insufficient for the spirit merchant or the banker: they would, indeed (viewed commercially), be quite useless to him.

We shall here give a series of special rules of calculation, for ready use in special commercial and handicraft callings; and shall in each case show the consistency between the abbreviated operation and the more lengthy process of ordinary arithmetic. In order to this, however, it may be necessary to recall to the mind of the reader a few of the fundamental properties of vulgar fractions, since they will be of frequent application in what follows.

On Vulgar Fractions.

1. The value of a vulgar fraction remains unaltered by whatever number we divide or multiply both numerator and denominator of that fraction: thus, $\frac{8}{12} = \frac{4}{6} = \frac{2}{3}$; that is, $\frac{8}{12}$ths may be replaced by $\frac{4}{6}$ths, or by $\frac{2}{3}$rds, whichever we please; but, as in the latter form, the fraction is expressed by smaller numbers than when it is written in either of the other forms, it is by *it* that we should replace $\frac{8}{12}$ in the final result of any operation. For instance, instead of writing, as the answer to a question, such a result as 4 ft. $7\frac{8}{12}$ in., we should put it in the simpler form, 4 ft. $7\frac{2}{3}$ in.; that is, four feet, seven inches, and two-thirds of an inch. The fraction is said to be in its *lowest terms* when no further simplication of it is possible; $\frac{2}{3}$ is $\frac{8}{12}$ in its lowest terms.

Again: by *multiplying* numerator and denominator, that is, the two *terms* of a fraction, by the same number, instead of *dividing*, we may exchange a simple for a more complicated

form, still preserving the actual value of the fraction: thus, $\frac{8}{12} = \frac{16}{24} = \frac{48}{72} = \frac{240}{360}$, &c. And in certain operations with fractions this change of form is absolutely necessary: fractions with their denominators *unlike* cannot be added or subtracted without such a change of form; for instance, we cannot add $\frac{2}{3}$ to $\frac{4}{5}$, nor subtract $\frac{2}{3}$ from $\frac{4}{5}$, till the fractions are replaced by equivalent fractions having a *common denominator*. The denominator *names* the *kind* of things, the *number* of which is denoted by the *numerator*. In the first fraction above, namely $\frac{2}{3}$, the *things* are *thirds*, and there are two of them; in the second ($\frac{4}{5}$) the *things* are *fifths*, and there are four of them. You cannot incorporate in one sum two things of one kind and four things of a different kind; both things *must* be reduced to the same denomination before you can either add or subtract; and this reduction to a common denominator may always be brought about in virtue of the privilege we have of multiplying the terms of a fraction by any number whatever. Thus, in the case of the two fractions here adduced, $\frac{2}{3}$ and $\frac{4}{5}$, we see, at once, that by multiplying the denominator of the first by 5 (the denominator of the *second*), and the denominator of the second by 3 (the denominator of the *first*), the product, in each case, will be the same number, namely, 15; hence the fractions are brought to a common denominator by multiplying the terms of the first by 5, and the terms of the second by 3; so that the two fractions may be replaced by the equivalent fractions $\frac{10}{15}$ and $\frac{12}{15}$, in which the things enumerated, or numbered, are the same in kind, namely, ten *fifteenths* and twelve *fifteenths*. And, of course, we can *now* add them together, or can subtract the less from the greater, thus:—

$$\frac{4}{5}+\frac{2}{3}=\frac{12}{15}+\frac{10}{15}=\frac{22}{15}; \text{ and } \frac{4}{5}-\frac{2}{3}=\frac{12}{15}-\frac{10}{15}=\frac{2}{15}.$$

Perhaps some of our readers may not clearly see how it is that the terms of a fraction may each of them be multiplied or divided by any number we please without causing a change of value in the fraction. A verification of the principle may therefore not be superfluous, and for this purpose a single instance will, we think, suffice.

Take the fraction with which we commenced, namely, $\frac{8}{12}$. What does this denote? It denotes the twelfth part of 8

things of some kind or other; as, for instance, of £8—the share, that is, of this £8 which would fall to each if equally divided amongst twelve persons. Is it not plain that this share, whatever it be, would be the very same if *half* the sum were divided amongst *half* the number of persons, or *one-fourth* the sum among *one-fourth* the number of persons? The share is, therefore, indifferently, £$\frac{8}{12}$, or £$\frac{4}{6}$, or £$\frac{2}{3}$, which is 13*s*. 4*d*.; and, in like manner, if we double, or treble, or quadruple, &c., the £8, and do the same by the number of persons, the share falling to each, that is, the value of the original fraction, *must* remain unaltered. This is a common-sense principle, and is obviously applicable to every vulgar fraction.

2. And, moreover, what is here said, as to the liberty we have of multiplying or dividing the numerator and denominator of a *fraction* by any number without causing a change of value in the fraction itself, obviously applies to the dividend and divisor in every common division operation; that is, we may, if we please, change in this way both of these, and then work with the results; for every dividend and divisor may be expressed *as* a fraction, the former being the numerator, and the latter the denominator.

If, instead of *dividing* one number by another, we have to *multiply* the two numbers together, we may still work with changed factors; but here, by whatever number we *divide* one factor, by that same number must we *multiply* the other; and by whatever number we multiply one, by that number must we divide the other. Suppose, for instance, the two factors were 16 and 3; it is plain that both may be changed, in this way, without affecting the product, thus:—

$$16 \times 3 = 8 \times 6 = 4 \times 12 = 2 \times 24 = 1 \times 48;$$
$$\text{also } 4 \times 15 = 12 \times 5 = 3 \times 20, \text{ \&c.}$$

From these obvious principles the rules for the multiplication and division of fractions are readily deduced. Suppose we have to multiply $\frac{2}{3}$ by $\frac{4}{5}$. Let us multiply the first fraction by 4: we must then *divide* the second by 4, and may then use them thus changed, being sure that $\frac{2}{3} \times \frac{4}{5} = \frac{8}{3} \times \frac{1}{5}$. Again, let us multiply the second of these by 5, then dividing the first by 5 (which first being already $8 \div 3$ must then be $8 \div 15$), we have $\frac{2}{3} \times \frac{4}{5} = \frac{8}{3} \times \frac{1}{5} = \frac{8}{15} \times \frac{1}{1}$, that is, $\frac{8}{15}$; so

that to get the product at once, we have only to multiply first the given numerators together, and then the given denominators, which is the general rule.

But suppose we have to *divide* instead of to multiply; then, by the above principle, $\frac{2}{3} \div \frac{4}{5} = \frac{2}{12} \div \frac{1}{5} = \frac{10}{12} \div \frac{1}{1} = \frac{10}{12}$, which is got at once by turning the divisor upside down (or reversing the terms of it), and *multiplying*: thus, $\frac{2}{3} \times \frac{5}{4} = \frac{10}{12}$; and this is the rule.

As it is not our intention to give here a *treatise* on vulgar fractions, we shall notice but one more elementary principle, which is, in fact, only a deduction from that considered in article 1, above.

3. When a quantity consists of a whole number and a fraction (in which case it is called a *mixed quantity*), it is sometimes necessary actually to add the whole number and fraction together into one sum, and thus bring the quantity out of the mixed form. If, for instance, the quantity proposed were $4\frac{2}{3}$, we should read it 4 *and* $\frac{2}{3}$; and we could not actually *add* these component parts into one sum till, as above, they were both expressed in the same denomination. In order to accomplish this, we give a fractional *form* to the 4, and write it thus, $\frac{4}{1}$; and then, by the general principle before explained, we have—

$$4 + \frac{2}{3} = \frac{4}{1} + \frac{2}{3} = \frac{12}{3} + \frac{2}{3} = \frac{14}{3};$$

and thus the *mixed quantity*, $4\frac{2}{3}$, is expressed in *one* denomination, namely *thirds*. As the numerator of the fraction $\frac{14}{3}$ is greater than the denominator, it is called an *improper fraction*, being equivalent to four whole units *and* the *proper* fraction $\frac{2}{3}$. To reduce an improper fraction to a whole, or mixed quantity, nothing more is necessary than to actually perform the division indicated; thus, executing the division implied in $\frac{14}{3}$, or $14 \div 3$, we find, for whole number, 4; but this is the third part of 12 only; so that the third part of the remaining 2 has still to be taken, the complete quotient being $4\frac{2}{3}$; that is, $14 \div 3 = 4\frac{2}{3}$.

Having made this digression upon so much of the doctrine of vulgar fractions as it is necessary that the reader should clearly understand before he proceeds to what follows, we shall now enter upon those compendious rules of computa-

tion by which, in special instances, numerical work may be shortened, and time and labour economized. But since certain of these compendious rules involve frequent use of fractional parts of the £ and of the shilling, we shall first give a Table of all such fractional parts as can be expressed in current coin; the numerator of each fraction being 1.

Table of Fractional Parts of £1 and of 1s.

Parts of £1.	Parts of 1s.	—— .
$\frac{1}{2}$ = 10s.	$\frac{1}{2}$ = 6d.	NOTE.—All these fractions of a shilling are, of course, *also* fractions of £1; found, in each case, by multiplying the denominator by 20. We thus have:—
$\frac{1}{3}$ = 6s. 8d.	$\frac{1}{3}$ = 4d.	
$\frac{1}{4}$ = 5s.	$\frac{1}{4}$ = 3d.	
$\frac{1}{5}$ = 4s.		
$\frac{1}{6}$ = 3s. 4d.	$\frac{1}{6}$ = 2d.	
$\frac{1}{8}$ = 2s. 6d.	$\frac{1}{8}$ = $1\frac{1}{2}$d.	£$\frac{1}{160}$ = $1\frac{1}{2}$d.
$\frac{1}{10}$ = 2s.		
$\frac{1}{12}$ = 1s. 8d.	$\frac{1}{12}$ = 1d.	
$\frac{1}{15}$ = 1s. 4d.		
$\frac{1}{16}$ = 1s. 3d.	$\frac{1}{16}$ = $\frac{3}{4}$d.	£$\frac{1}{320}$ = $\frac{3}{4}$d.
$\frac{1}{20}$ = 1s.		
$\frac{1}{24}$ = 10d.	$\frac{1}{24}$ = $\frac{1}{2}$d.	£$\frac{1}{480}$ = $\frac{1}{2}$d.
$\frac{1}{30}$ = 8d.		
$\frac{1}{32}$ = $7\frac{1}{2}$d.		
$\frac{1}{40}$ = 6d.		
$\frac{1}{48}$ = 5d.	$\frac{1}{48}$ = $\frac{1}{4}$d.	£$\frac{1}{960}$ = $\frac{1}{4}$d.
$\frac{1}{60}$ = 4d.		
$\frac{1}{64}$ = $3\frac{3}{4}$d.		
$\frac{1}{80}$ = 3d.		*** We thus see that the shilling (48 farthings) has *nine* different divisors or factors, and that the pound (960 farthings) has *twenty-seven.*
$\frac{1}{96}$ = $2\frac{1}{2}$d.		
$\frac{1}{120}$ = 2d.		
$\frac{1}{192}$ = $1\frac{1}{4}$d.		
$\frac{1}{240}$ = 1d.		

PROBLEM 1.

To find how many articles of the same kind, may be bought for a given number of POUNDS *(without shillings or pence), when the price of one is an* EVEN *number of shillings.*

By the general rule at p. 51, we should bring the given pounds into shillings, in order that both sums may be of the

same denomination; we should then divide the number of shillings, thus obtained, by the number of shillings in the price of *one* of the articles; the quotient would be the number of articles required. The bringing the pounds into shillings would require a multiplication by 20; but as, here, the divisor is an *even* number, we take advantage of this circumstance, and divide *half* the dividend by *half* the divisor; that is, we work by the following special rule, observing that a number is multiplied by 10 by simply annexing to it a cipher.

RULE.—Annex a cipher to the given number of pounds, and then divide by half the given number of shillings.

EXAMPLES.

1. If a yard of cloth cost 8*s.*, how many yards may be bought for £16?
   $160 \div 4 = 40$ yards.
2. How many yards, at 6*s.* per yard, can be bought for £48?
   $480 \div 3 = 160$ yards.
3. How much sugar, at 30*s.* per cwt., can be bought for £80?

$$\begin{array}{r} 15)\ 800 \\ \hline 53\tfrac{1}{3} \text{ cwt.} = 53 \text{ cwt. } 1 \text{ qr. } 9\tfrac{1}{3} \text{ lb.} \end{array}$$

Here the fraction which completes the quotient is $\frac{5}{15}$, which, in its lowest terms, is $\frac{1}{3}$; and $\frac{1}{3}$ cwt. is $\frac{4}{3}$ qr. $= 1\frac{1}{3}$ qr. $= 1$ qr. $+ \frac{28}{3}$ lb. $= 1$ qr. $9\frac{1}{3}$ lb. And this little additional work may be readily executed mentally. In the following example, however, the use of the pen will be found necessary.

4. If 1 cwt. cost 26*s.*, how much can be bought for £80?
   The work, at length, is as follows:—

$$\begin{array}{l} 13)\ 800 \\ \hline \quad 61 \text{ cwt., } 7 \text{ remainder.} \\ \qquad\qquad\quad 4 \\ \hline \qquad 13)\ 28 \text{ qrs.} \\ \hline \qquad\qquad 2 \text{ qrs., } 2 \text{ rem.} \\ \qquad\qquad\qquad 28 \\ \hline \qquad\qquad 13)\ 56 \text{ lbs.} \\ \hline \qquad\qquad\qquad 4\tfrac{4}{13} \text{ lbs.} \end{array}$$

Ans. 61 cwt. 2 qr. $4\frac{4}{13}$ lbs.

5. How many cwt. of butter, at 42*s.* per cwt., can be bought for £126? *Ans.* 60 cwt.
6. How many tons of coals, at 24*s.* a ton, can be bought for £25? *Ans.* 20 tons $16\frac{2}{3}$ cwt.
7. How many barrels of ale, at 70*s.* per barrel, can be bought for £27? *Ans.* $7\frac{5}{7}$, or 7 bar. $25\frac{5}{7}$ gal.
8. How many gallons of brandy, at 32*s.* per gallon, can be bought for £34? *Ans.* $21\frac{1}{4}$ gallons.

NOTE.—In some of the examples to which the foregoing rule is applicable. a mode of proceeding even more easy and convenient may be employed: we refer to those instances in which the price, in shillings, of the single article, ends with a cipher; as 30*s.*, 50*s.*, 70*s.*, &c. Thus, taking example 3, where the price of 1 cwt. is 30*s.*, this price in *pounds*, is $£\frac{30}{20} = £\frac{3}{2}$: therefore $\frac{3}{2}$ becomes the divisor of the 80: but since the quotient remains the same, by whatever number both dividend and divisor are multiplied (p. 55), we may here take the *double* of both; that is, 160 for dividend, and 3 for divisor; when the work will be simply this—

$3)\ 160$ $\overline{53\frac{1}{3}}$ cwt. In like manner if the price had been 70*s.*, then instead of dividing 800 by 35, we may divide twice 80 by 7 (which is easier) and we shall get the same result. Both methods are here exhibited.	$35)\ 800\ (22\frac{30}{35} = 22\frac{6}{7}$; or $7)\ 160$ $\quad 70 \qquad\qquad\qquad\qquad \overline{22\frac{6}{7}}$ $\quad \overline{100}$ $\quad 70$ $\quad \overline{30}$

And similarly in all such cases; the general precept being this:—

Suppress the final 0 in the price, and divide twice the given number of pounds by what is left. For, expressing the price in pounds,—

$$30s. = £\frac{30}{20} = £\frac{3}{2};\ 50s. = £\frac{50}{20} = £\frac{5}{2};\ 70s. = £\frac{70}{20} = £\frac{7}{2};$$
$$90s. = £\frac{90}{20} = £\frac{9}{2};$$

and so on, for 110*s.*, 130*s.*, &c.; the multiplier for the given number of pounds being always 2, and the divisor the number of shillings, when the final 0 in that number is expunged. Of course, we do not notice here the cases in which the price is 20*s.*, 40*s.*, 60*s.*, &c., because as these sums express £1, £2, £3, &c., respectively, we should merely have to divide the given number of pounds by the number which denotes the price of the single article.

There are cases, too, in which the number of shillings in the price is *odd*, which deserve special mention; the cases, namely, in which that number terminates with a 5; as 15,

25, 35, 45, &c. Every such number is divisible by 5; and since 20 is *also* divisible by 5, we have—

$$15s. = £\frac{15}{20} = £\frac{3}{4};\ 25s. = £\frac{25}{20} = £\frac{5}{4};\ 35s. = £\frac{35}{20} = £\frac{7}{4};\ \&c.;$$

the multiplier of the given number of pounds being always the fifth part of 20 (namely, 4), and the divisor, the fifth part of the number of shillings; it being remembered (as already explained at p. 56) that to *divide* by a fraction we merely turn that fraction upside down (or reverse its terms), and *multiply*: thus, if we are required to find how many articles at 35*s.* each can be bought for £21, we have—

$$21 \div \frac{7}{4} = 21 \times \frac{4}{7} = 12, \text{ the number required.}$$

In like manner, if the price of one were 75*s.*, the number purchasable for £21 would be—

$$21 \div \frac{15}{4} = 21 \times \frac{4}{15} = 7 \times \frac{4}{5} = \frac{28}{5} = 5\tfrac{3}{5}$$

The general precept, in all such cases, is this, namely—Divide 4 times the number of pounds by a fifth part of the number of shillings. So that, in these two examples, all that need be put upon paper is

$$\text{1st. } \frac{21 \times 4}{7} = 12. \qquad \text{2nd. } \frac{21 \times 4}{15} = \frac{28}{5} = 5\tfrac{3}{5}.$$

And similarly in every instance in which the price of the single article, in shillings, is a number ending with the figure 5.

Problem 2.

The price of one article being given, to find the price of twelve.

It is evident that 12 things, at any *number of pence* each, will cost just as much as that number of things at 12 pence, or 1 shilling each; hence the

Rule.—Call the pence which *one* costs shillings; and we shall thus have the price of *twelve.*

Examples.

1. If a pound of sugar cost 7*d.*, what will 12 lbs. cost? *Ans.* 7*s.*
2. If a yard cost $6\frac{1}{2}d.$, what will 12 yds. cost? *Ans.* $6\frac{1}{2}s. = 6s.\ 6d.$

3. If a yard cost $9\frac{1}{4}d.$, what will 12 yds. cost? *Ans.* $9\frac{1}{4}s. = 9s.\ 3d.$
4. If a yard cost $5\frac{3}{4}d.$, what will 12 yds. cost? *Ans.* $5\frac{3}{4}s. = 5s.\ 9d.$
5. If a pound cost $1s.\ 4\frac{1}{2}d.$, what will 12 lbs cost?
   $1s.\ 4\frac{1}{2}d. = 16\frac{1}{2}d.$; and $16\frac{1}{2}s. = 16s.\ 6d.$, the *Ans.*
6. If a piece of calico cost $5s.\ 3\frac{3}{4}d.$, what would 12 such pieces cost?
   $5s.\ 3\frac{3}{4}d. = 63\frac{3}{4}d.$; and $63\frac{3}{4}s. = £3\ 3s.\ 9d.$, the *Ans.*

[The reader need scarcely be reminded that $\frac{1}{2}s. = 6d.$; $\frac{1}{3}s. = 4d.$; $\frac{1}{4}s. = 3d.$; $\frac{3}{4}s. = 9d.$; $\frac{2}{3}s. = 8d.$; $\frac{1}{6}s. = 2d.$; $\frac{1}{8}s. = 1\frac{1}{2}d.$; as per Table p. 57.]

7. What is the price of 12 oz. of silver, at $4s.\ 7\frac{3}{16}d.$ per oz.? As the price per oz., in *pence,* is to be regarded as so many *shillings,* we have to determine what $\frac{3}{16}$ of a shilling is.* Multiplying therefore the fraction by 12, we have—

$$\frac{3}{16}s. = \frac{36}{16}d. = \frac{9}{4}d. = 2\tfrac{1}{4}d., \text{ so that, since}$$

$4s.\ 7d., = 55d$; and that $55s. = £2\ 15s.$, therefore $55\frac{3}{16}s. = £2\ 15s.\ 2\frac{1}{4}d.$

Problem 3. (Converse of Prob. 2.)

The price of twelve being given in shillings, to find the price of one.

The price of one will of course be the twelfth part of the price of twelve; and since the twelfth part of every shilling is a penny, the rule is as follows:—

Rule.—As many shillings as twelve cost, so many pence will one cost.

Examples.

1. If 12 pigeons cost $8s.$, what is the price of one? *Ans.* $8d.$
2. If 12 yards cost $16s.$, what is the price of 1 yd? *Ans.* $16d. = 1s.\ 4d.$
3. If 12 pairs of socks cost $4s.\ 8d.$, what will one pair cost?
   The cost of 12, in shillings, is $4\frac{2}{3}s.$; hence the cost of 1 is $4\frac{2}{3}d.$
4. If 12 gallons cost £1, what is the price per gallon?
   Ans. $20d. = 1s.\ 8d.$
5. If a dozen pairs of gloves cost £1 $7s.$, what is that per pair?
   Ans. $27d. = 2s.\ 3d.$
6. If I give £1 $17s.\ 9d.$ for 12 lbs of tea, what does it cost me per lb.?
   £1 $17s.\ 9d. = 37\frac{3}{4}s.$; hence the cost per lb. is $37\frac{3}{4}d. = 3s.\ 1\frac{3}{4}d.$

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* See, however, the Table at page 57 for $\frac{1}{16}s.$, which is 3 farthings, and therefore $\frac{3}{16}s.$ is 9 farthings, or $2\frac{1}{4}d.$

PROBLEM 4.

The price of one being given in pence, to find the price of any number which is a multiple of 12; *that is, which contains* 12 *without remainder.*

RULE.—Call the pence *shillings*, and then multiply by the number of *twelves*. For the price of 1 *twelve* is twelve times that of 1.

EXAMPLES.

1. What is the price of 24 yards of calico, at $3\frac{3}{4}d.$ per yard?
   $3\frac{3}{4}s. = 3s.\ 9d.$, which multiplied by 2, gives $7s.\ 6d.$, the *Ans.*
2. What are 36 lbs. of sugar worth, at $4\frac{1}{2}d.$ per lb.?
   $4\frac{1}{2}s. = 4s.\ 6d.$, and three times this is $13s.\ 6d.$, the *Ans.*
3. Required the cost of 72 lb. of meat, at $9\frac{1}{4}d.$ per lb?
   $9\frac{1}{4}s. = 9s.\ 3d.$, and six times this is £2 $15s.\ 6d.$, the *Ans.*
4. 96 door-locks, at $3s.\ 7\frac{1}{4}d.$ each?
   $3s.\ 7\frac{1}{4}d. = 43\frac{1}{4}d.$, and $43\frac{1}{4}s. =$ £2 $3s.\ 6d.$: this $\times\ 8 =$ £17 $8s.$
5. 120 pairs of gloves, at $2s.\ 3\frac{1}{4}d.$ a pair?
   $2s.\ 3\frac{1}{4}d = 27\frac{1}{4}d.$, and $27\frac{1}{4}s. \times 10 = 275s. =$ £13 $15s.$
6. 120 gallons of rum, at $13s.\ 10d.$ per gallon?
   $$13s.\ 10d. = 166d.,\ \text{and}\ 166s. \times \frac{10}{20} = £\frac{166}{2} = £83.$$

NOTE.—As in this example there are 10 twelves, it is obviously only necessary to divide the number of pence (regarded as so many pounds) in the price of one gallon, by 2. We might have done the same in the preceding example: thus, £27 $10s. \div 2 =$ £13 $15s.$ A *general* rule may often be advantageously departed from in *particular* examples coming under it.

7. 132 quarters of barley, at £1 $13s.\ 9d.$ per quarter?
   $33s.\ 9d. = 405d.$, and $405s. \times 11 = 4455s. =$ £222 $15s.$
8. 108 yards of cloth at $2s.\ 9\frac{1}{2}d.$ per yard?
   $33\frac{1}{2}s. \times 9 = 301\frac{1}{2}s. =$ £15 $1s.\ 6d.$
9. 84 yards of silk velvet, at $9s.\ 8\frac{3}{4}d.$ per yard?
   $116\frac{3}{4}s. = 116s.\ 9d.$; which, $\times\ 7$, is $817s.\ 3d. =$ £40 $17s.\ 3d.$

PROBLEM 5.

The price of one being given, in pence, to find the price of any number which is not a multiple of 12.

RULE 1.—Find the number of *twelves* in the proposed number, and reserve the remainder.

2. Compute for this multiple of 12, as in last problem;

then multiply the price of one by the reserved remainder, and *add* the product to the former result. Or, increase the aforesaid multiple of 12 by a unit, and compute by *it;* then multiply the price of one by what the remainder *wants* of 12, and *subtract.*

EXAMPLES.

1. What is the price of 25½ stone of wheat, at 17½*d.* per stone?

Here there are *two* twelves in the proposed number, with 1½ remainder; hence, proceeding by the Rule,—

17*s.* 6*d.* × 2 =	£1	15*s.*	0*d.*
then 17½*d.* × 1½ = {		1	5½, for the 1.
			8¾, for the ½.
	£1	17*s.*	2¼*d.*, the *Ans.*

2. 137 lbs. of worsted, at 17½*d.* per lb? Here 137 ÷ 12 = 11, 5 *rem.*

17*s.* 6*d.* × 11 =	£9	12*s.*	6*d.*
and 1*s.* 5½*d.* × 5 =		7	3½
	£9	19*s.*	9½*d.*, the *Ans.*

3. 104 yards of cloth, at 8*s.* 6¾*d.* per yard? 104 ÷ 12 = 8, 8 *rem.*

As the remainder here is only 4 less than 12, we shall increase the multiple (8) of 12, by 1, and then employ this 4.

8*s.* 6¾*d.* = 102¾*d.*; and 102¾*s.* = £5 2*s.* 9*d.*

£5 2*s.* 9*d.* × 9 =	£46	4*s.*	9*d.*
8*s.* 6¾*d.* × 4 =	1	14	3 *Subtract.*
	£44	10*s.*	6*d.*, the *Ans.*

4. 76½ gallons of rum, at 14*s.* 8½*d.* per gallon? 76½ ÷ 12 = 6, 4½ *rem.*

14*s.* 8½*d.* = 176½*d.*; and 176½*s.* = £8 16*s.* 6*d.*

£8 16*s.* 6*d.* × 6 =	£52	19*s.*	0*d.*
14*s.* 8½*d.* × 4½ = {	2	18	10 for the 4.
		7	4¼ for the ½.
	£56	5*s.*	2¼*d.*, the *Ans.*

5. 90 lbs of tobacco, at 3*s.* 6½*d.* per pound? *Ans.* £15 18*s.* 9*d.*
6. 47 cwt of flour, at 16*s.* 8½*d.* per cwt? *Ans.* £39 5*s.* 3½*d.*
7. 52 acres of land, at £1 3*s.* 6*d.* per acre? *Ans.* £61 2*s.*

The converse of this problem, which is—From the price of any number of articles, not a multiple of 12, to find the price of a single article,—does not admit of being worked

otherwise than by the general Rule at page 51. But when the given number *is* a multiple of 12, the following compendious method may be employed.

Problem 6. (Converse of Prob. 4.)

The price, in shillings, of a specified number of articles, being given (the number being a multiple of 12), *to find the price of one article.*

Rule.—Regard the shillings in the price as so many *pence*, and divide this number of pence by the number of twelves in the specified number of articles: the quotient will be the price of a single article.

For to get the price of one, by the ordinary rule, we must divide the price of the entire number of articles by that number. *Here* the dividend would be *shillings*, and the divisor a multiple of 12. But since (p. 55) we may take a twelfth part of each, we may regard the shillings in the dividend as *pence* (a penny being $\frac{1}{12}$ of 1s.), and the divisor as $\frac{1}{12}$th of the divisor actually given: and hence the rule.

Examples.

1. If 48 pairs of scissors cost £1 4*s*., what will one pair cost?
   As there are 4 twelves in 48, therefore 24*d*. $\div$ 4 = 6*d*., the *Ans.*
2. 72 yards of cloth cost £3 6*s*.; what is the cost per yard?
   72 = 12 $\times$ 6; therefore 66*d*. $\div$ 6 = 11*d*., the *Ans.*
3. 48 articles for £1 16*s*.: required the cost of one?
   48 = 12 $\times$ 4; therefore 36*d*. $\div$ 4 = 9*d*., the *Ans.*
4. 60 finger-plates for £7 10*s*.; what is the cost of one?
   60 = 12 $\times$ 5, therefore 150*d*. $\div$ 5 = 30*d*. = 2*s*. 6*d*., the *Ans.*
5. If 84 articles cost £7 13*s*. 6*d*., what is the cost of one?
   84 = 12 $\times$ 7; and £7 13*s*. 6*d*. = 153½*s*.; therefore,
   153½*d*. $\div$ 7 = 21¾*d*. + $\frac{5}{7}$*f*. = 1*s*. 10*d*., very nearly; the *Ans.*
6. If 48 articles cost £1 5*s*. 9*d*., what is the cost of one?
   48 = 12 $\times$ 4; and £1 5*s*. 9*d*. = 25¾*s*., and 25¾*d*. $\div$ 4 = 6½*d*. nearly; the *Ans.*

Problem 7.

To calculate the price of any number of articles, when the price of one of them is less than a shilling.

Rule.—Regard the number of articles as so many *pence*, and multiply this number by the *number* of pence in the price of one article.

This rule, it will be seen, differs but little in its expression

from that in common use; but instead of implicitly following the ordinary method, it will often be better, as here directed, to change the number of articles into so many pence, and the number of pence, in the price of one, into so many articles.

EXAMPLES.

1. Find the price of 48 lbs. at 9*d.* per lb.
   Here 4*s.* × 9 = £1 16*s.*, the *Ans.*
2. 84 lbs. at 7*d.* per lb? 7*s.* × 7 = £2 9*s.*, the *Ans.*
3. 132 lbs. at 11*d.* per lb.? 11*s.* × 11 = £6 1*s.*, the *Ans.*
4. 300 lbs., at 7*d.* per lb?
   300*d.* = £1 5*s.*, and £1 5*s.* × 7 = £8 15*s.*, the *Ans.*
5. 65¼ oz., at 5*d.* per oz?
   65¼*d.* = 5*s.* 5¼*d.*, which × 5 = £1 7*s.* 2¼*d.*, the *Ans.*
6. 99⅛ yards, at 4*d.* per yard?
   99⅛*d.* = 8*s.* 3⅛*d.*, 4 times which is £1 13*s.* 0½*d.*, the *Ans.*
7. 183⅝ yds., at 10*d.* per yd.?
   183⅝*d.* = 15*s.* 3⅝*d.*, and 10 times this is £7 13*s.* 0¼*d.*, the *Ans.*

[If the reader feel any difficulty about the fraction here, let him remember that $\frac{5}{8} \times 10 = \frac{5}{4} \times 5 = \frac{25}{4} = 6\frac{1}{4}$. See page 55.]

8. 96 yds. at 10¾*d.* per yard? 8*s.* × 10¾ = 86*s.* = £4 6*s.*, the *Ans.*
9. 75 lbs. at 9¾*d.* per lb? 75*d.* = 6*s.* 3*d.*; and *ten* times this is—

	£3	2*s.*	6*d.*
¼ of 6*s.* 3*d.* =		1	6¾ (*Subtract.*)
Ans.	£3	0*s.*	11¼*d.*

10. What will 2½ dozen of edging come to, at 3¾*d.* per yard?
    Take the price per yard at 4*d.*; then—

30*d.* =	2*s.*	6*d.*
		4
	10	0 = price of 2½ doz. at 4*d.*
¼ of 30*d.* =		7½ (*Subtract.*)
	9*s.*	4½*d.* = price at 3¾*d.*

We may proceed otherwise as follows: $3\frac{3}{4} = \frac{12}{4} + \frac{3}{4} = \frac{15}{4}$ (p. 56).

Then, $\frac{15 \times 30}{4} = \frac{15 \times 15}{2} = \frac{225}{2} = 112\frac{1}{2}$ (pence) = 9*s.* 4½*d.*;

or, which is a little shorter, $\frac{15 \times 30}{4} = \frac{450}{4} = 112\frac{1}{2}$.

We shall now give a few miscellaneous examples for the practice of the reader in the foregoing rules; and one or two of the more complicated of these we shall solve at length, chiefly as instances of the proper management of vulgar fractions; observing, however, that here, as well as in most of the worked examples that have preceded, certain easy steps and operations, actually put down, an expert calculator may readily work mentally, and so reduce the space occupied, and, in a slight degree, the time expended.

Miscellaneous Examples.

1. Required the price of $47\frac{1}{4}$ dozen of Brussels lace, at 9*s*. $10\frac{3}{16}$*d*. per yard.

Here we have to perform this operation, namely, $118\frac{3}{16}s. \times 47\frac{1}{4}$; and we proceed thus:—

$$\tfrac{3}{16}s. = \frac{9 \times 12}{16}d. = \frac{27}{4}d. = 6\tfrac{3}{4}d. \quad \text{Also, } 47 = 9 \times 5 + 2.$$

Price of 1 doz. =	118*s*.	$6\frac{3}{4}$*d*. × 2.	
(See Rule, p. 64.)		9	
	1067	$0\frac{3}{4}$	
		5	
	5335	$3\frac{3}{4}$	= price of 45 doz.
	237	$1\frac{1}{2}$	= " 2 doz.
	29	$7\frac{11}{16}$*	= " $\frac{1}{4}$ doz. (*Add.*)
	5602*s*.	$0\frac{15}{16}$*d*.	= price of $47\frac{1}{4}$ doz. = £280 2*s*. 1*d*.

We see that the denomination of the fraction last arrived at in the foregoing work is *sixteenths:* we must therefore change the denomination of each of the preceding fractions into sixteenths before they can all three be added together; these changes are seen at a glance to be $\frac{3}{4} = \frac{12}{16}$, and $\frac{1}{2} = \frac{8}{16}$; so that the sum of the three fractions is $\frac{31}{16} = 1\frac{15}{16}$.

2. $76\frac{3}{4}$ dozen of blonde lace, at 9*s*. $5\frac{7}{32}$*d*. per yard?

Here the work to be performed is $113\frac{7}{32}s. \times 76\frac{3}{4}$.

$$\text{Now, } \tfrac{7}{32}s. = \frac{7 \times 12}{32}d. = \frac{21}{8}d. = 2\tfrac{5}{8}d. \quad \text{Also, } 76\tfrac{3}{4} = 11 \times 7 - \tfrac{1}{4}.$$

* $\frac{1}{4}$ of $30\frac{3}{4}$*d*. is $7d. + \frac{1}{2}d. + \frac{3}{16}d. = 7d. + \frac{8}{16}d. + \frac{3}{16}d. = 7\frac{11}{16}d.$

Price of 1 doz. = 113*s.* $2\frac{5}{8}$*d.* $\quad$ [$\frac{5}{8} \times 11 = \frac{55}{8} = 6\frac{7}{8}$.]
$\quad\quad\quad\quad\quad\quad\quad\quad\quad$ 11

1245 $\quad 4\frac{7}{8}$
$\quad\quad\quad\quad$ 7

8717 $\quad 10\frac{1}{8}$ $\quad$ = price of 77 doz.
$\quad$ 28 $\quad 3\frac{21}{32}$ $\quad$ = $\quad$ " $\quad$ $\frac{1}{4}$ doz. (*Subtract.*)

2,0) 868,9*s.* $\quad 6\frac{15}{32}$*d.** = price of $76\frac{3}{4}$ doz.

or, £434 9*s.* $\quad 6\frac{15}{32}$*d.* $\quad$ [$\frac{15}{32}$*d.* = $\frac{1}{2}$*d.* very nearly.]

*** The statements within the brackets and in the foot-notes are needful hints only to those whose acquaintance with fractions is but slight.

3. $23\frac{1}{2}$ dozen of French lace, at 2*s.* $11\frac{5}{16}$*d.* per yard?

$\frac{5}{16}$*s.* = $\frac{5 \times 12}{16}$*d.* = $3\frac{3}{4}$*d.* And 35*s.* $3\frac{3}{4}$*d.* × (24 − $\frac{1}{2}$) is worked thus:

35*s.* $\quad 3\frac{3}{4}$*d.* = price of 1 doz.
$\quad\quad\quad$ 24 $\quad$ (See the work within brackets below.)

847 $\quad 6$ $\quad$ = $\quad$ " $\quad$ 24 doz.
$\quad$ 17 $\quad 7\frac{7}{8}$ $\quad$ = $\quad$ " $\quad$ $\frac{1}{2}$ doz. (*Subtract.*)

829*s.* $10\frac{1}{8}$*d.* = $\quad$ " $\quad$ $23\frac{1}{2}$ doz.

= £41 9*s.* $10\frac{1}{8}$*d.*, the answer. The $\frac{1}{8}$*d.* would, of course, be disregarded.

[$\frac{3}{4}$*d.* × 24 = 1*s.* 6*d.*; and 3*d.* × 24 = 6*s.*]

[35 × 24 = 70 × 12.]

[$\frac{1}{2}$ of $3\frac{3}{4}$*d.* = $1\frac{1}{2}$*d.* + $\frac{3}{8}$*d.* = $1\frac{4}{8}$*d.* + $\frac{3}{8}$*d.* = $1\frac{7}{8}$*d.*]

4. Required the cost of 150 silk mantles, at £1 3*s.* 9*d.* each?
   Ans. £178 2*s.* 6*d.*
5. If 11 dozen of wine glasses cost £4 19*s.*, what is the cost of one?
   Ans. 9*d.*
6. What is the value of $19\frac{1}{2}$ reams of paper, at 7*s.* $9\frac{3}{4}$*d.* per ream?
   Ans. £7 12*s.* $4\frac{1}{8}$*d.*
7. What is the value of $13\frac{3}{4}$ gallons of rum, at 15*s.* $9\frac{1}{2}$*d.* per gallon?
   Ans. £10 17*s.* $1\frac{5}{8}$*d.*
8. Required the price of 85 lbs. of beef, at $9\frac{3}{4}$*d.* per lb.?
   Ans. £3 9*s.* $0\frac{3}{4}$*d.*
9. If £1 4*s.* $9\frac{1}{2}$*d.* be paid for $42\frac{1}{2}$ lbs. of beef, what is the cost per lb.?
   Ans. 7*d.*
10. If £3 3*s.* $7\frac{3}{4}$*d.* be paid for $23\frac{1}{2}$ yards of silk, what is the cost per yard?
    Ans. 2*s.* $8\frac{1}{2}$*d.*
11. What is the price of $68\frac{1}{2}$ lbs. of worsted, at $17\frac{1}{2}$*d.* per lb.?
    Ans. £4 19*s.* $10\frac{3}{4}$*d.*

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* $\frac{1}{8} - \frac{21}{32} = \frac{4}{32} - \frac{21}{32}$; and, borrowing $\frac{32}{32}$, or 1, $\frac{36}{32} - \frac{21}{32} = \frac{15}{32}$; and the 1 borrowed must be carried to the 3.

12. What is the price of two dozen yards of Nottingham lace, at 2*s.* 8$\frac{1}{8}$*d.* per yard? *Ans.* £3 4*s.* 3*d.*
13. What is the value of 12 oz. of silver, at 4*s.* 7$\frac{3}{16}$*d.* per oz.? *Ans.* £2 15*s.* 2$\frac{1}{4}$*d.*
14. What is the price of 16$\frac{1}{4}$ doz. of lace, at 5*s.* 7$\frac{3}{8}$*d.* per yard? *Ans.* £54 9*s.* 10$\frac{1}{8}$*d.*
15. Calculate the value of 27$\frac{3}{4}$ doz. of lace, at 3*s.* 2$\frac{9}{32}$*d.* per yard. *Ans.* £53 2*s.* 3$\frac{21}{32}$*d.*
16. Find the value of 127$\frac{1}{2}$ doz. of lace, at 2*s.* 1$\frac{15}{16}$*d.* per yard. *Ans.* £165 7*s.* 0$\frac{3}{8}$*d.*
17. If a dozen yards cost £5 10*s.* 4$\frac{1}{2}$*d.*, what is the cost per yard? *Ans.* 9*s.* 2$\frac{3}{8}$*d.* [$\frac{3}{8}$*d.* = $\frac{1}{4}$*d.* + $\frac{1}{2}$*f.*].
18. If a dozen articles cost £2 7*s.* 8*d.*, what is the cost of one? *Ans.* 3*s.* 11$\frac{2}{3}$*d.* [$\frac{2}{3}$*d.* = $\frac{1}{2}$*d.* + $\frac{2}{3}$*f.*].

[Those sums of money, in the answers above, which involve fractions of a penny smaller than $\frac{1}{4}$*d.*, cannot be paid in existing coin; but the utmost practical accuracy will be attained by replacing every such small fraction by $\frac{1}{4}$*d.*, if the fraction equal or exceed half a farthing, and by omitting it altogether if it fall short of half a farthing. But, in purchasing articles in dozens, or in any large numbers, the fraction of a penny, in the price of the *single* article, must be retained in its integrity, as a small fraction of a penny, by numerous repetitions, may accumulate to a very appreciable sum.]

In the foregoing problems and examples, the articles concerned are reckoned in dozens; and, whenever the number of them is not an exact number of dozens, it has been shown how the excess or defect is to be calculated and allowed for. But in certain commercial transactions, the commodities are sold by the *gross* (twelve dozen), or by the *score* (twenty); and in others, by the 100 (five score), or by the 120 (six score). The following problems relate to purchases in which the articles are enumerated in one or other of these ways.

Problem 8.

To find the price of a gross, the price of one of the articles being given in pence.

Rule.—Regard the pence in the price of one article as so many shillings. Multiply these shillings by twelve, and the product will be the price of a gross.

For, by taking the pence as so many shillings, the price is,

in effect, multiplied by 12; so that, by again multiplying by 12, the product will be 144 times the price of one article; that is, it will be the price of twelve dozen, or a gross.

EXAMPLES.

1. A gross, at 8¼*d.* each article, is 8*s.* 3*d.* × 12 = 99*s.* = £4 19*s.*
2. A gross, at 9½*d.* each, is 9*s.* 6*d.* × 12 = 114*s.* = £5 14*s.*
3. A gross, at 11¾*d.* each, is 11*s.* 9*d.* × 12 = 141*s.* = £7 1*s.*
4. A gross, at 13½*d.* each, is 13*s.* 6*d.* × 12 = 162*s.* = £8 2*s.*
5. A gross, at 19¼*d.* each, is 19*s.* 3*d.* × 12 = 231*s.* = £11 11*s.*
6. A gross, at 23¾*d.* each, is £1 3*s.* 9*d.* × 12 = £14 5*s.*

PROBLEM 9.

To find the price of a score, the price of one article being given in shillings.

RULE.—Regard the shillings as so many pounds; and we shall thus have the price, in pounds, of a score, as is obvious. [See the remarks within brackets on next page.]

EXAMPLES.

1. A score, at 19*s.* 9*d.* each, is £19¾ = £19 15*s.*
   [9*d.* = ¾*s.*, and £¾ = 15*s.*]
2. A score, at 11*s.* 6*d.* each, is £11½ = £11 10*s.*
3. A score, at 39*s.* 9*d.* each, is £39¾ = £39 15*s.*
4. A score, at £2 7*s.* 6*d.* each, is £47 10*s.*
5. Sixty chairs, at £1 2*s.* 9*d.* each, is £22¾ × 3 = £68¼ = £68 5*s.*
6. One hundred spoons, at 11*s.* 4*d.*? £11⅓ × 5 = £56⅔ = £56 13*s.* 4*d.*

In the foregoing examples, the price of the single article is expressed in shillings and pence only, without farthings. If farthings enter, we must increase the price of a score by an additional 5*d.* for each farthing; because 20 farthings = 5*d.* Thus, taking the examples above—

1. The price, at 19*s.* 9¼*d.*, is £19 15*s.* 5*d.*
2. The price, at 11*s.* 6½*d.*, is £11 10*s.* 10*d.*
3. The price, at 39*s.* 9¾*d.*, is £39 16*s.* 3*d.*
4. The price, at £2 7*s.* 6¾*d.*, is £47 11*s.* 3*d.*
5. The price, at £1 2*s.* 9½*d.*, is £68 7*s.* 6*d.*
6. The price, at 11*s.* 4¼*d.*, is £56 15*s.* 5*d.*

Should the number of articles be exactly TWELVE SCORE, or 240, we may compute somewhat differently. The price of *one* score being as many pounds as there are *shillings* in the

price of the single article, the price of *twelve* score must be as many pounds as there are *pence* in the price of the single article; so that we have only got to convert this latter price into pence in order to get the number of pounds in the price of twelve score: thus,

7. 240 yds., at 19¼*d.* per yd., is £19 5*s.*

[At this price per yard, it would be more troublesome to find the price of 20 yards than to find the price of 240. Following the foregoing directions, we should express the price of a score, at 1*s.* 7¼*d.*, thus:

$$£1\frac{7}{12} + 5d.\text{; and } £\frac{7}{12} \text{ being } \frac{140}{12}s. = 11s.\ 8d.,$$

the price is £1 12*s.* 1*d.* We thus see that when the *pence* connected with the shillings, in the price of a single article, are neither ¼ th, ½, nor ¾ths, of a shilling, the calculation of the price of a score may be a little troublesome; and that it will, in general, be easier, in such a case, to calculate, as above, for 12 score; and then to divide by 12: thus,

$$\begin{array}{l} \quad\ \ £ \quad\ s. \\ 12)\ 19 \quad 5 = \text{price of 12 score.} \\ \hline £1 \quad 12s. \quad 1d. \quad ,, \quad 1 \text{ score.} \end{array}$$

And this fact deserves the special attention of persons engaged in calculations of this kind.]

8. 240 articles at 7*s.* 7¾*d.* each? £91¾ = £91 15*s.* *Ans.*

For one score or 20, we have,

$$\begin{array}{l} \quad\ \ £ \quad s. \\ 12)\ 91 \quad 15 \\ \hline £7 \quad 12s. \quad 11d. \end{array}$$

By the Rule, the price of 20 is calculated thus—

$$£7\tfrac{7}{12} + 1s.\ 3d. = £7\ 11s.\ 8d. + 1s.\ 3d. = £7\ 12s.\ 11d.$$

9. What is the price of 243 quarters of oats, at 54*s.* 9¼*d* per qr.?

$$\begin{array}{rl} 54s.\ 9\tfrac{1}{4}d. = 657\tfrac{1}{4}d.\text{; and} & \\ £657\tfrac{1}{4} = 657 \quad 5s. & = \text{price of 240.} \\ £2\ 14s.\ 9\tfrac{1}{4}d. \times 3 = \quad 8 \quad 4 \quad 3\tfrac{3}{4}d. & = \text{price of 3.} \\ \hline £665 \quad 9s. \quad 3\tfrac{3}{4}d. & = \text{price of 243.} \end{array}$$

Otherwise:

£54¾ + 5*d*. =	£54	15*s*.	5*d*. one score.
			12
	£657	5*s*.	0*d*.
£2 14*s*. 9¼*d*. × 3 =	8	4	3¾*d*.
	£665	9*s*.	3¾*d*.

We need scarcely observe, in reference to the second method, that the multiplication of the *pence* by 12 need never be actually performed; since we see, at once, that the product is the same number of shillings as the number of pence.

10. What is the price of 301 yards of silk, at 17*s*. 9½*d*. per yard?

17*s*. 9½*d*. = 213½*d*.

£213½ =	£213	10*s*.		= price of 240.
¼ of £213 10*s*. =	53	7	6*d*	= price of 60.
		17	9½*d*.	= price of 1.
	£267	15*s*.	3½*d*.	= price of 301.

Otherwise:

£17¾ + 10*d*. =	£17	15*s*.	10*d*., one score.
			10
Price of 200 =	177	18	4
Price of 100 =	88	19	2
Price of 1 =		17	9½
Price of 301 =	£267	15*s*.	3½*d*.

11. 240 yards of cloth, at 16*s*. 7¾*d*. per yard? *Ans*. £199 15*s*.
12. 260 articles, at 23*s*. 7¼*d*. each? *Ans*. £306 17*s*. 1*d*.
13. 280 yards, at 27*s*. 9¾*d*. per yard? *Ans*. £389 7*s*. 6*d*.
14. 400 yards of silk velvet, at 21*s*. 7¾*d*. per yd.? *Ans*. £432 18*s*. 4*d*.
15. 720 lbs., at 7¼*d*. per lb.? *Ans*. £21 15*s*.
16. 960 stone, at 10½*d*. per stone? *Ans*. £42.
17. 1440 lbs., at 3*s*. 9*d*. per lb.? *Ans*. £270.
18. 1680 lbs., at 5*s*. 5½*d*. per lb.? *Ans*. £458 10*s*.
19. 967 lbs., at 4½*d*. per lb.? *Ans*. £18 2*s*. 7½*d*.
20. 1199¾ lbs., at 5*s*. 5½*d*. per lb.? *Ans*. £327 8*s*. 7⅝*d*.

PROBLEM 10.

To find the price of a score, when the price of one article is given in pence.

This problem has been, in substance, anticipated in the observations within the brackets at page 70, but as no *Rule* is formally given in that place, we supply the rule here.

RULE I.—Regard the pence as so many pounds, and divide that sum by 12: the result will be the price of a score.

EXAMPLES.

1. A score, at $5\frac{1}{4}d.$ each, is £5 5*s.* ÷ 12 = 105*s.* ÷ 12 = 8*s.* 9*d.*
2. A score, at $13\frac{1}{2}d.$ each, is £13 10*s.* ÷ 12 = 270*s.* ÷ 12 = 22*s.* 6*d.*
3. Three score, at $7\frac{3}{4}d.$, is £7 15*s.* × 3 ÷ 12 = £23 5*s.* ÷ 12 = £24 ÷ 12, *less* 1*s.* 3*d.* = £1 18*s.* 9*d.* [15*s.* ÷ 12 = 1*s.* 3*d.*]
   Or: £7 15*s.* × $\frac{3}{12}$ = £7 15*s.* ÷ 4 = £1 18*s.* 9*d.*

As there is little or no advantage in this, over the common method, we shall not exemplify it further. Indeed, the common method, when slightly modified as follows, seems to be the preferable of the two.

RULE II.—Multiply the pence by 20, and add 5*d.* for every farthing: the result will be the price in *pence:* thus, taking the preceding examples, in order.

1. 5*d.* × 20 + 5*d.* = 105*d.* = 8*s.* 9*d.*
2. 13*d.* × 20 + 10*d.* = 270*d.* = 22*s.* 6*d.*
3. 7*d.* × 20 + 15*d.* = 155*d.* = 12*s.* 11*d.*, which multiplied by 3 gives £1 18*s.* 9*d.*

NOTE.—When the price of the single article consists of shillings and pence, the *pence* may be calculated for by one or other of these rules; and then the shillings, taken as *pounds*, be prefixed: for example, if the price of one article were 3*s.* $5\frac{1}{4}d.$, the price of a score of the articles would be £3 8*s.* 9*d.* (See Ex. 1 above.) Again: in Ex. 2, the price of 1 is 1*s.* $1\frac{1}{2}d.$: at $1\frac{1}{2}d.$ *only*, the price of 20 is seen at once to be 2*s.* 6*d.*: hence, at 1*s.* $1\frac{1}{2}d.$, the price of a score is £1 2*s.* 6*d.*

PROBLEM 11.

To find the price of 100 *articles, when the price of one of them is given.*

RULE.—1. Multiply the shillings (if there be any) in the given price, by 5; the result will be *pounds.*

2. For every farthing in the pence, take twice as many shillings, and once as many pence; their sum added to the pounds will be the price required.

The reason is this:—Twenty times any number of shillings are so many pounds; and therefore 100 times that number of shillings must be 5 times that number in pounds. Again: taking the farthings for so many pence, is the same as multiplying the farthings by 4; and taking each of these farthings for 2*s*., or 96 farthings, is the same as multiplying them by 96; so that, working in this way, we multiply the farthings by 4 + 96, that is, by 100, as we are required to do.

Examples.

1. 100 articles at 2*s*. 3¼*d*. each?

In the pence here, there are 13 farthings: hence, by the rule, we have,

	£	*s*.	*d*.
For the 2*s*. . . .	10	0	0
For the 13 *far*. .	1	6	0
		1	1
Ans.	£11	7*s*.	1*d*.

2. 100 at 17*s*. 10¼*d*. each?

	£	*s*.	*d*.
For the 17*s*. . . .	85	0	0
For the 41 *far*. .	4	2	0
		3	5
Ans.	£89	5*s*.	5*d*.

3. 100 yards, at 5¾*d*. per yd?

	£	*s*.	*d*.
For 23 *far*. . . .	2	6	0
		1	11
Ans.	£2	7*s*.	11*d*.

4. 100 articles at 7*s*. 6¾*d*. each?

	£	*s*.	*d*.
For the 7*s*. . . .	35	0	0
For the 27 *far*. .	2	14	0
		2	3
Ans.	£37	16*s*.	3*d*.

5. 100 at 14½*d*. each?
£5 + 20*s*. + 10*d*. =
£6 0*s*. 10*d*. *Ans*.

6. 100 copy-books, at 4½*d*. each? *Ans*. £1 17*s*. 6*d*.
7. 100 quarts of vinegar at 1*s*. 3¾*d*. per quart? *Ans*. £6 11*s*. 3*d*.
8. 100 articles at £2 7*s*. 6*d*. each? *Ans*. £239 10*s*.
9. 100 at 11*s*. 4*d*. each? *Ans*. £283 6*s*. 8*d*.
10. 100 at 2*s*. 4¼*d*. each? *Ans*. £11 15*s*. 5*d*.

If we have to perform the *reverse* operation, namely, from having the price of 100 articles, to find the price of one of them, the common method of working is as convenient and as expeditious as can be desired; that is, taking the converse of Ex. 4, the method exhibited in the margin; which gives for result, 7*s*. 6¾*d*. (See Problem 13.)

£	*s*.	*d*.
37	16	3
20		
7,56		
12		
6,75		
4		
3,00		

Problem 12.

To find the price of 120 *articles, the price of one of them being given.*

Rule.—Half the number of *pence* in the price of one, taken as so many *pounds*, will be the price in £ of 120. For, as already shown (page 70), the *whole* number of pence, taken as so many pounds, is the price of 240.

Examples.

1. What is the price of six score lambs, at 12*s.* 6*d.* each?
   £150 ÷ 2 = £75, *Ans.*
2. 120 articles, at 3*s.* 8¾*d.* each? £44 15*s.* ÷ 2 = £22 7*s.* 6*d.*, *Ans.*
3. 120, at 11*s.* 5¼*d.* each? £137 5*s.* ÷ 2 = £68 12*s.* 6*d.*, *Ans.*
4. 120, at £1 2*s.* 7½*d.*? £120 + ½ (£31 10*s.*) = £135 15*s.*, *Ans.*
5. What is the price of six score flower-pots, at 4*d.* each? *Ans.* £2.
6. What is the price of six score brass finger-plates, at 2*s.* 6*d.* each? *Ans.* £15.
7. 120 pair of gloves, at 2*s.* 3½*d.* per pair? (See Ex. 5 p. 62.) *Ans.* £13 15*s.*
8. 120 gallons of rum, at 13*s.* 10*d.* per gal? (See Ex. 6 p. 62.) *Ans.* £83.

For the *reverse* problem, that is, to find the price of one from the price of 120 being given, we may proceed thus: take 5 times the given price, and, for the moment, regard the product as the price of 100 articles, and compute for *one*, as shown above, and then divide the result by 6. Thus, taking the converse of Ex. 2, the work is that in the margin. [See, however, Prob. 13.]

£	s.	d.
22	7	6
		5
111	17	6
20		
22,37		
12		
4,50		
4		
2,00		
6) 22*s.* 4½*d.*		
3*s.* 8¾*d.*		

The reason of this is, that $\frac{5}{6}$ths of 120 is 100, so that any sum divided by 120 is the same as $\frac{5}{6}$ths of that sum divided by 100; that is, the quotient in the former case is $\frac{5}{6}$ths of the quotient in the latter case.

It is as well to mention here that when, in either of the two reverse operations in the margin above, the figure cut off for farthings leaves a significant number on the right, instead of ciphers as in the foregoing results, that number in the first case expresses so many hundredths of a farthing, and in the second case so many 120ths.: we thus get the price of the single article to

the minutest accuracy, and can then increase or diminish the exact price, thus determined, by a farthing, according as this overplus fraction is greater or less than half a farthing; that is, according as the number cut off to the right is greater or less than 50, in the first case (p. 73), or greater or less than 60, in the second case. But whenever such extreme accuracy is considered to be unnecessary, and the price of the 100 or of the 120 is in *pounds*, the price of *one*, within a farthing of error, may be quickly found by the following rule.

Problem 13.

The price of 100 *or of* 120 *being given in pounds, to find the price of one either exactly, or to the nearest farthing.*

Rule.—1. For 100. Take the price in pounds as so many shillings, and divide by 5. [For this is the same as dividing by 20 and 5.]

2. For 120. Take the price in pounds as so many shillings, and divide by 6. [For this is the same as dividing by 20 and 6.]

Examples.

1. If 100 articles cost £28 10*s*., what is the cost of one?
   28*s*. 6*d*. $\div$ 5 = 5*s*. 8$\frac{2}{5}$*d*. exactly.
2. If 100 cost £37 16*s*. 3*d*., what is the cost of one? [This is worked to strict accuracy in the margin at page 73.]

That the given sum may be expressed in pounds (*nearly*), without an inconvenient fraction, we may regard it as £37 15*s*. = £37$\frac{3}{4}$; then 37*s*. 9*d*. $\div$ 5 = 7*s*. 6$\frac{3}{5}$*d*., exact to the nearest farthing; for $\frac{3}{4}$ differs from $\frac{3}{5}$ only by $\frac{3}{4} - \frac{3}{5} = \frac{15}{20} - \frac{12}{20} = \frac{3}{20}$ths of a penny: we therefore replace $\frac{3}{5}$*d*. by $\frac{3}{4}$*d*.

3. If 120 cost £22 7*s*. 6*d*. what will be the cost of one? (See Example 2, p. 74.)

We see by the Table (p. 57), that 6*s*. 8*d*. $\doteq$ £$\frac{1}{3}$, and that 10*d*. = £$\frac{1}{24}$. Also that $\frac{1}{3}$*s*. + $\frac{1}{24}$*s*. = 4$\frac{1}{2}$*d*.: hence 22*s*. 4$\frac{1}{2}$*d*. $\div$ 6 = 3*s*. 8$\frac{3}{4}$*d*., the *Ans.*

4. If 120 cost £68 12*s*. 6*d*., required the cost of one? (See Ex. 3, p. 74.)

Taking the sum to be £68$\frac{1}{2}$, we have 68*s*. 6*d*. $\div$ 6 = 11*s*. 5*d*., a farthing short; but we have taken the given cost £$\frac{1}{8}$ too little; and $\frac{1}{8}$*s*. = 1$\frac{1}{2}$*d*., $\frac{1}{6}$th of which gives the wanting farthing.

Note.—It is plain that in order to convert the shillings and pence in the given sum into a convenient fraction of a pound, we need never increase or diminish these shillings and pence by more than (nor even by so much as) 2*s*. 6*d*. ($\frac{1}{8}$th of a pound), so that we shall never have to allow more than $\frac{1}{4}$*d*. for the excess or defect, whether there be 100 articles or 120; for in the former case $\frac{1}{5}$th of 1$\frac{1}{2}$*d*. differs from a farthing

by only $\frac{1}{4}$th of a farthing, and in the latter case $\frac{1}{6}$th of $1\frac{1}{2}d.$ is a farthing exactly. In Example 3, above, since 7*s.* 6*d.* is three half-crowns, it is £$\frac{3}{8}$, so that the given sum is exactly £$22\frac{3}{8}$, and therefore the *exact* price of the single article is, as stated, 22*s.* $4\frac{1}{2}d.$ ÷ 6 = 3*s.* $8\frac{3}{4}d.$ In like manner, in Example 4, the given sum is exactly £$68\frac{3}{8}$; and 68*s.* $7\frac{1}{2}d$ ÷ 6 = 11*s.* $5\frac{1}{4}d.$, the price exactly. We have worked this example, and Example 2, as above, in order that the reader may clearly see how inconsiderable is the effect, upon the result, by taking the given sum either in excess or in defect to the extent of even half-a-crown; for which extreme sum only a farthing is to be allowed as correction of the error.

It will be of service to the retailer to recollect that if he buy articles by the long hundred (six score) he pays for each article as many farthings as he lays out half-crowns; and that if he buy by the common hundred (five score) he pays at the rate of $1\frac{1}{2}d.$ for *five* articles, for every half-crown he lays out. For 120 farthings = 30*d.* = 2*s.* 6*d.*; and $1\frac{1}{2}d.$ × 20 = 30*d.* = 2*s.* 6*d.*; and in 100 there are 20 *fives.* [Ex. 7, below: £13 15*s.* = 110 half-crowns; and 110 far. = 2*s.* $3\frac{1}{2}d.$]

5. If 100 cost £56 13*s.* 4*d.*, what is the cost of one? *Ans.* 11*s.* 4*d.*
6. If 100 cost £6 11*s.* 3*d.*, what does one cost? *Ans.* 1*s.* $3\frac{3}{4}d.$
7. If 120 cost £13 15*s.*, what is the cost of one? *Ans.* 2*s.* $3\frac{1}{2}d.$
8. If 120 cost £135 10*s.*, what is the cost of one? *Ans.* £1 2*s.* $7\frac{1}{2}d.$

Problem 14.

To find the price of 1000, *the price of one being given in* PENCE.

Rule.—Regard the price in pence as *pounds*, and multiply these pounds by $4\frac{1}{6}$: the product will be the price of 1000.

For the pence, taken as pounds, gives the price of 240, (page 70). Four times 240 is 960; and $\frac{1}{6}$th of 240, that is, 40 added to this, gives 1000. [It may be useful to recollect that 1000, at 1*d.*, amount to £4 3*s.* 4*d.*, and therefore, at 2*d.*, to £8 6*s.* 8*d.*; and so on.]

Examples.

1. At $1\frac{3}{4}d.$ per yard, what will 1000 yards cost?
   £$1\frac{3}{4}$ is £1 15*s.*, which × $4\frac{1}{6}$ = £7 5*s.* 10*d.*, the *Ans.*
2. At 7*s.* 9*d.* per yard, 1000 yards is
   £93 × $4\frac{1}{6}$ = £372 + £15 10*s.* = £387 10*s.*

Note.—In this example we have written down, first the product by 4, and have then connected with it the product by $\frac{1}{6}$, making two separate amounts, which are then added into one sum. But in the first example we have written this final sum at once, since we could readily determine it *without* thus calculating the two component parts

of it separately. We took, for the multiplier 4 only, not the £1 15s., but £1¾, and wrote £7 for the product; we then divided the *equivalent* of this £1¾, namely, 35s. by 6, and subjoined the resulting 5s. 10d. to the previously written £7. And advantage should always be thus taken of every little peculiarity favourable to the abbreviation of the figure-work.

3. At 2s. 7½d. per yard, what will 1000 yards cost?
   £31¼ = £31 10s., which × 4⅙ is £126 + £5 5s. = £131 5s., *Ans.*
   Here 31½ × 4 = 126.
4. At 14s. 7d. per gallon, what will 1000 gallons cost?
   £175 × 4⅙ = £700 + £29 3s. 4d. = £729 3s. 4d. *Ans.*
5. 1000 yards, at 3½d.? *Ans.* £14 11s. 8d.
6. 1000 gallons, at 3s. 10½d.? *Ans.* £193 15s.
7. 1000 at 1s. 3¾d. each? *Ans.* £65 15s.
8. 1000 at 5¼d. each? *Ans.* £21 17s. 6d.
9. 1000 at 3s. 7¾d. each? *Ans.* £182 5s. 10d.
10. 1000 at 1s. 11¾d. each? *Ans.* £98 19s. 2d.

NOTE.—We might compute this example for 2s.; and knowing that 960 farthings make £1, and that 40 make 10d., we should have to subtract £1 0s. 10d. from the result.

PROBLEM 15 (Converse of Prob. 14).

The price of 1000 *being given in* POUNDS, *to find the price of one.*

RULE.—Regard the price in pounds as so many pence, and divide these pence by 4⅙. Or, which is the same thing, multiply the pounds, taken as pence, by 6, and divide the result by 25. For $4\frac{1}{6} = \frac{25}{6}$; and to divide by a fraction is to turn the fraction upside down (or to reverse its terms) and multiply; so that to divide by 4⅙, or by $\frac{25}{6}$, is to multiply by $\frac{6}{25}$.

EXAMPLES.

1. If 1000 cost £25, what is the cost of one?
   25d. × $\frac{6}{25}$ = 6d., the *Ans.*
2. If 1000 cost £387 10s., what is the cost of one?
   387½d. × 6 = 2325d.
   5) 2325d.
   5) 465
   93d. = 7s. 9d., *Ans.*

3. If 1000 cost £131 5s., what is the cost of one?
   131¼d. × 6 = 787½d.
   5) 1575 halfpence.
   5) 315
   63 = 31½d. = 2s. 7½d., *Ans.*

4. If 1000 cost £729 3*s.* 4*d.*, what will one cost?

$729\frac{1}{6}d. \times 6 = 4375d.$

$$\begin{array}{r} 5)\ 4375 \\ \hline 5)\ 875 \\ \hline \textit{Ans. } 175d. = 14s.\ 7d. \end{array}$$

5. If 1000 cost £182 5*s.* 10*d.*, what will one cost?

Omit the 10*d.* for the present; then we have—

$182\frac{1}{4}d. \times 6 = 1093\frac{1}{2}d.$

$$\begin{array}{r} 5)\ 2187 \text{ halfpence.} \\ \hline 5)\ 437,2 \text{ rem.} \\ \hline 87,2 \text{ rem.} \end{array}$$

that is, $43\frac{1}{2}d. + \frac{12}{25}$ of 1 halfpenny or $\frac{24}{25}$ far.

Now 1000th part of the omitted 10*d.* is $\frac{1}{100}d. = \frac{4}{100}$ far., and $\frac{24}{25}$ far. $= \frac{96}{100}$ far.

The sum of these two is $\frac{100}{100}$ far. = 1 farthing: hence the price required is $43\frac{3}{4}d. = 3s.\ 7\frac{3}{4}d.$ (See Ex. 9, p. 77.)

It is obvious that this supplementary work for the accurate estimate of the influence of the omitted 10*d.*, and of the fraction $\frac{24}{25}f.$, upon the cost price of one of the articles, might have been dispensed with. The antecedent result, namely, $43\frac{1}{2}d. + \frac{24}{25}f.$, is sufficiently conclusive that the sought price, per article, is either $43\frac{3}{4}d.$ *exactly*, or else this sum to the nearest farthing. No account at all need ever be taken of the odd pence in the wholesale price; nor even of the shillings, below 10*s.*, if the object be only to find the cost price, per article, to the nearest farthing. For we have just seen that the influence of 10*d.* on this cost price is but $\frac{4}{100}f.$: so that the influence of 12 times 10*d.*, or 10*s.*, would be only $\frac{48}{100}f.$, which is *less* than half a farthing. Hence when the prime cost of 1000 articles is so many pounds and shillings, if the shillings do not exceed 10*s.*, we may disregard them, and may calculate the prime cost of a single article to the nearest farthing, by taking account of the *pounds* only in the cost of the 1000; and if the shillings *do* exceed 10*s.*, then we have only to add 1 to the number of pounds, and to calculate as above.

The principal object in finding the prime cost of each one of a large number of articles, purchased wholesale, is to guide the retail dealer as to what he ought to charge, per article, selling them singly, in order to realize what he considers a reasonable profit upon his outlay: the way to find the selling price, per article, with a view to such profit, will be shown in the next problem.

Let us now return to Ex. 5, disregarding not only the 10*d.*, but also the 5*s.*: the work will then be as follows:—

$182d. \times 6 = 1092d.$; this $\div 5 = 218d.$, with 2 for remainder; and

218*d.* ÷ 5 = 43*d.*, with 3 for remainder: hence the complete remainder, arising from the division by 25, is 17; so that the complete result is, $43\frac{17}{25}d. = 43\frac{3}{4}d.$, to the nearest farthing; that is, it is impossible that this result can be so much as a farthing in error; for $\frac{17}{25}d.$ is itself more than $\frac{1}{2}d.$, it is in fact, $\frac{1}{2}d.$ and $\frac{4\frac{1}{2}}{25}d.$, that is, $\frac{18}{25}f.$, besides; and then there is the omitted 5*s.* 10*d.* unaccounted for.

The reader is expected to work the subjoined examples in accordance with what has now been said, always assuming *pounds* only, for the cost price, without shillings or pence.

Thus, in Examples 6, 8, and 10, the cost price of the 1000 is to be regarded as £165, £388, and £18, respectively; and in the other examples the odd shillings and pence are to be omitted.

6. If 1000 articles cost £164 18*s.*, what is the cost of one, to the nearest farthing? *Ans.* 3*s.* 3½*d.*
7. If 1000 cost £325 9*s.*, what is the cost of one, to the nearest farthing? *Ans.* 7*s.* 6*d.*
8. If 1000 cost £387 16*s.*, what is the cost of one, to the nearest farthing? *Ans.* 7*s.* 9*d.*
9. If 1000 cost £729 6*s.* 6*d.*, what is the cost of one, to the nearest farthing? *Ans.* 14*s.* 7*d.*
10. If 1000 cost £17 18*s.* 9*d.*, what is the cost of one, to the nearest farthing? *Ans.* 4¼*d.*

Problem 16.

From knowing the cost-price of any number of articles, to find the selling-price, per article, so that a certain amount of profit may be realized.

A trader accustomed to purchase the articles in which he deals by the 100, or the 120, or the 1000, &c., can form a pretty close guess, from practice, of the prime cost of one of the articles from his outlay for the entire number; but even should his approximation be comparatively wide of the truth, it will still suffice for his purpose. In proof of this, let us return to Example 5, of last problem, where £182 5*s.* 10*d.* is supposed to be paid for 1000 articles; and let us assume that the purchaser estimates the prime cost, per article, at about 3*s.* 6*d.*: his business will then be to find, by problem

14, the price of 1000 at this rate; that is, he will have to calculate thus:

$$£42 \times 4\tfrac{1}{6} = £168 + £7 = £175.$$

He would in this way find that 3*s*. 6*d*. per article is *less* than the cost price; that, in fact, he has paid, for the 1000, more by £7 5*s*. 10*d*. than he would have paid if the price had been only 3*s*. 6*d*. per article: his aim would therefore now be gradually to increase this 3*s*. 6*d*. till it amounts to a retail price sufficiently high to cover the outlay and give a reasonable profit besides. Now as 1000 farthings is £1 0*s*. 10*d*., it is only necessary to see how many times this sum, when added to £175, will produce an amount *above* the outlay, £182 5*s*. 10*d*., sufficient for the profit proposed. It is plain, at a glance, that 6 times £1 0*s*. 10*d*. will not be sufficient; taking 7 times, we have, at the rate of 3*s*. 7¾*d*. each, the price of 1000 = £175 + £7 5*s*. 10*d*. = £182 5*s*. 10*d*. It thus so happens, in this particular case, that the cost price, per article, is an exact sum, without fractions of a farthing, namely, 3*s*. 7¾*d*.; and we may here notice, that whenever such is the case, we may, in this manner, *always* arrive at the cost price.

Adding now farthing after farthing to the 3*s*. 7¾*d*., we see that the profit on the 1000, at 3*s*. 8*d*. each, is £1 0*s*. 10*d*.; at 3*s*. 8¼*d*., it is £2 1*s*. 8*d*.; at 3*s*. 9*d*. (that is, adding five farthings), it is £5 4*s*. 2*d*., and so on; £1 0*s*. 10*d*. being the additional profit for every additional farthing. If it had so happened that 3*s*. 7¾*d*. was *not* the exact price, as would have been the case if the outlay had been (say) £182 only, then, with the profits here determined, there would have been, in each case, 5*s*. 10*d*. profit besides.

It is obvious that whatever be the number of articles purchased wholesale for a given sum, the profit to the retailer upon his outlay, by selling the articles singly, at any assigned price, may in this way always be easily determined. If the estimated cost price, per article, be but a few pence, we may, as above, readily find the profit to be realized by increasing this estimated cost price by any number of farthings; or if the cost price be considerable, by any number of pence, or even shillings; the general rule being this:—

Rule.—Having roughly estimated the prime cost of the

single article, find, by the suitable rule (among the rules already given), the exact cost of the entire number, at this estimated price.

Then, since every farthing, halfpenny, penny, &c., added to the price of one article, adds as many farthings, halfpence, pence, &c., as there are articles, to the price of the whole, we can easily find what must be the increase, in the above estimated cost price of one, in order that the price of the whole, at that rate per article, may *exceed* the actual outlay by the desired amount of profit; as in the following examples.

Examples.

1. If a dozen articles cost £2 7*s*. 8*d*., what is the lowest retail price, per article, that will produce a profit upon the outlay of not less than 10*s*. 6*d*.? (See Ex. 18, p. 68.)

Take the estimated cost price, per article, at 4*s*.; then—

at 4*s*., the price of 12 would be		£2 8*s*.
„ 4*s*. 6*d*.	,, ,,	£2 14*s*.
„ 4*s*. 9*d*.	,, ,,	£2 17*s*.
„ 4*s*. 10½*d*.	,, ,,	£2 18*s*. 6*d*.

and £2 18*s*. 6*d*. *less* £2 7*s*. 8*d*. = 10*s*. 10*d*.; so that, to gain this profit, the articles must be sold at 4*s*. 10½*d*. each. Of course, from 4*s*. to 4*s*. 10½*d*. might be reached at once by adding 10½*s*. to the £2 8*s*.

2. If 150 articles cost £178 2*s*. 6*d*., what is the lowest price at which each must be sold, so that the retailer may realize at least £12 profit upon his outlay? (See Ex. 4, p. 67.)

Take the estimated cost price, per article, at £1; then, bearing in mind that 150*s*. = £7 10*s*., we see that

at £1, the cost would be	£150			
„ £1 5*s*. ,, ,,	£187	10*s*.		
„ £1 5*s*. 6*d*. ,, ,,	£191	5*s*.		
	£178	2*s*.	6*d*.	(Subtract.)
	£13	2*s*.	6*d*.	Profit at £1 5*s*. 6*d*.

If we take 150*d*. = 12*s*. 6*d*. from this there will remain £12 10*s*.; hence for *this* profit, the retail price must be £1 5*s*. 5*d*.; or, deducting 6*s*. 3*d*. (the half of 12*s*. 6*d*.) from the £12 10*s*., there will remain £12 3*s*. 9*d*., the profit when the retail price is £1 5*s*. 4½*d*.; and by the deduction of another farthing from the retail price, the profit is reduced to £12 0*s*. 7½*d*. And in this way may the increase or decrease of profit, consequent upon any increase or decrease in the retail price per article, be always readily found.

3. If 19½ reams of paper be purchased for £7 12*s*. 4*d*., at what price, per ream, must it be sold, in order that about 30*s*. profit upon the outlay may be realized? (See Ex. 6, p. 67.)

Take the estimated cost price at 8*s*. per ream: then,

	£	s.	d.
at 8*s*., the cost price of 19½ reams would be	7	16	0
„ 9*s*. „ „ „	8	15	6
„ 9*s*. 6*d*. „ „ „	9	5	3
Subtract	7	12	4
Profit, at 9*s*. 6*d*. per ream,	£1	12*s*.	11*d*.

Taking 19½*d*. from this, the profit, at 9*s*. 5*d*. per ream, will be £1 11*s*. 3½*d*, which is probably near enough; but if the retail price be further reduced ½*d*., the profit would be 9¾*d*. less, namely, £1 10*s*. 5¾*d*. And at 9*s*. 4¼*d*., it would be £1 10*s*. 1*d*. Of course there is another way of working questions of this kind: we may add the proposed profit to the outlay, and then, either by Prob. 6, or by common division of this sum by the number of articles, find the selling price of one of them.

But by proceeding otherwise than as above, fractions of a farthing—things having no representation in current coin—are not unlikely to occur in the result; and the neglect of the fraction, though in an individual case of but very trifling consequence, yet, in a large number of sales, by the single article, such neglect may cause a deficiency of considerable amount.

For variety, we shall here work the foregoing example by ordinary division.

£	s.	d.	
7	12	4	Cost price of 19½ reams.
1	10	0	Profit to be made.
9	2	4	Selling price of the 19½ reams.

```
     9
    20
   ---
   182
    12
  ----
  2188
     2
  ----
39) 4376 (112 8/39 d.
    39
    --
     47
     39
     --
      86
      78
      --
       8
```

[Both divisor and dividend are doubled, in order to do away with the fraction ½ in the former.]

Therefore reducing the fraction of a penny to that of a farthing, we have, for the selling price, 9*s*. 4*d*. + $\frac{32}{39}$*f*., or 9*s*. 4¼*d*. all but $\frac{7}{39}$*f*.

4. If 83 yards of lace cost £53 19*s*., what must it be sold at per yard for the profit on the outlay to be £5 9*s*.? *Ans*. 14*s*. $3\frac{3}{4}d$.
5. If 107 lbs. of tea cost £21 3*s*., at what rate per lb. must it be sold, so that £4 14*s*. profit may be gained? *Ans*. 4*s*. 10*d*.
6. If 23 tons of potatoes be purchased for £56 12*s*., at what price, per ton, must they be sold, to produce a profit of £11 4*s*. on the outlay? *Ans*. £2 19*s*. all but $\frac{1}{2}d$.

NOTE.—In here concluding the foregoing series of problems, it may be well to state that although the rules proposed for the working of them are all specially adapted to the ready computation of the several kinds of arithmetical examples considered; yet that a case or two, properly coming under one or other of these problems, may occasionally occur in which the ordinary process of common arithmetic may answer quite as well as the *generally* shorter method here prescribed. It must always be left for the judgment and penetration of the computer to decide beforehand, whether, in the particular case he is to deal with, there is likely to be time and trouble saved by working according to the special rule, or by the ordinary method of common arithmetic. The last example above, for instance, may be quite as readily worked out by this ordinary method as by any special rule: thus:—

```
     £   s.
     56  12 Prime cost.
     11   4 Profit.
     ------
23)  67  16 (£2
     46
     --
     21
      20
     ---
     436 (19s.
     23
     ---
     206
     207
     ---
     —1
     ---
```

Selling price £2 19*s*., *less* $\frac{1}{23}s. = \frac{12}{23}d. = \frac{1}{2}d.$

*** Of course, $\frac{12}{23}d.$ could not be estimated at other than $\frac{1}{2}d$.

If another shilling be added to the profit, the division would leave no remainder; so that for the profit to be £11 5*s*. the selling price must be exactly £2 19*s*. per ton.

We shall now proceed to apply the preceding rules to mercantile and commercial transactions of a specific character; as also to those mechanical or handicraft employments in which calculation is necessary in order to estimate correctly the value of the work done, or of the materials supplied. In these applications, we may sometimes repeat, in substance, a rule, of wider import, already given in the

preceding pages. It may be convenient that this should now and then be done, rather than that the computer should continually have to turn to back references.

Calculations useful in the Wool Trade.

In this trade the weights used, although bearing the same names, are not always the same, as to the number of pounds represented by them, in different parts of the country. In some places the stone, usually 14 lbs., is a weight of 15 lbs.; in other places, a weight of 16 lbs.; and the weights of higher denominations—multiples of the stone—vary of course in like proportion. The usual weights for wool are as follows:—

Table of Weights for Wool.

7 lbs.	make	1 Clove.	6 Tod 1 Stone			*lbs.*
2 Cloves (or 14 lbs.)	„	1 Stone.		make	1 Wey =	182.
			2 Weys	„	1 Sack =	364.
2 Stone (or 28 lbs.)	„	1 Tod.	12 Sacks	„	1 Last =	4368.

In some places the weights are these, namely.

15 lbs.	make	1 Stone.	8 Tod (or 240 lbs.)	make	1 Pack.
2 Stone (or 30 lbs.)	„	1 Tod.	In Ireland, 16 lbs.	„	1 Stone.

Since a pack of wool weighs 240 lbs., it follows, that at 15 lbs. to the stone, it weighs exactly 16 stone; at 16 lbs. to the stone, it weighs exactly 15 stone; but at 14 lbs. to the stone, it weighs 17 stone 2 lbs.

Problem 1.

The price of 1 lb. being given in pence, to find the price of a stone of 14 lbs., of 15 lbs., or of 16 lbs.

Rule I.—Regard the pence as so many shillings; to this sum add $\frac{1}{6}$th of itself, if the stone be 14 lbs.; $\frac{1}{4}$th of itself, if the stone be 15 lbs.; and $\frac{1}{3}$rd, if it be 16 lbs.

For, by regarding the pence as so many shillings, you, in effect, multiply the price of 1 lb. by 12; that is, you get the value of 12 lbs.; and $\frac{1}{6}$th of this must be the value of 2 lbs.; $\frac{1}{4}$th, the value of 3 lbs.; and $\frac{1}{3}$rd, the value of 4 lbs.; so that

these portions severally added to the price of 12 lbs., must give the price of 14 lbs., 15 lbs., and 16 lbs., respectively, Or,

RULE II.—Having taken the price of 1 lb., in *pence*, as so many shillings, add to this amount the price of 2 lbs., 3 lbs., or 4 lbs., according as the stone is 14 lbs., 15 lbs., or 16 lbs.

EXAMPLES. (Stone of 14 lbs.)

1. If 1 lb. of wool cost 17*d.*, what will a stone of 14 lbs. cost?

		s.	*d.*				*s.*	*d.*
By Rule I.	6)	17	0	= cost of 12 lbs.	By Rule II.		17	0
		2	10	„ 2 lbs.		1*s.* 5*d.* × 2 =	2	10
	Ans.	19*s.*	10*d.*	„ 14 lbs.		*Ans.*	19*s.*	10*d.*

2. If 1 lb. cost 23½*d.*, what will a stone of 14 lbs. cost?
This example requires merely that we calculate the price at 2*s.*,—which would be 28*s.* per stone, and then subtract 14 times ½*d.*, or 7*d.*, leaving for answer 27*s.* 5*d.*

3. If 1 lb. cost 3*s.* 9½*d.*, what will a stone cost?
The price, in pence, is 45½*d.*; and regarding every penny as a shilling, we have 45½*s.*, that is, £2 5*s.* 6*d.* for the price of 12 lbs.; therefore,

	£	*s.*	*d.*				£	*s.*	*d.*
6)	2	5	6	= Price of 12 lbs.	Or,		2	5	6
		7	7	= „ 2 lbs.		3*s.* 9½*d.* × 2 =		7	7
	£2	13*s.*	1*d.*	= „ 14 lbs.			£2	13*s.*	1*d.*

4. If 1 lb. cost 5*s.* 6¼*d.*, what will a stone cost? *Ans.* £3 17*s.* 3½*d.*
5. If 1 lb. cost 1*s.* 10¾*d.*, what will a stone cost? *Ans.* £1 16*s.* 6½*d.*

Stone of 15 lbs.

6. What will a stone of 15 lbs. cost, at 2*s.* 1¾*d.* per lb.?
The price per stone in pence is 25¾*d.*; and 25¾*s.* = £1 5*s.* 9*d.*, the cost of 12 lbs.

4)	£1	5*s.*	9*d.*	Or,	£1	5*s.*	9*d.*	= cost of 12 lbs.
		6	5¼	2*s.* 1¾*d.* × 3 =		6	5¼	= „ 3 lbs.
Ans.	£1	12*s.*	2¼*d.*		£1	12*s.*	2¼*d.*	= „ 15 lbs.

7. If 1 lb. cost 13½*d.*, what will 15 lbs. cost? *Ans.* 16*s.* 10½*d.*
8. If 1 lb. cost 2*s.* 7¼*d.*, what will 7 stone cost? *Ans.* £13 13*s.* 5¼*d.*
9. If 1 lb. cost 1*s.* 9½*d.*, what will 9 stone cost? *Ans.* £12 1*s.* 10½*d.*

Stone of 16 lbs.

10. If 1 lb. cost 2*s.* 5½*d.*, what will a stone of 16 lbs. cost?

The price per stone, in pence, is 29½*d.*; and 29½*s.* = £1 9*s.* 6*d.*, the cost of 12 lbs.

3) £1 9*s.* 6*d.*	Or,	£1 9*s.* 6*d.* =	cost of 12 lbs.
9 10	2*s.* 5½*d.* × 4 =	9 10 =	„ 4 lbs.
Ans. £1 19*s.* 4*d.*		£1 19*s.* 4*d.* =	„ 16 lbs.

11. If 1 lb. cost 17¾*d.*, what will a stone of 16 lbs. cost? *Ans.* £1 3*s.* 8*d.*
12. If 1 lb. cost 13½*d.*, what will a stone cost? *Ans.* 18*s.*
13. If 1 lb. cost 16¼*d.*, what will 3 stone cost? *Ans.* £3 5*s.*

The reverse problem, namely: Having the price of a stone, to find the price of 1 lb., may be readily enough worked by common division. For example:—

At £2 13*s.* 1*d.* per stone of 14 lbs., what is the cost of 1 lb.?

```
14) 53s.  1d. (3s. 9½d. Ans.
    42
    --
    11
    12
    ---
    133 (9d.
    126
    ---
      7d. = 28f. (2 farthings.
```

Problem 2.

The price of 1 lb. being given in pence, to find the price of a pack.

Rule.—Regard the pence as so many pounds (£), and you will have at once the price of the pack.

For there are 240*d.* in £1, and 240 lbs. in 1 pack; so that by changing the *pence*, in the price of 1 lb., into so many *pounds*, we in effect multiply the price of 1 lb. by 240, and thus get the price of 240 lbs.

Examples.

1. The price of a pack of wool, at 15¾*d.* per lb., is £15¾ = £15 15*s.*
2. The price of a pack of wool, at 17¼*d.* per lb., is £17¼ = £17 5*s.*
3. The price of 7 packs, at 23¾*d.* per. lb., is £166 5*s.*

PROBLEM 3. (CONVERSE OF PROB. 2.)

The price of a pack of wool being given in pounds (£), to find the price of 1 lb., or of 1 stone, clove, &c.

RULE.—As many *pounds* as a pack costs, so many *pence* will 1 lb. cost, as is obvious.

EXAMPLES.

1. If a pack of wool cost £15 15*s*., what is that per lb., and per clove?

£15 15*s*. = £15¾; therefore, the price per lb. is 15¾*d*. = 1*s*. 3¾*d*.; and the price per clove is 7 times this; namely, 9*s*. 2¼*d*.

2. If a pack of wool cost £17 5*s*., what is the cost per stone (14 lb.)?

£17 5*s*. = £17¼; therefore, the price per lb. is 17¼*d*. = 1*s*. 5¼*d*.;

and the price per stone, 14 times this, is = $\left\{ \begin{array}{rrr} & 17s. & 3d. \\ & 2 & 10\frac{1}{2} \end{array} \right.$

(Prob. 1, p. 84.) Hence the price of a stone is = £1 0 1½*d*.

3. If a pack of wool cost £23 15*s*., what would be the cost of a stone of 15 lbs.?

£23 15*s*. = £23¾; therefore, the price per lb. is 23¾*d*. = 1*s*. 11¾*d*.; and the price per stone (15 lbs.) is (Prob. 1, p. 84) = $\left\{ \begin{array}{rrr} £1 & 3s. & 9d. \\ & 5 & 11\frac{1}{4} \end{array} \right.$

Ans. £1 9*s*. 8¼*d*.

or, since the price of 1 lb. is 2*s*. all but ¼*d*., therefore the price of 15 lbs. is 30*s*. *less* 3¾*d*. = 29*s*. 8¼*d*.

4. If a pack of wool cost £25 15*s*., what will 1 lb. cost? *Ans*. 2*s*. 1¾*d*.
5. If a pack of wool cost £16 5*s*., what will a stone of 16 lbs. cost? *Ans*. £1 1*s*. 8*d*.

CALCULATIONS IN REFERENCE TO WHEAT, OATS, &C.

In order to find the price of any number of bushels, quarters, &c., from the price of one bushel, quarter, &c., being known, the direct way is to multiply the price of one by the number. This is the method of common arithmetic, and in certain cases is the most convenient method; as, for instance, when the number (or multiplier) is below 12. For higher multipliers, special rules, for arriving at the result more expeditiously, have been given in the preceding pages: the

application of these rules is still further seen in the following examples.

[Although, in working these examples, we have usually referred to the rules conformably to which the operations have been performed, yet such formal reference will but seldom be necessary if the computer bear in mind, 1st, that whenever we reckon articles by the *dozen*, we take the price of the single article in *pence*, and then replace the pence by that number of *shillings:* we thus get the price of 1 dozen. 2nd. Whenever we reckon articles by the *score*, we take the price of the single article in *shillings*, and replace these shillings by so many *pounds:* we thus get the price of 1 score. 3rd. Whenever the articles are reckoned by the 240, we take the price of the single article in *pence*, and replace the pence by *pounds*, and thus get the price of 240. It will, however, be sufficient, *in all cases*, to regard the given number of articles, in each case, as consisting of *dozens*, with part of a dozen over; or of *scores*, with part of a score over; the articles, over and above the complete dozens or scores, to be computed for *separately*. In numbers higher than the number 20, it is optional whether we compute by the dozen or by the score.]

Examples.

1. What is the cost of 18 bushels of corn, at 5*s*. 4*d*. per bushel?

As 18 is 12 + 6, we first calculate the price of a dozen (p. 60), and then add half that price for the 6. Thus:—Taking 64*d*. as 64*s*., and adding the half to it, we have 96*s*. = £4 16*s*., the cost required. But the work would be easy enough by the ordinary method: thus, 5*s*., × 18 = 90*s*., to which adding a third of 18*s*. for the 4*d*., the result is 96*s*.

2. 23 quarters, at £2 9*s*. 3*d*. per quarter?

Here expressing the price in shillings, and then counting these as pounds (Rule, p. 69), we have £49¼ =

	£49	5*s*.	0*d*.	for 20 qrs.
For 3 qrs.	7	7	9	
Price of 23 qrs.	£56	12*s*.	9*d*.	

3. 26 bushels, at 4*s*. 10½*d*. per bushel? [4*s*. 10½*d*. = 58½*d*.]

	£	*s*.	*d*.
Price of 12 bush. =	2	18	6
” ” ” =	2	18	6
” 2 ” =		9	9
	£6	6*s*.	9*d*.

Or thus: (See Table, p. 57)
4*s*. 10½*d*. = 4⅞*s*.; and

£4⅞ = £4	17*s*.	6*d*.	= price of	20
also 1	9	3	= ”	6
£6	6*s*.	9*d*.	= ”	26

4. 30 quarters of barley, at 27*s.* 9*d.* per qr.? [27*s.* 9*d.* = $27\frac{3}{4}$*s.*]

	£	*s.*	*d.*	
£$27\frac{3}{4}$ =	27	15	0	= price of 20 (Rule, p. 69.)
One half =	13	17	6	= „ 10
	£41	12*s.*	6*d.*	= „ 30

5. 23 quarters, at 29*s.* 5*d.* per quarter? [29*s.* 5*d.* = $29\frac{1}{3}$*s.* + 1*d.*]

	£	*s.*	*d.*	
£$29\frac{1}{3}$ =	29	6	8	= price of 20, at 29*s.* 4*d.*
	4	8	0	= „ 3 „
		1	11	= „ 23 at 1*d.*
	£33	16*s.*	7*d.*	= „ 23 „ 29*s.* 5*d.*

Or thus: 24, at 353 *pence*, amount to the same as 353 at 24*d.*, or 2*s.*; that is, to 706*s.* = £35 6*s.* 0*d.*
from which take 1 9 5

There remains £33 16*s.* 7*d.*

6. 37 quarters, at 25*s.* 10*d.* per quarter? [25*s.* 10*d.* = $25\frac{3}{4}$*s.* + 1*d.*]

	£	*s.*	*d.*	
£$25\frac{3}{4}$ =	25	15	0	= price of 20, at 25*s.* 9*d.*
	12	17	6	= „ 10 „
	9	0	3	= „ 7 „
		3	1	= „ 37 at 1*d.*
	£47	15*s.*	10*d.*	= „ 37 at 25*s.* 10*d.*

Or thus: 36 at 310*d.* = 310 at 3*s.* = 930*s.* =

£	*s.*	*d.*	
46	10	0	= price of 36
1	5	10	= „ 1
£47	15*s.*	10*d.*	= „ 37

NOTE.—The last two examples may be otherwise worked, thus:

Ex. 5. £$29\frac{5}{12}$ = £29 8*s.* 4*d.*, price of 20
4 8 3 „ 3

£33 16*s.* 7*d.* „ 23

Ex. 6. 25*s.* 10*d.* = $25\frac{5}{6}$*s.* (See Table, p. 57.)

	£	*s.*	*d.*	
£$25\frac{5}{6}$ =	25	16	8	= price of 20
	12	18	4	= „ 10
	9	0	10	= „ 7
	£47	15*s.*	10*d.*	= „ 37

7. **32 quarters of wheat, at £2 11s. 10d. per qr.?**

The price per qr. here is $51\frac{3}{4}$s. + 1d.; therefore, (Rule, p. 69.)

£$51\frac{3}{4}$ = £51	15s.	0d. =	price of 20, at £2 11s. 9d.		
25	17	6 =	„	10 „	„
5	3	6 =	„	2 „	„
	2	8 =	„	32 at 1d.	
£82	18s.	8d. =	„	32 „ £2 11s. 10d.	

[This example may also be worked as in the NOTE, p. 89, the price per quarter here being $51\frac{5}{6}$s.]

As already observed, it is not in *every* case that the *special* rule, for working examples really coming under that rule, has any marked advantage over the *general* rule, taught in books of arithmetic. The merchant or trader, accustomed to calculations peculiar to his own calling, will see, at a glance, whether, in any particular case that may come before him, the special or the general rule will be the more convenient for him to use. By way of comparison, we shall take the last three examples, and work them each by the general rule (p. 34).

Ex. 5.

	£	s.	d.	
	1	9	5	× 3
			5	
	7	7	1	
			4	
Price of 20 =	£29	8	4	
„ 3 =	4	8	3	
„ 23 =	£33	16s.	7d.	

Ex. 6.

	£	s.	d.	
	1	5	10	× 1
			6	
	7	15	0	
			6	
Price of 36 =	46	10	0	
„ 1 =	1	5	10	
„ 37 =	£47	15s.	10d.	

Ex. 7.

£	s.	d.
2	11	10
		8
20	14	8
		4
£82	18s.	8d. = *Ans.*

The special rule has the more decided advantage when the

number to be computed for is an *exact* number of dozens, or scores, as in each of the examples following.

8. Required the price of 40 qrs. of oats, at 42*s.* 6*d.* per qr.?
   £42½ × 2 = £85, *Ans.*
9. 60 qrs., at 52*s.* 9*d.* per qr.? £52⅜ × 3 = £158¼ = £158 5*s.*, *Ans.*
10. 80 qrs., at 54*s.* 6*d.* per qr.? £54½ × 4 = £218, *Ans.*
11. 36 bushels, at 5*s.* 4*d.* per bushel? 64*s.* × 3 = 192*s.* = £9 12*s.* *Ans.*
12. 48 bushels, at 4*s.* 10½*d.* per bushel?
    58½*s.* × 4 = 234*s.* = £11 14*s.*, *Ans.*

NOTE.—5 bushels of corn is called a *load* of corn, but a cart-load is 40 bushels. The *load* has reference to porterage; the cart-load to horse carriage. It may also be noticed here, that the *sack* varies in measure with the commodities measured. When coals were measured by the bushel, *three* bushels went to the sack: in corn of all descriptions, *four* bushels (not heaped up, as with coals) make a sack; but of flour, there are *five* bushels to the sack. A *quarter* of corn, or of malt, is *two* sacks, that is eight bushels, as already stated; and a cart-load is *ten* sacks, or *five* quarters.

The following Table will be found useful to retail corn-dealers.

TABLE.

From the price of a Quart to find the price of a Bushel.

Quart.	Bushel.	Quart.	Bushel.	Quart.	Bushel.
d.	*s.* *d.*	*d.*	*s.* *d.*	*d.*	*s.* *d.*
¼	8	2	5 4	3¾	10 0
½	1 4	2¼	6 0	4	10 8
¾	2 0	2½	6 8	4¼	11 4
1	2 8	2¾	7 4	4½	12 0
1¼	3 4	3	8 0	4¾	12 8
1½	4 0	3¼	8 8	5	13 4
1¾	4 8	3½	9 4	5¼	14 0

For every additional farthing in the price of a quart, another 8*d.* must be added to the price of the bushel; for there are 32 quarts (8 gallons) in a bushel, and 32 farthings make 8*d.* If the dealer purchase by the quarter (8 bushels), and retail by the bushel, then every farthing added to the bushel adds 2*d.* to the quarter; so that as many twopences as he proposes for his profit on the prime cost of the quarter, so many farthings must he add to the prime cost of the bushel. But the prime cost per quart, or per bushel, may

not be readily ascertainable; it may involve a fraction of a farthing: thus, in the Table above, if, for instance, instead of 5*s.* 4*d.*, the corn were 5*s.* 6*d.* per bushel, the price of the quart would contain such a fraction. In this case, the profit, at 2¼*d.* per quart, would be 2*d.* *less* than 8*d.*, that is, it would be 6*d.*; and at 2½*d.*, it would be 6*s.* 8*d.* *less* 5*s.* 6*d.*, that is, 1*s.* 2*d.*, and so on.

Calculations in reference to Flour.

A sack of flour weighs 2½ cwt., or 20 stone, or 280 lbs., and as, in certain calculations with which the flour-factor and the retail dealer ought to be familiar, this number 280 and its subdivisions very frequently enter, it will facilitate the numerical work if the following particulars be preserved in the memory, namely, 280*d.* = 23*s.* 4*d.*; 280 farthings = 5*s.* 10*d.*, or 70*d.*; and 280 *sevenths* of a farthing = 10*d.*; so that 280 *fourteenths* of a farthing = 5*d.*

That these are relations useful to be borne in mind the reader may convince himself even now, at the outset, for he here perceives that at 1*d.* per lb., a sack of flour will cost £1 3*s.* 4*d.*, and at a farthing per lb., 5*s.* 10*d.* He sees, moreover, that for every farthing added to the price of the lb., 5*s.* 10*d.* is added to the price of the sack; that every *seventh* of a farthing added to the price of the lb., adds 10*d.* to the price of the sack; and, *consequently*, that a *whole* farthing added to the price of *seven* lbs. (half a stone), adds 10*d.* to the price of the sack; while the additional farthing to the price of a *stone* increases the price of the sack by 5*d.* And thus the retailer of flour in small quantities may readily estimate his profit, per sack, upon every advance of one farthing in the price, per lb. or per stone.

We shall now give a few examples showing the application of such of the rules, in the foregoing pages, as are available in business transactions in flour, and shall then supply the *additional* rules which these special transactions require, and which, to the retail dealer in particular, will be found to be of essential service.

Examples in the Purchase of Flour.

1. What will 13 stone of flour come to, at 1*s.* 3*d.* per stone?

1*s.* 3*d.* = 15*d.*, therefore (Rule, p. 60) 15*s.* + 1*s.* 3*d.* = 16*s.* 3*d.*, *Ans.*

2. What will 14 stone of flour come to, at 1*s*. 4*d*. per stone?

The price of 2 stone is 2*s*. 8*d*., therefore (Rule, p. 60) 16*s*. + 2*s*. 8*d*. = 18*s*. 8*d*., *Ans*.

3. What will 30 sacks of flour cost, at 29*s*. 3*d*. per sack?

			£	s.	d.
By the Rule, p. 69, the price of 20 sacks is £29¼		=	29	5	0
"	10 "	=	14	12	6
	30	=	£43	17*s*.	6*d*.

4. What is the price of 20 sacks of flour, at 27*s*. 9*d*. per sack? *Ans*. £27 15*s*.
5. What is the price of 60 sacks, at 32*s*. 3*d*. per sack? *Ans*. £96 15*s*.
6. What is the price of 80 sacks, at 35*s*. 10*d*. per sack? *Ans*. £143 6*s*. 8*d*.
7. What is the price of 38 sacks, at 34*s*. 8*d*. per sack? *Ans*. £65 17*s*. 4*d*.

Problem 1.

The price of 1 lb. of flour being given, to find the price of a sack.

Rule I.—Multiply 5*s*. 10*d*. by the number of farthings in the price of 1 lb. Or,

Rule II.—Annex a cipher to the number of farthings in the price of 1 lb., and call the result *pence*. Multiply by 7, and the product will be the price of a sack.

Examples.

1. At 1¾*d*., and 2¾*d*., per lb., what will be the corresponding prices of a sack of flour?

By Rule I.

1. At 7 farthings per lb. : 5*s*. 10*d*.	2. At 11 farthings : 5*s*. 10*d*.
7	11
Ans. 40*s*. 10*d*.	*Ans*. 64*s*. 2*d*.

By Rule II.

1. At 7 farthings: 70*d*. = 5*s*. 10*d*.	2. At 11 farthings: 110*d*. = 9*s*. 2*d*.
7	7
Ans. 40*s*. 10*d*.	*Ans*. 64*s*. 2*d*.

The reason of Rule I. is pretty obvious: 5*s*. 10*d*. is the price of a sack at a farthing per lb.; consequently, at any number of farthings per lb., the price of a sack must be 5*s*. 10*d*. multiplied by that number.

Rule II. is explained thus: By annexing a cipher to the price in farthings, we, in effect, multiply that price by 10, and by calling the result *pence*, we virtually again multiply by 4; that is, the price is now multiplied by 40; and then the final multiplication by 7 completes the multiplication of the price of 1 lb. by the number 280, and thus gives the price of 280 lbs. This second rule is a little the more convenient of the two when the number of farthings in the price of 1 lb. exceeds 12.

2. What is the price of a sack of flour, at $3\frac{1}{4}$*d*. per lb.?

At 13*f*., by Rule 2, 130*d*. = 10*s*. 10*d*.; and this $\times$ 7 = 75*s*. 10*d*.

3. What is the price of a sack of flour, at $2\frac{1}{4}$*d*. per lb.? *Ans*. 52*s*. 6*d*.

4. What is the price of a sack of flour, at $3\frac{1}{2}$*d*. per lb.? *Ans*. 81*s*. 8*d*.

Problem 2. (Converse of Prob. 1.)

The price of a sack of flour being given, to find the price of 1 *lb.*

Rule.—Divide the price of the sack, in *pence*, by 70; the result will be the number of *farthings* per lb.

Examples.

1. A sack of flour costs 40*s*. 10*d*., another costs 64*s*. 2*d*.; required the respective prices per lb.?

One sack costs 490 pence, the other 770 pence; hence, by the Rule:

7,0) 49,0	7,0) 77,0
7*f*. = $1\frac{3}{4}$*d*. per lb.	11*f*. = $2\frac{3}{4}$*d*. per lb.

The truth of the above rule will be seen from the following considerations. The price of 1 lb., in farthings, will be obtained by dividing the price, in farthings, of a sack, by 280. But this is the same as dividing successively by 4 and by 70. Now by regarding the *pence-price* of a sack as so many *farthings*, we in reality take a fourth part of that price, or divide it by 4; and then again by dividing by 70, the result

must be the same number of farthings that we should get by dividing at once by 280. And this is the Rule.

The price of a sack may, no doubt, be such that the 280th part of it may not be an exact number of farthings; in such a case, a fraction of a farthing will be unavoidable, as in the example next following, namely:—

2. If a sack of flour cost 41*s.* 10*d.*, what does it cost per lb.?

Here the price, in pence, is 502*d.*, and $502 \div 70 = 7\frac{12}{70}$; so that the *accurate* price of 1 lb. is 7 farthings and 12 seventieths, that is (p. 53), 6 thirty-fifths of a farthing besides. Such a price could not be paid in existing coin. But if 12 pence were taken from the cost of the sack (12 being the *remainder*, or overplus, after the division by 70), that is, if the price per sack were reduced to 40*s.* 10*d.*, then the price per lb. would be *exactly* 7 farthings. The retail dealer, knowing this, can readily find his exact profit per sack by increasing the 7 farthings by 1 farthing, then by another farthing, and so on; every additional farthing to the price of a lb. adding 5*s.* 10*d.* to the selling price of the sack: thus, at 8 farthings, or 2*d.*, per lb., the profit on the sack, at 41*s.* 10*d.*, would be 4*s.* 10*d.*: at $2\frac{1}{4}$*d.* per lb., the profit would be 10*s.* 8*d.*, and so on.

Problem 3.

The price of a stone of flour being given, to find the price of a sack.

Rule.—The price of a stone, in farthings (regarded as so many *pence*), multiplied by 5, will give the price of a sack.

For by regarding the number of *farthings*, in the price of a stone, as so many *pence*, and then multiplying by 5, we, in effect, multiply the price by 4, and the product by 5; that is, we multiply by 20.

Note.—If the price of the stone be in *shillings*, without any pence, then the price of the sack is just so many *pounds*, as is obvious, and no calculation is required; but if the price of the stone comprise shillings and pence, then, instead of turning the shillings into farthings, it is better to write, for these shillings, so many *pounds*, and to deal with the odd pence *only* as the Rule directs.

Examples.

1. If a stone of flour cost $15\frac{3}{4}$*d.*, what will a sack cost?

$15\frac{3}{4}$*d.* = 63*f.*; and 63*d.* = 5*s.* 3*d.*, which × 5 = 26*s.* 3*d.*, *Ans.*

Or (Note above), $3\frac{3}{4}$*d.* = 15 farthings; and 15*d.* = 1*s.* 3*d.*, which × 5 = 6*s.* 3*d.*; and prefixing to this £1, for the *shilling*, we have £1 6*s.* 3*d.* for the price of the sack.

2. If a stone of flour cost $11\frac{1}{4}d.$, what will a sack cost?
   $11\frac{1}{4}d. = 45f.$; and $45d. = 3s.\ 9d.$, which $\times\ 5 = 18s.\ 9d.$, *Ans.*
3. If a stone of flour cost $13\frac{1}{2}d.$, what will a sack cost?
   (NOTE, p. 95). *Ans.* £1 2*s.* 6*d.*
4. If a stone of flour cost 1*s.* $7\frac{1}{2}d.$, what will a sack cost?
   (NOTE, p. 95). *Ans.* £1 12*s.* 6*d.*

[The last two examples may be worked by aid of the Table (p. 57), thus: Ex. 3. $13\frac{1}{2}d. = 1\frac{1}{8}s.$; and $£1\frac{1}{8} = £1\ 2s.\ 6d.$ Ex. 4. $1s.\ 7\frac{1}{2}d. = 1\frac{5}{8}s.$; and $£1\frac{5}{8} = £1\ 12s.\ 6d.$ But reference to a *Table* for such particulars may always be rendered unnecessary by a very trifling amount of calculation: thus, in the present case,—

$$1\tfrac{1}{2}d. = \frac{1\frac{1}{2}}{12}s. = \frac{3}{24}s. = \frac{1}{8}s.\ \text{(p. 53); and } 7\tfrac{1}{2}d. = \frac{7\frac{1}{2}}{12}s. = \frac{15}{24}s. = \frac{5}{8}s.$$

Also $£\frac{1}{8} = \frac{20}{8}s. = 2s.\ 6d.$; and $£\frac{5}{8} = \frac{100}{8}s. = 12s.\ 6d.$ And similarly in all cases.]

PROBLEM 4. (CONVERSE OF PROB. 3.)

The price of a sack of flour being given, to find the price of a stone.

RULE.—Divide the number of *pence* in the price of a sack (regarding these pence as so many *farthings*) by 5, and the result will be the price of a stone. For the price of a stone is

$$\frac{\text{pence in sack}}{20}d. = \frac{\text{pence in sack} \times 4}{20}f. = \frac{\text{pence in sack}}{5}f.,$$

which is the Rule.

EXAMPLES.

1. The price of a sack of flour being 26*s.* 3*d.*, required the price of a stone?
   $26s.\ 3d. = 315d.$; and $315f. \div 5 = 63f. = 15\frac{3}{4}d.$, *Ans.*
2. If a sack of flour cost 18*s.* 9*d.*, what will a stone cost?
   $18s.\ 9d. = 225d.$; and $225f. \div 5 = 45f. = 11\frac{1}{4}d.$, *Ans.*
3. If a sack of flour cost 22*s.* 6*d.*, what will a stone cost? *Ans.* $13\frac{1}{2}d.$
4. Required the cost of a stone of flour, at 32*s.* 6*d.* per sack?
   Ans. 1*s.* $7\frac{1}{2}d.$
5. If a sack of flour cost 24*s.*, at what price per stone must it be sold in order that the retailer may gain about 5*s.* profit on the sack?

[It may be instructive to work this example at length.]
The selling price must be at the rate of 29*s.* per sack: $29s. = 348d.$; and $348f. \div 5 = 69\frac{3}{5}f. = 17\frac{1}{4}d. + \frac{3}{5}f.$ This is the exact price at which a stone must be sold, in order that the gain, per sack, may be just 5*s.*

The gain would also be exactly this sum if the cost price of the sack were 3*d*. less than 24*s*., and the selling price, per stone, 17$\frac{1}{4}$*d*.; seeing that the *remainder* from the foregoing division by 5 is 3. Hence, disregarding the fraction $\frac{3}{5}$*f*., the gain, at 17$\frac{1}{4}$*d*. per stone, is 5*s*. – 3*d*., that is, 4*s*. 9*d*.; the addition of a farthing to this price would add 5*d*. to the price per sack, that is, at 17$\frac{1}{2}$*d*. per stone, the profit, at 24*s*. per sack, would be 5*s*. 2*d*.

As the remainder, arising from dividing the farthings by 5, is always so many *fifths* of a farthing, it is well to remember that one-fifth of a farthing added to the price of the stone, adds 1*d*. to the price of the sack; thus, the $\frac{3}{5}$*f*., added to the 17$\frac{1}{4}$*d*. per stone above, raises the price of the sack from 24*s*. – 3*d*. to 24*s*.

CALCULATIONS RESPECTING CARRIAGE OF GOODS BY RAILWAY, ETC.

The highest weight-denomination in the goods carried may be the cwt., or it may be the ton: special rules for the two cases here follow.

PROBLEM 1.

To find the charge for carriage at a given sum per cwt.

RULE.—Call the number of cwts. so many shillings: we shall thus get the cost of carriage of these cwts., at the rate of 1*s*. per cwt.; and for the additional smaller weights the charge, at this rate, would be, for $\frac{1}{4}$ cwt., 3*d*.; for 14 lbs. (a stone), 1$\frac{1}{2}$*d*.; and for 7 lbs., $\frac{3}{4}$*d*. Add these several charges together, and multiply the amount by the number of shillings per cwt., taking parts for the odd pence if there be any, as in the following worked examples.

NOTE.—If the charge be *pence* only, without any shillings, we are still, as here directed, to compute the charge at 1*s*., and then, taking parts for the pence, to add together these parts *only*. For instance, in Ex. 2, page 98, if the charge for carriage be only 7*d*. per cwt., instead of 1*s*. 7*d*., the work would be as here annexed.

At 1*s*. .	13*s*.	10$\frac{1}{2}$*d*
„ 6*d*. .	6	11$\frac{1}{4}$
„ 1*d*. .	1	2
„ 7*d*. .	8*s*.	1$\frac{1}{4}$*d*.

EXAMPLES.

1. What will the carriage of 17 cwt. 3 qrs. 21 lbs. come to, at 3*s*. per cwt.?

   17*s*. + 9*d*. + $\frac{9}{4}$*d*. = 17*s*. 11$\frac{1}{4}$*d*., the charge at 1*s*. per cwt.;
   and 17*s*. 11$\frac{1}{4}$*d*. × 3 = £2 13*s*. 9$\frac{3}{4}$*d*., the charge at 3*s*., *Ans*.

2. What will the carriage of 13 cwts. 3 qrs. 15 lbs. come to, at 1*s*. 7*d*. per cwt.?

[The 15 lbs. must be estimated as 1 stone (14 lbs.).]

.Charge at 1*s*. per cwt. $= 13s. + 9d. + 1\frac{1}{2}d.$	$=$	13*s*.	$10\frac{1}{2}d.$; and
$\frac{1}{2}$ of this (for 6*d*.)	$=$	6	$11\frac{1}{4}$
$\frac{1}{6}$ of this last (for 1*d*.)	$=$	1	2
		Ans. 21*s*.	$11\frac{3}{4}d.$

The charge made in this case would, no doubt, be 22*s*.

If the cost of carriage had been 2*s*. 7*d*. per cwt., we should have computed exactly as above, and then, to the result here arrived at, should have added 13*s*. $10\frac{1}{2}d.$, making the charge 35*s*. $10\frac{1}{4}d.$; and if the charge had been 3*s*. 7*d*. per cwt., we should have added *twice* 13*s*. $10\frac{1}{2}d.$, instead of once that sum; and so on, since this sum is the charge at 1*s*. per cwt. But when the charge is two or more shillings, besides odd pence, a little time and trouble may be saved by writing *above* the total charge at 1*s*. as many times this charge as there are extra shillings, and *then* adding all the amounts up; thus, taking the charge for the foregoing weight of goods at 3*s*. 7*d*. per cwt., we should recommend working as in the margin; the 13*s*. $10\frac{1}{2}d.$ being found, as above, and written down *first*; twice this sum being then placed immediately *above* it, and the half immediately below it.

s.	*d*.
27	9
13	$10\frac{1}{2}$
6	$11\frac{1}{4}$
1	2
49*s*.	$8\frac{3}{4}d.$

[It is obvious that if instead of the cost of *carriage* being so much per cwt., the cost of the goods themselves were at that rate, the foregoing method of computation would be equally applicable to the finding of the cost of the whole.]

Problem 2.

When the charge for carriage is by the ton.

Rule.—Call the tons so many pounds (£), and the cwts. so many shillings; then, as in last problem, allow as follows: for $\frac{1}{4}$ cwt., 3*d*.; for 14 lbs., $1\frac{1}{2}d.$; and for 7 lbs., $\frac{3}{4}d.$, and compute as before. [The Rule applies as well to the *cost* of the goods as to the carriage (see Ex. 3, p. 100).]

If the charge per ton be *more* than £1, multiply the cost at £1 by the number of pounds (£), taking parts for the

shillings and pence, as in last problem; but if it be *less* than £1, the parts for the shillings are, alone, to be added together.

EXAMPLES.

1. What is the charge for the carriage of 7 tons 5 cwts. 3 qrs. 14 lbs., from London to Manchester, at the rate of 25*s*. 3*d*. per ton?

	£	s.	d.
Charge for 7 tons, at £1	7	0	0
,, 5 cwts. ,,		5	0
,, 3 qrs. ,,			9
,, 14 lbs. ,,			1½
Total charge at £1 per ton	£7	5	10½
For 5*s*. = ¼ £	1	16	5½
3*d*. = $\frac{1}{20}$ 5*s*.		1	10
Total charge at 25*s*. 3*d*.	£9	4	2

The work admits of contraction: it has been spread out, as in the above form, merely to exemplify the rule in full detail. It is pretty obvious, however, that we may write down the total charge at the rate of £1 per ton, *at once*, so that the last four lines *only* comprehend all the work that is really requisite.

[It is scarcely necessary to state, in proof of the rule, that since 20 cwts. = 1 ton, the charge for 1 cwt., at 20*s*. per ton, must be 1*s*. If in the above example the charge per ton had been £2 5*s*. 3*d*., or £3 5*s*. 3*d*., or &c., instead of £1 5*s*. 3*d*.; we should have multiplied the total charge at £1 by the number of these *additional* pounds per ton; and, writing the product *above* the £7 5*s*. 10½*d*., the work would have stood as here annexed, supposing the rate of carriage to have been £3 5*s*. 3*d*. per ton.]

£	s.	d.
14	11	9
7	5	10½
1	16	5½
	1	10
£23	15*s*.	11*d*.

2. What is the charge for the carriage of 9 tons 9 cwts. 1 qr. 16 lbs. from York to Newcastle, at the rate of 17*s*. 6*d*. per ton?

By the Rule, the charge, at £1 per ton, is	£9	9*s*.	4¾*d*.*
,, 2*s*. 6*d*. = ⅛ £	1	3	8 (*Subtract.*)
,, 17*s*. 6*d*.	£8	5*s*.	8¾*d*.†

[The charge per ton here being *less* than £1 by 2*s*. 6*d*., the part for this deficiency is subtracted.]

* ¼*d*. being allowed for the 2 lbs. above 14 lbs.

† This would no doubt be regarded as £8 5*s*. 9*d*.

3. 215 tons 17 cwts. 3 qrs. 9 lbs., at £9 11*s*. 6¼*d*. per ton?

	£	*s*.	*d*.	
	1727	2	8	(at £8 per ton.)
By the Rule, the charge, at £1 per ton, is	215	17	10	
,, 10*s*. = ½£ is	107	18	11	
,, 1*s*. = $\frac{1}{10}$ 10*s*. is	10	15	10½	
,, 6*d*. = ½ 1*s*. is	5	7	11¼	
,, ¼*d*. = $\frac{1}{24}$ 6*d*. is		4	6	= 108*d*. ÷ 2
Charge at £9 11*s*. 6¼*d*. =	£2067	7*s*.	8¾*d*.	

This would of course be considered as £2067 7*s*. 9*d*.

[In reference to the last item above, namely, 4*s*. 6*d*., it will be noticed that the 24th part of 108*s*. (which £5 7*s*. 11¼*d*. may be taken for) is the *half* of 108 *pence*.]

4. 45 tons 3 cwts. 3 qrs. 17 lbs. of goods were booked at Manchester for Edinburgh, at 1¼*d*. per ton per mile (distance 272 miles); the Lancashire Company carried the goods 80 miles; the York and Newcastle Company, 76 miles; the Newcastle and Berwick, 64 miles; and the Berwick and Edinburgh, 52 miles. What is the amount due to each Company?

The weight carried is 903 cwt. 101 lbs.

```
                     903
(See Rule p. 15.)   903
                   903
                     101
                   ------
                   101237 lbs.
                        5 = No. of farthings per 2240 lbs. (1 ton.)
                   ------
                   506185 ÷ 2240 = 226f. = 56½d. = 4s. 8½d.
```

[It is plain that as many times as 2240 lbs. is contained in 5 times the whole weight in lbs., so many farthings will the carriage of that weight cost per mile. As, however, every number terminating in a 0 is divisible by 5, the number 2240 is divisible by 5, the quotient being 448; so that the multiplication by 5, as above, might have been dispensed with; it would have been sufficient merely to have divided 101237 by 448. (See p. 55.) And it is worthy of remembrance, not only that every number terminating in 0 is divisible by 5, but also that the quotient from this division is always found by merely multiplying the number itself, after expunging the 0, by 2; thus, $2240 \div 5 = 224 \times 2 = 448$; $1370 \div 5 = 137 \times 2 = 274$; $63920 \div 5 = 6392 \times 2 = 12784$;

and so on, generally; because expunging the 0 is dividing by 10, the result being just *half* what the division by 5 would be.]

Proceeding with the arithmetical work, we have now to calculate the separate charges at 4*s.* 8½*d.* per mile, first for 80 miles, then for 76 miles, then for 64 miles, and lastly for 52 miles. The work for the first of these charges may be computed as follows:—

	£	s.	d.
4 at £4 [that is, 80 at 4*s.*] =	16	0	0
8 at 6*s.* 8*d.* [that is, 80 at 8*d.*] =	2	13	4
½ at 6*s.* 8*d.* [that is, 80 at ½*d.*] =		3	4
Charge for the Lancashire Company	£18	16*s.*	8*d.*

Now the charge for 4 miles, being 4 times 4*s.* 8½*d.*, is 18*s.* 10*d.*, and the charge for 12 miles is 3 times this, or £2 16*s.* 6*d.* Subtracting therefore the first of these sums from £18 16*s.* 8*d.*, the second of them from the remainder, and then again from the next remainder, the computation is continued thus:—

	£	s.	d.		
Charge for the Lancashire Company	18	16	8	For 80	miles.
(*Subtract*)		18	10	„ 4	„
„ York and Newcastle „	£17	17*s.*	10*d.*	„ 76	„
(*Subtract*)	2	16	6	„ 12	„
„ Newcastle and Berwick „	£15	1*s.*	4*d.*	„ 64	„
(*Subtract*)	2	16	6	„ 12	„
„ Berwick and Edinburgh „	£12	4*s.*	10*d.*	„ 52	„
And the charge for the entire distance is	£64	0*s.*	8*d.*	„ 272	„

[The foregoing example does not, in strictness, belong to the present problem, inasmuch as *mileage* is the most important part of the railway charge: but we have thought that an instance of this kind would be instructive to the reader, and we could find a no more appropriate place for it than under the head of the present division of our subject. We have worked it above, with all the detail that a novice might require; but in this, as also indeed in most of the worked examples in this book, there is a fulness of detail which, in actual business, would be for the most part,

suppressed. What in real practice would be but mentally supplied, *must* appear before the eye in a printed book.

The above calculation will suffice to show the Goods-Manager, at a Railway Station, how to compute the charge for any weight of goods, at any price per ton, per mile. It will also be a guide for Ticket-Clerks, in cases where several Companies are concerned in the carriage of the same goods; and will likewise be of service to computers in the Clearance-House.]

5. What is the charge for the carriage of 37 tons 13 cwts. 2 qrs. 18 lbs. from Bristol to Liverpool, at the rate of £1 13*s.* 10*d.* per ton? *Ans.* £63 14*s.* 11¼*d.*
6. Required the charge for 43 tons 1 cwt. 1 qr., from Paddington to Edinburgh, at the rate of £2 7*s.* 9*d.* per ton? *Ans.* £102 16*s.* 3*d.*
7. Required the charge for 84 tons, 3 cwts. 3 qrs. 20 lbs., from Manchester to Aberdeen, at 52*s.* 9*d.* per ton? *Ans.* £222 1*s.* 4¼*d.*

Calculations Useful in the Spirit Trade.

[It will be found convenient in this business to recollect that 63 farthings make 1*s.* 3¾*d.*, and 63 halfpence 2*s.* 7½*d.*]

Problem 1.

From the price of a gallon, to find the price of a hogshead.

Rule I.—Multiply £3 3*s.* by the number of *shillings* in the price per gallon; 5*s.* 3*d.* by the number of *pence;* and 1*s.* 3¾*d.* by the number of *farthings;* and add the results.

Rule II.—Regard the price, in *shillings*, per gallon as so many *pounds* and parts of a pound, and multiply it by 3: multiply the price, *as given*, also by 3, and add the results.

Of course, there will always be as many *shillings* (and parts) in the latter product as there are *pounds* (and parts) in the former.

Examples.

1. If the price of a gallon be 9*s.* 7¾*d.*, what is the price of a hogshead?

By Rule I.	By Rule II.
£3 3*s.* × 9 = £28 7*s.*	Take the price per gal. at 9*s.* 8*d.*
5*s.* 3*d.* × 7 = £ 1 16*s.* 9*d.*	3 £9⅔ + 3 (9*s.* 8*d.*) = £30 9*s.* 0*d.*
1*s.* 3¾*d.* × 3 = 3*s.* 11¼*d.*	Subtract 63 far. = 1*s.* 3¾*d.*
Ans. £30 7*s.* 8¼*d.*	*Ans.* £30 7*s.* 8¼*d.*

[Note.—3 × £9⅔ = £29; and 3 (9*s.* 8*d.*) = 29*s.* The notation 3 (9*s.* 8*d.*) means 3 *times* 9*s.* 8*d.*]

The reason of Rule I. will appear evident from considering that £3 3*s*. = 63 shillings; 5*s*. 3*d*. = 63 pence; and 1*s* 3¾*d*. = 63 farthings; and that the shillings in the price of a hogshead must be equal to 63 times the shillings in the price of a gallon; the pence in the price of the former, equal to 63 times the pence in the price of the latter; and the farthings in the price of the former, equal to 63 times the farthings in the price of the latter, because 63 gallons make 1 hogshead. In the above, each denomination in the price of a gallon—shillings, pence, and farthings—is multiplied by 63, and the products added.

As to Rule II., it is sufficient to notice that by taking the shillings in the price of a gallon as so many pounds, and multiplying by 3, we get the price of three-score, that is, of 60 gallons; to which the price of 3 gallons being added, the amount must be the price of 63 gallons.

It may be observed here that, whatever be the capacity of the cask,—whether it contain 63, 54, 36, &c. gallons, the mode of computation is similar, as respects Rule I. Thus, suppose the cask contains 54 gallons, at 9*s*. 7¾*d*. per gallon; then to find the price of the whole, we have only to remember that 54*s*. = £2 14*s*.; 54*d*. = 4*s*. 6*d*.; and 54*f*. = 1*s*. 1½*d*., and to proceed, after the model above, as follows:—

	£	*s*.	*d*.
£2 14*s*. × 9 =	24	6	0
4*s*. 6*d*. × 7 =	1	11	6
1*s*. 1½*d*. × 3 =		3	4½
Ans.	£26	0*s*.	10½*d*.

For the first of the sums here added is 9 times 54*s*., which is, of course, the same as 54 times 9*s*.; the second, is 7 times 54 pence, this being the same as 54 times 7 pence; and the third, 3 times 54 farthings is the same as 54 times 3 farthings; so that the total amount of these three separate sums must be 54 times 9*s*. 7¾*d*. And the work would be similar for any other number of gallons,—the rule being perfectly general and quite irrespective of all peculiarity as to the composition of the number proposed. The number 54, taken above, is employed solely for the purpose of illustration; it

	115*s*.	9*d*.
		9
2)	1041	9
2,0)	52,0	10½
	£26 0*s*.	10½*d*.

happens to be a number favourably constituted for computing by *dozens*, as it is composed of 9 *half-dozens;* so that the work might be perhaps more expeditiously performed as in the margin after replacing 9*s.* 7¾*d.* by 115*s.* 9*d.* (Rule p. 62.)

It may be as well to add here that, although the present problem concerns *gallons* only, yet that this *first* rule equally applies, whatever be the nature of the single article, and whatever be the number of articles. We shall now give a few unworked examples for exercise in the two rules above.

2. If the price of a gallon be 6*s.* 3*d.*, what is the price of a hogshead?
   Ans. £19 13*s.* 9*d.*
3. If the price of a gallon be 14*s.* 9*d.*, what is the price of a hogshead?
   Ans. £46 9*s.* 3*d.*
4. If a gallon cost 13*s.* 4½*d.*, what will a hogshead cost?
   Ans. £42 2*s.* 7½*d.*
5. If a gallon cost 11*s.* 5¾*d.*, what will 76 gallons cost?
   Ans. £43 12*s.* 5*d.*

NOTE.—As a tun is equal to four hogsheads, the price of a tun is found by multiplying the price of a hogshead by 4.

PROBLEM 2. (CONVERSE OF PROB. 1.)

From the price of a hogshead to find the price of a gallon.

The most convenient way of solving this problem is to proceed by the method of common arithmetic, namely:

RULE.—Divide the price of the hogshead by 63 (or by 7 and 9); the quotient will be the price of a gallon. For example:

If the price of a hogshead be £46 9*s.* 3*d.*, what will be the price of a gallon?

```
      £  s. d.                    £  s.  d.
      46 9  3           or,   7) 46  9   3
      20                        ----------
      ---                       9) 6 12  9
 63)  929 (14s. 9d. Ans.         ----------
      63                          14s. 9d. Ans.
      ---                         -------
      299
      252
      ---
       47
       12
      ---
      567 (9d.
      567
      ---
```

As this is the process of ordinary arithmetic, additional examples may not be necessary; but if the reader desire further exercise in the present problem he can take the reverse of the examples in the problem preceding, just as we have here taken the reverse of Example 3.

[The retailer of spirits can easily determine his profit upon the hogshead by recollecting that every penny added to the price of the gallon adds 5*s*. 3*d*. to the price of the hogshead. But the usual practice is to secure profit, not by increasing the cost price of the article, but by reducing its strength by diluting it. In this way the profit, per gallon, which accrues from adding a certain quantity of water, is not quite so readily estimated. Suppose, for example, that the prime cost per gallon is 12*s*., and that the retailer adds a pint of water to the gallon; then he has got 9 pints of the diluted spirits for 12*s*.; a gallon of it therefore (8 pints) costs him only eight-ninths of 12*s*., that is, 12*s*. *less* one-ninth of 12*s*.; so that, as $\frac{1}{9}$th of 12*s*. is 1*s*. 4*d*., this is the profit he receives upon every gallon of the mixture, provided he sell it at the cost price of the unreduced article.

The general principle may be expressed thus:—

Whatever fraction the water, added to the gallon of spirits, is of the *whole mixture*, that fraction of the cost price of the spirits is the profit upon a gallon of the mixture. Thus, if a pint of water be added to the gallon of spirits, the profit, per gallon, *of the mixture*, will be $\frac{1}{9}$th of the cost of the gallon of spirits; if a quart be added, the profit, per gallon, will be $\frac{1}{5}$th the cost of the gallon of spirits; if a gallon be added, the profit, per gallon, of the mixture, will be *half* the cost of the gallon of spirits; and so on.

It may be useful to make this matter the subject of two or three distinct problems].

Problem 3.

The prime cost of a gallon of spirits being given, to find how much pure water must be added in order that an assigned amount of profit may be secured upon the outlay; the selling price, per gallon, being the same as the cost price of a gallon of the unadulterated article.

The water costs nothing, but when incorporated with the spirits it fetches the cost price of the spirits, per gallon.

Hence, this cost price, *multiplied* by the fraction which the water is of a gallon, is the profit upon the whole mixture; and consequently the profit, *divided* by the price of a gallon of the spirits, must be equal to this fraction of a gallon of water. The rule is therefore this:—

RULE.—Divide the proposed profit by the given price per gallon; the quotient will be the fraction which expresses the portion of a gallon of water to be added.

EXAMPLES.

1. If a gallon of spirits cost 12*s*., how much water must be added to it in order that the mixture, at 12*s*. per gallon, may yield a profit of 2*s*.?

   2*s*. $\div$ 12*s*. $= \frac{1}{6}$ of a gallon $= \frac{8}{6}$ pints $= 1\frac{1}{3}$ pints;

   so that a pint and one-third of a pint of water must be added. This may be easily verified as follows: $\frac{1}{6}$ gallon of water being added, the measure of the mixture is $1\frac{1}{6}$ gallons; this, at 12*s*. per gallon, amounts to 12*s*. $\times\ 1\frac{1}{6} =$ 14*s*., which gives a profit of 2*s*. upon the outlay per gallon.

2. If a gallon of spirits cost 13*s*. 6*d*., how much water must be added to secure a profit of 3*s*. upon this 13*s*. 6*d*.?

   3*s*. $\div$ 13*s*. 6*d*. $= 6 \div 27 = \frac{6}{27}$, the fraction of a gallon $= \frac{48}{27}$ pints $=$ 1 pint and $\frac{21}{27}$ths of a pint, that is, $\frac{7}{9}$ths of a pint. We shall verify this result, as in the former example. The measure of the mixture (replacing $\frac{6}{27}$ by the equivalent fraction $\frac{2}{9}$) is $1\frac{2}{9}$ gallon, which, at 13*s*. 6*d*. per gallon, amounts to 27 sixpences $\times\ 1\frac{2}{9} =$ 27 sixpences + 6 sixpences $=$ 13*s*. 6*d*. + 3*s*.; so that the profit is 3*s*.

3. How much water must be added to a gallon of spirits at 15*s*. to secure a profit of 3*s*. 6*d*.? *Ans*. $\frac{7}{30}$ of a gallon.
4. How much water must be added to a gallon of spirits at 14*s*. 6*d*. to secure a profit of 2*s*. 9*d*.? *Ans*. $\frac{11}{58}$ of a gallon.
5. How much water must be added to a gallon of spirits at 18*s*. 4*d*. in order that the profit upon it may be 3*s*. 8*d*.? *Ans*. $\frac{1}{5}$ of a gallon.

[In the above problem the amount of profit is given to determine the quantity of water; but if the quantity of water be given to determine the amount of profit upon the sale of the whole mixture, then, whatever be the quantity of water, the profit will obviously be the worth of that quantity of unmixed spirits. How to find the profit *per gallon* of the mixture has been sufficiently explained in the directions immediately preceding the present problem.]

PROBLEM 4.

1. *A given quantity of sweetened water being added to a gallon of spirits, to find the profit on the prime cost of the gallon, the selling price being at the rate of the cost price per gallon.*
2. *To find how much sweetened water must be added to a gallon of spirits in order that the profit upon that gallon may be a given sum, the selling price being the same as the cost price of the unadulterated spirits per gallon.*

The first part of this problem requires no special rule; it is plain that whatever be the quantity of sweetened water added, the profit will be the cost price of that quantity of pure spirits *less* the price of the sugar. For the second part of the problem we give the following rule:—

RULE.—Divide the proposed profit by the prime cost of the gallon of spirits *diminished* by the cost of the sugar used in sweetening a gallon of water; the quotient will be the quantity (or fraction of a gallon) of the sweetened water to be added.

For if a *whole* gallon of the sweetened water were to be added, the profit upon the mixture would be the cost price of a gallon of the pure spirits, diminished by the cost price of a gallon of the sweetened water; consequently whatever *fraction* of a gallon be added, the profit will be found by multiplying the cost of a gallon of spirits *less* the cost of a gallon of the sweetened water, by that fraction; and therefore the fraction itself will be the quotient arising from dividing the proposed profit by the cost of a gallon of the spirits diminished by the cost of a gallon of the sweetened water; and this is the rule.

EXAMPLES.

1. If a gallon of spirits cost 12*s*., how much sweetened water must be added to it, in order that the mixture, at 12*s*. per gallon, may yield a profit of 2*s*., the cost of the sugar being 8*d*. for a gallon of the sweetened water?

$$2s. \div 11s.\ 4d. = 2s. \div 11\tfrac{1}{3}s. = \tfrac{6}{34} = \tfrac{3}{17} \text{ of a gallon.}$$

Or, reducing at once to *fourpences*, of which a shilling contains three, we have $6 \div 34 = \frac{3}{17}$, which is the portion of a gallon to be added to every gallon of pure spirits, in order that the profit on that gallon may be 2*s*. Or, 3 gallons (or 3 measures of any kind) of the sweetened water must be added to 17 gallons (or 17 like measures) of the spirits, in

order that 2*s*. profit may be realized upon every gallon of pure spirits in the mixture. If the water were unsweetened, then to produce the same profit, per gallon of spirits, 3 gallons (or measures) of pure water must be added to 18 gallons (or measures) of spirits; or 1 of the former to 6 of the latter. (Ex. 1 Prob. 3.) As in the example here referred to, we shall now verify the foregoing conclusion.

The selling price of the whole $1\frac{3}{17}$ gallons is $12s. + \frac{3}{17}\,12s.$, that is, $12s. + \frac{36}{17}s. = 12s. + 2s. + \frac{24}{17}d.$ And the cost price is $12s. + \frac{3}{17}\,8d. = 12s. + \frac{24}{17}d.$; hence, subtracting this amount from the former, 2*s*. is the amount of profit on the purchased gallon.

2. If a gallon of spirits cost 13*s*. 6*d*., how much sweetened water must be added to it so that the reduced spirits, at the same price per gallon, may yield a profit of 3*s*.; the cost of the sugar in a gallon of the sweetened water being 10*d*.?

   Ans. $\frac{9}{38}$ of a gallon; or 9 measures of the water to 38 of the spirits.

3. If a gallon of spirits cost 25*s*. 10*d*., and a gallon of sweetened water cost 9*d*., how much of the latter must be added to the former, so that the mixture, at 25*s*. 10*d*. per gallon, may yield a profit of 3*s*. 6*d*. upon the outlay?

   Ans. $\frac{6}{43}$ of a gallon; or 6 measures of the water to 43 of the spirits.

[It will of course be remembered, in all cases coming under the present problem, that the predetermined profit is that upon each gallon of *pure spirits* in the mixture, and not the profit upon each gallon of the mixture itself. To find *this* profit is the object of the next problem.]

Problem 5.

A given quantity of water, sweetened or unsweetened, being added to a gallon of spirits, to find the profit upon a gallon of the mixture, if sold at as much per gallon as the undiluted article cost.

1. If pure water only be added to the gallon, the profit, upon the sale of the whole mixture, will obviously be equal to the cost of an equal measure of the undiluted spirits. Hence we have only to divide this profit upon the whole mixture by the number of gallons in it, in order to find the profit upon a single gallon of it.

2. If the water added be sweetened, the profit upon the whole mixture will be the cost of an equal measure of spirits *less* the cost of the sugar in the water added; so that, by dividing this profit by the number of gallons in the mixture,

the result must be the profit on a single gallon of that mixture. Hence, whether the water be sweetened or unsweetened, the rule is as follows:—

RULE.—Find the profit upon the sale of the whole mixture, or take this profit if it be already assigned (Problems 3, 4), and divide it by the number of gallons in the whole; the quotient will be the profit per gallon on the diluted spirits.

EXAMPLES.

1. If a gallon of spirits cost 12*s.*, $\frac{1}{6}$ of a gallon of pure water must be added to it in order that the profit upon the whole mixture may be 2*s.* (Ex. 1, Prob. 3); what is the profit upon each gallon of the spirits so diluted?

Here the whole mixture measures $1\frac{1}{6}$ gallons, the profit on which is 2*s.*; therefore 2*s.* $\div 1\frac{1}{6}$, or $\frac{12}{7}$*s.* = 1*s.* $8\frac{4}{7}$*d.* is the profit per gallon on the diluted spirits.

2. The quantity of pure water which must be added to a gallon of spirits, at 13*s.* 6*d.*, in order that the profit upon the whole mixture may be 3*s.*, is $\frac{2}{9}$ of a gallon (Ex. 2, Prob. 3); what is the profit, per gallon, on the diluted spirits? *Ans.* 2*s.* $5\frac{5}{11}$*d.*

3. The quantity of sweetened water, worth 10*d.* per gallon, to be added to a gallon of spirits, at 13*s.* 6*d.*, in order that the profit upon the whole mixture may be 3*s.*, is $\frac{9}{38}$ of a gallon (Ex. 2, Prob. 4); what is the profit, per gallon, on the diluted spirits? *Ans.* 2*s.* $5\frac{5}{47}$*d.*

[The reader is recommended to verify these two results; that is, to prove first that if the profit upon a gallon of the mixture be 2*s.* $5\frac{5}{11}$*d.*, the profit upon $1\frac{2}{9}$ gallons will be 3*s.*; and secondly, if the profit upon a gallon of the mixture be 2*s.* $5\frac{5}{47}$*d.*, that the profit upon $1\frac{9}{38}$ gallons will be just the same, namely, 3*s.* It may be well to give a hint as to the best way of doing this. Take the latter case, but, to get rid of the fraction, multiply $1\frac{9}{38}$ by 38; we thus have 47; and 2*s.* $5\frac{5}{47}$*d.* $\times$ 47 = 94*s.* + 240*d.* = 114*s.*, which divided by 38 gives 3*s.*, as it ought to do.]

PROBLEM 6.

To find how much water, sweetened or unsweetened, must be added to each gallon of spirits, in order that the profit upon a gallon of the mixture may be an assigned sum.

RULE.—1. Add the proposed profit upon a gallon of the mixture to the cost of a gallon of the water. [If unsweetened

this cost is of course nothing, and there is then to be no addition.]

2. Subtract the result from the cost of a gallon of the spirits, and divide the proposed profit by the remainder; the quotient will be the portion of a gallon of the diluting liquid to be added to each gallon of spirits.

EXAMPLES.

1. How much pure water must be added to a gallon of spirits, at 13*s*. 6*d*., in order that the profit upon the mixture may be at the rate of 2*s*. $5\frac{5}{11}$*d*. per gallon? We proceed by the Rule as follows:—

13*s*. 6*d*. − 2*s*. $5\frac{5}{11}$*d*. = 162*d*. − $29\frac{5}{11}$*d*. = $132\frac{6}{11}$*d*. We have therefore to execute the division of $29\frac{5}{11}$ by $132\frac{6}{11}$; and in order to accomplish this, without the fractions, we multiply both dividend and divisor by 11 (p. 55); we thus have, 324 ÷ 1458, or $\frac{324}{1458} = \frac{2}{9}$ of a gallon of water, as we otherwise know it ought to be (see Ex. 2, p. 109). The $\frac{2}{9}$ is the fraction $\frac{324}{1458}$ in its lowest terms; for both numerator and denominator of that fraction are divisible by 162, the numerator giving 2 for quotient, and the denominator giving 9.

In this example the water is unsweetened, and therefore costs nothing. We shall now work an example in which the water is sweetened, and therefore involves an outlay; we shall then give the reasoning from which the rule is deduced.

2. How much sweetened water, worth 10*d*. a gallon, must be added to each gallon of spirits, at 13*s*. 6*d*., in order that the profit upon the mixture may be at the rate of 2*s*. $5\frac{5}{47}$*d*. per gallon?

2*s*. $5\frac{5}{47}$*d*. + 10*d*. = 3*s*. $3\frac{5}{47}$*d*. = $39\frac{5}{47}$*d*.; then, by the Rule, 162*d*. − $39\frac{5}{47}$*d*. = $122\frac{42}{47}$*d*.; and by this the profit per gallon, namely, $29\frac{5}{47}$*d*., is to be divided. In order to readily perform the division, take 47 times dividend and divisor; we shall then have 1368 ÷ 5776; or, dividing each number by 152, the fraction is $\frac{9}{38}$, the portion of a gallon of sweetened water to be added to each gallon of spirits. (See Ex. 3, p 109.)

[The method of reducing a fraction to its lowest terms is taught in all books of common arithmetic. The Table at p. 30 will also show what the factors are of any number within its limits.]

The foregoing rule is deduced from the following considerations, to which it is necessary, for the clear perception of its truth, that the reader give careful attention.

1. The profit upon the whole mixture consists entirely of the profit upon the measure of diluting liquid which is added to the gallon of spirits; it being sold at the same price as an equal measure of the spirits costs.

2. Suppose, for convenience, we represent the unknown fraction of a gallon which measures the quantity of the diluting liquid to be added to the gallon of spirits, by the symbol q;* then this portion of a gallon sells for the qth part of the cost of a gallon of the spirits; that is, it *sells* for the cost of a gallon of the spirits multiplied by q, whatever fraction q may represent; but it *costs* the qth part of the cost of a gallon of the diluting liquid; that is, it costs the value of a gallon of the diluting liquid multiplied by the fraction q. Hence, the profit upon the *whole* mixture is what remains after subtracting this product from the former.

3. But we want to know the profit, not upon the whole mixture, but upon only a gallon of it; let us then imagine the excess above a gallon to be taken away; we shall thus take away from the profit on the *whole* the profit on the portion abstracted; and since the quantity of the mixture subtracted is just equal to the quantity of the diluting liquid that was previously added, the *profit* thus taken away from the *whole profit* is a qth part of the profit *per gallon*, and the remainder is the profit on the remaining gallon of the mixture. The qth part of the profit, per gallon, is, of course, expressed by multiplying the profit upon the whole gallon by the fraction q.

4. It thus appears that the profit upon this remaining gallon of the mixture is to be found by subtracting from the cost of a gallon of the spirits when multiplied by q, the cost of a gallon of the diluting liquid when multiplied by q, and then further subtracting the profit, per gallon, of the mixture when multiplied by q:—each item is to be multiplied by q.

The inference therefore is, that if we add the profit upon a gallon of the mixture to the cost of a gallon of the diluting liquid, subtract the sum from the cost of a gallon of the spirits, and multiply the remainder by the fraction q, the product will be the profit, per gallon, on the mixture.

5. But if the product of two factors be divided by either factor, the quotient must be the other factor. Hence, if we divide the profit, per gallon, on the mixture, by the above-

* The reader may replace the symbol q by any fraction he please, say, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{2}{7}$, &c.; and then, substituting this fraction throughout the following reasoning for the letter q, he will find, choose whatever fraction he may, that the inference is the same. See the NOTE on this Prob. at the end.

mentioned remainder, the quotient will be the fraction q; that is, the fraction of a gallon of the diluting liquid which must be added to a gallon of the pure spirits, in order that the assigned profit, on a gallon of the mixture, may be realized. And hence the rule.

3. How much pure water must be added to a gallon of spirits, at 12*s.*, in order that the profit upon a gallon of the mixture may be 1*s.* 8$\frac{4}{7}$*d.*?

Ans. $\frac{1}{6}$ of a gallon; or 1 gallon of water to 6 gallons of spirits.

4. How much sweetened water, worth 9*d.* per gallon, must be added to a gallon of spirits, at 15*s.* 9*d.*, in order that the profit, per gallon, on the mixture may be 2*s.* 6*d.*?

Ans. $\frac{1}{5}$ of a gallon; or 1 gallon of the water to 5 gallons of the spirits.

Problem 7.

To find the proportion in which spirits, at two different prices per gallon, must be mixed in order that the compound may cost an assigned intermediate price per gallon.

Rule.—Divide the difference between the higher price and the intermediate price by the difference between the intermediate price and the lower price; the quotient will be the quantity of the *cheaper* spirits to be added to a gallon of the dearer.

Or, divide the difference between the intermediate price and the lower price by the difference between the higher price and the intermediate price; the quotient will be the quantity of the *dearer* spirits to be added to a gallon of the cheaper.

Examples.

1. What quantity of spirits, at 9*s.* 6*d.* per gallon, must be mixed with a gallon of spirits at 15*s.*, so that the cost of the compound may be 13*s.* 6*d.* per gallon?

15*s.* − 13*s.* 6*d.* = 1*s.* 6*d.* = 18*d.*: 13*s.* 6*d.* − 9*s.* 6*d.* = 4*s.* = 48*d.*; and $\frac{18}{48} = \frac{3}{8}$, the part of a gallon to be added; or 3 gallons of the inferior spirits to 8 of the superior, as may be thus verified: 3 gallons at 9*s.* 6*d.* cost 28*s.* 6*d.*, and 8 gallons at 15*s.* cost 120*s.*; so that the entire 11 gallons in the compound cost 148*s.* 6*d.*, and therefore the cost of a single gallon is 148*s.* 6*d.* ÷ 11 = 13*s.* 6*d.*, as it ought to be.

2. What quantity of pure water must be added to a gallon of spirits worth 12*s.*, in order that the mixture may cost at the rate of 10*s.* per gallon?

Here the inferior spirit (so to call it) is worth nothing. The first of

the two differences is 12*s.*—10*s.* = 2*s.*; the second is 10*s.* itself; therefore 2*s.* ÷ 10*s.* = $\frac{1}{5}$, the part of a gallon of water to be added. The measure of the whole mixture is therefore $1\frac{1}{5}$ gal., and its cost 12*s.*: hence its cost per gallon is 12*s.* ÷ $1\frac{1}{5}$, or 60*s.* ÷ 6 = 10*s.*, as it ought to be. And there must be 1 gallon of water to every 5 gallons of spirits.*

3. How much sweetened water, at 9*d.* per gallon, must be added to a gallon of spirits, at 15*s.* 9*d.*, so that the compound may cost at the rate of 13*s.* 3*d.* per gallon?

 15*s.* 9*d.* − 13*s.* 3*d.* = 2*s.* 6*d.*, and 13*s.* 3*d.* — 9*d.* = 12*s.* 6*d.*
 2*s.* 6*d.* ÷ 12*s.* 6*d.* = $2\frac{1}{2} \div 12\frac{1}{2} = \frac{5}{25} = \frac{1}{5}$ [See Ex. 4, Prob. 6].

4. How much spirits, at 15*s.* per gallon, must be mixed with a gallon at 9*s.* 6*d.*, in order that the compound may cost at the rate of 13*s.* 6*d.* per gallon?

 13*s.* 6*d.* — 9*s.* 6*d.* = 4*s.*: 15*s.* − 13*s.* 6*d.* = 1*s.* 6*d.*: then

48*d.* ÷ 18*d.* = $\frac{48}{18} = \frac{8}{3} = 2\frac{2}{3}$ gallons, the quantity required. Let us verify this: The cost of the whole $3\frac{2}{3}$ gallons is 9*s.* 6*d.* + 15*s.* × $2\frac{2}{3}$ = 9*s.* 6*d.* + 40*s.* = 49*s.* 6*d.*, and therefore the cost of 1 gallon is 49*s.* 6*d.* ÷ $3\frac{2}{3}$, or (multiplying each by 3), 148*s.* 6*d.* ÷ 11 = 13*s.* 6*d.*, as it ought to be.

And in this way may the truth of the Rule be tested, and satisfactorily proved, in every individual example to which it is applied. To prove it *generally*, without reference to particular examples, like the proof of the preceding Rule (Prob. 6), would be rather a tedious business without the aid of *Algebra*, a branch of science which the readers of this work are not to be presumed to be acquainted with. (See, however, p. 123.)

It is evident that the proportion in which the two kinds of spirits are to be mixed, so that the cost price of a gallon of the compound may be a predetermined sum, being found, as in the foregoing examples, the retailer has only got to increase this cost price by what he proposes for profit, per gallon, of the mixture, in order to know the proper retail price per gallon.

As such calculations as the above, concerning the mixing of spirits, must be the same, whatever be the ingredients mixed, we shall continue the subject under a more general head.

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* It will be seen that the present Rule is but a more general form of that at p. 109 (Prob. 6). What here is inferior *spirits*, is there pure or sweetened *water*. A Rule still more general is given at p. 115.

CALCULATIONS USEFUL IN THE MIXING OF TEAS, SUGARS, SPIRITS, GRAIN, &c.

PROBLEM 1.

When given measures (or weights) of ingredients, at different prices, are mixed together, to find the price of a single measure (or weight) of the mixture.

RULE.—Multiply the price of one measure (or weight) of each ingredient by the number of measures (or weights) in it, and add all the products together; add also the different measures (or weights) themselves together, and divide the former sum by the latter; the quotient will be the price of a single measure (or weight) of the mixture.

EXAMPLES.

1. If 4 cwt. of sugar, at 56*s*. per cwt., 7 cwt. at 43*s*., and 5 cwt. at 37*s*., be mingled together: what will 1 cwt. of the mixture be worth ?

$$\begin{array}{l} \quad s. \qquad\quad s. \\ 56 \times 4 = 224 \\ 43 \times 7 = 301 \\ 37 \times 5 = 185 \\ \overline{16)} \;\; \overline{710}s. \; (44\tfrac{3}{8}s. = 44s.\ 4\tfrac{1}{2}d., \textit{Ans.} \end{array}$$

It is plain that in every case, as well as here, the worth of the whole mixture must be the worth of all the ingredients. In the present instance the worth of the whole 16 cwt. in the mixture is found to be 710*s*.; and therefore the worth of 1 cwt. of it must be the 16th part of 710*s*., namely, 44*s*. 4½*d*.

2. If 27 bushels of wheat at 5*s*. 6*d*. per bushel, the same quantity of rye at 4*s*. per bushel, and 14 bushels of barley at 3*s*. per bushel be mixed together, what will be the worth of a bushel of the mixture ?

$$\begin{array}{l} 27 \text{ at } 5s.\ 6d. = 148s.\ 6d. \\ 27 \;,,\; 4 \quad 0 \;\; = 108 \quad 0 \\ 14 \;,,\; 3 \quad 0 \;\; = \;\; 42 \quad 0 \\ \overline{68)} \qquad\qquad \overline{298 \quad 6} \; (4s.\ 4\tfrac{1}{2}d. + \tfrac{12}{17}f.\ \textit{Ans.} \end{array}$$

If the mixture be sold at 4*s*. 5*d*. per bushel, as it no doubt would be, the gain upon the whole 68 bushels would be 68 times 1 farthing plus 68 times $\frac{5}{17}$*f*.; that is, 68*f*. + 20*f*. = 88*f*. = 1*s*. 10*d*.

3. If 5 gallons of wine at 7*s*. per gallon, 9 gallons at 8*s*. 6*d*., and 14½ gallons at 5*s*. 10*d*. be mixed together, what will a gallon of the mixture be worth ? *Ans*. 6*s*. 10½*d*.

4. If 20 bushels of wheat at 5*s.*, 36 bushels of rye at 3*s.*, and 40 bushels of barley at 2*s.* be mixed together, what will be the worth of a bushel of the mixture? *Ans.* 3*s.*
5. If 20 gallons at 5*s.* 4*d.*, 12 at 5*s.*, 30 at 6*s.*, and 20 at 4*s.* 6*d.* be mixed together, what will the mixture be worth per gallon? *Ans.* 5*s.* $3\frac{3}{4}d. + \frac{25}{41}f.$

PROBLEM 2.

The price of each of the several ingredients being given, to find in what proportions they may be mixed together, in order that the composition may bear a given price.

RULE.—Write the prices of the ingredients one under another in a column, commencing with either the lowest price or the highest, and proceeding in order, leaving a small space to separate the prices, each of which is higher than the proposed intermediate price, from those which are lower; and to the left of the column, against this space, write the given intermediate price.

2. Link together, by a curve or connecting line, each number above the space with one or other of the numbers below it, and each number below the space with one or other of the numbers above it.

3. Against the first number in the column of numbers write the difference between the number with which it is linked and the isolated number to the left of the space; and if it be linked to more numbers than one, write the sum of *all* the differences; and do the same with every number in the column. The numbers thus written against the several prices will express what quantities, at those prices, may be mixed together.

NOTE.—When more than two ingredients are to be mixed, they may be mixed in different proportions, and yet be worth the assigned price per gallon, pound, cwt., &c., as will be sufficiently seen in the following examples.

EXAMPLES.

1. In what proportion may teas, at 4*s.* 8*d.*, 4*s.*, 3*s.* 8*d.*, and 2*s.* 6*d.* per lb., be mixed, in order that the mixture may be worth 3*s.* 10*d.* per lb?

	56	16 at 4*s.* 8*d.*
46	48	2 „ 4*s.*
	44	2 „ 3*s.* 8*d.*
	30	10 „ 2*s.* 6*d.*

Proof.

16	at 56*d.*	=	896*d.*
2	„ 48	=	96
2	„ 44	=	88
10	„ 30	=	300
30)			1380 (46*d.* = 3*s.* 10*d.*

Otherwise,

46	56	2 at 4*s*. 8*d*.	
	48	16 „ 4*s*.	
	44	10 „ 3*s*. 8*d*.	
	30	2 „ 2*s*. 6*d*.	

Proof.

2 at 56*d*.	=	112*d*.
16 „ 48	=	768
10 „ 44	=	440
2 „ 30	=	60
30)		1380 (46*d*.

Hence the mixture may be in either of these two proportions; or, taking only half of each of the ingredients, the desired compound will be produced by mixing together 8 lbs. of the highest price tea, 1 lb. of the next highest, 1 lb. of the next, and 5 lbs. of the cheapest; or otherwise, by taking 1 lb. of the highest price, 8 lbs. of the next price, 5 lbs. of the next, and 1 lb. of the cheapest. And generally, in all cases of the kind, the several weights or measures of the component ingredients being determined, as above, they may all be multiplied or divided each by the same number,—any number we please; since the resulting quantities will be to one another still in the same proportion; and therefore however large the quantities of the separate ingredients in either of the sets determined by the foregoing process may be, they may all be reduced, by division, so as to come within any limits, as to quantity, which we may choose to fix. We see, however, that the proportions of the distinct ingredients in the different sets found by the Rule, may *themselves* be very different:—no common multiplier or divisor applied to the several quantities in the first set above, will give the several quantities of the second set. (See the Remarks at page 121.)

2. In what proportion may whiskies at 16*s*., 18*s*., and 22*s*. per gallon be mixed, so that the compound may be worth 20*s*. per gallon?

20	16	2 at 16*s*.
	18	2 at 18*s*.
	22	4 + 2 = 6 at 22*s*.

Or, taking the half of each, 1 gallon at 16*s*., 1 gallon at 18*s*., and 3 gallons at 22*s*.; the whole number of gallons being 5, and their value 16*s*. + 18*s*. + 66*s*. = 100*s*., which, divided by 5, gives 20*s*., the price of 1 gallon of the mixture, as it ought to do. And in a similar manner may the accuracy of the general rule, as applied to any particular case, be proved.

3. In what proportion may spirits at 16*s*., 14*s*., 9*s*., and 8*s*. per gallon be compounded in order that the mixture may be worth 10*s*. per gallon? *Ans.* 1 gal. at 16*s*., 2 at 14*s*., 6 at 9*s*., and 4 at 8*s*.

4. In what proportion may raisins at 4*d.*, 6*d.*, and 10*d.* per lb. be mixed, so that the compound may be worth 8*d.* per lb.?
Ans. 1 lb. at 4*d.*, 1 at 6*d.*, and 3 at 10*d.*

5. Four sorts of cheap wines at 1*s.* 6*d.*, 1*s.* 8*d.*, 2*s.*, and 2*s.* 4*d.* per quart, respectively, are to be mixed together, so that the compound may be worth 1*s.* 10*d.* per quart: required the quantity of each sort that may be used?
Ans. 1 quart at 1*s.* 6*d.*, 3 at 1*s.* 8*d.*, 2 at 2*s.*, and 1 at 2*s.* 4*d.*
Or 3 at 1*s.* 6*d.*, 1 at 1*s.* 8*d.*, 1 at 2*s.*, and 2 at 2*s.* 4*d.*

[From what is here shown, the reader will readily perceive that the special Rule given in the last article (p. 112) is comprehended in the general Rule above, and that the examples already worked by that Rule may be very conveniently solved by this; we shall here select two of them, and give the solution of each in the margin.

1. Spirits at 12*s.* per gallon are to be mixed with as much water as will reduce the value to 10*s.* per gallon; required the proportion of water to the spirits? (Ex. 2, Prob. 7.) The proportion must be 5 gallons of spirits to 1 gallon of pure water, the cost of the water being 0.

10 | 12 — 10 at 12*s.*,
   | 0 — and 2 at 0;
or 5 at 12*s.*, and 1 at 0.

2. Spirits at 15*s.* 9*d.* per gallon are to be diluted with sweetened water, at 9*d.* per gallon, in such proportion that the mixture may be worth 13*s.* 3*d.* per gallon; required the proportion? (Ex. 3, Prob. 7.)]

13*s.* 3*d.* | 15*s.* 9*d.* — 12½
   | 9*d.* — and 2½,
or 25 and 5.

Problem 3.

When one of the ingredients is limited to a certain quantity, and the prices only of the other ingredients are given, to find how much of each of these latter may be mixed with the given fixed quantity of the former, in order that the whole mixture may be worth a proposed intermediate price, per pound, gallon, &c.

Rule.—1. Arrange the several prices in column as before; and, as before, take the difference between each and the intermediate price.

2. Then, as the difference written against the price of the given quantity is to any other of the differences, so is the given quantity itself to the quantity against the price of which that other difference is written. Or, which is the same thing, multiply the given quantity by the difference written against the price of any other of the ingredients, and divide the product by the difference written against the

price of the ingredient whose quantity is given: the quotient will be the quantity to be taken of that other ingredient.

Examples.

1. A grocer proposes to mix 20 lbs. of coffee, at 15*d.* per lb., with other coffees at 16*d.*, 18*d.*, and 22*d.* per lb., so that the mixture may be worth 17*d.* per lb.: how much of each of the three latter may he use?

17	15	5	Then, per Rule,	lbs.
	16	1		5 : 1 :: 20 : 4 lbs. at 16*d.*
	18	1		5 : 1 :: 20 : 4 lbs. at 18*d.*
	22	2		5 : 2 :: 20 : 8 lbs. at 22*d.*

Or,

20 lbs. ÷ 5 = 4 lbs., 20 lbs. ÷ 5 = 4 lbs., 40 lbs. ÷ 5 = 8 lbs.

Otherwise.

17	15	1	Then,	
	16	5		20 lbs. × 5 = 100 lbs. at 16*d.*
	18	2		20 lbs. × 2 = 40 lbs. at 18*d.*
	22	1		20 lbs. × 1 = 20 lbs. at 22*d.*

[Other suitable mixtures may be obtained by linking the pairs differently.]

Verification of the first set of results.

4 at 16*d.*	=	64*d.*
4 „ 18	=	72
8 „ 22	=	176
and 20 „ 15	=	300
36)		612 (17*d.* per lb.

Verification of the second set of results.

100 at 16*d.*	=	1600*d.*
40 „ 18	=	720
20 „ 22	=	440
and 20 „ 15	=	300
180)		3060 (17*d.*

2. How much wine at 5*s.*, 5*s.* 6*d.*, and 6*s.* per gallon, may be mixed with 3 gallons at 4*s.* per gallon, so that the mixture may be worth 5*s.* 4*d.* per gallon?

64	48	8		
	60	2 × 3 = 6, which ÷ 8 =	¾ at 60*d.*	= 45*d.*
	66	4 × 3 = 12, „	= $\frac{3}{2}$ „ 66	= 99
	72	16 × 3 = 48, „	= 6 „ 72	= 432
			Also 3 „ 48	= 144
			11¼	720*d.*

Multiplying divisor and dividend each by 4 . . 45) 2880 (64*d.*

Here the work is arranged so as to exhibit the required results, and the verification of them, in a more compact form. The answer is ¾ gal. at 5*s.*, 1½ gallons at 5*s.* 6*d.*, and 6 gallons at 6*s.*

Otherwise.

$$\begin{array}{cr|lcrl}
 & 48 & 8 + \ 2 = 10, & & & \\
 & 60 & 2 + \ 8 = 10, & \text{which} \times 3 \text{ and} \div 10 & = 3 \text{ at } 60d. & = 180d. \\
64 & 66 & 4 + 16 = 20, & ,, & = 6 \ ,, \ 66 & = 396 \\
 & 72 & 16 + \ 4 = 20, & ,, & = 6 \ ,, \ 72 & = 432 \\
 & & & \text{Also} & 3 \ ,, \ 48 & = 144 \\
 & & & & \overline{18)} & \overline{1152} \ (64
\end{array}$$

Therefore the desired mixture may also be prepared by adding to the 3 gallons at 4*s*., 3 at 5*s*., 6 at 5*s*. 6*d*., and 6 at 6*s*. We would recommend that in these examples the truth of the results be always tested and verified in this way. (See the *General Remarks* at page 121.)

3. A grocer wishes to mix teas at 6*s*., 5*s*., and 3*s*. per lb., respectively, with 20 lbs. at 2*s*., so that he may afford to sell the mixture at 4*s*. per lb.: what quantities will suffice for the purpose?
   Ans. Either 20 lbs. at 6*s*., 10 lbs. at 5*s*., and 10 lbs. at 3*s*., or 20 lbs. at 6*s*., 40 lbs. at 5*s*., and 40 lbs. at 3*s*. [See NOTE below.]

4. Spirits at 7*s*. and 4*s*. per gallon, respectively, are to be mixed with 40 gallons of other spirits at 12*s*. per gallon: what quantities will suffice, in order that the compound may be fairly charged at 8*s*. per gallon? *Ans.* 32 gallons of each.

NOTE.—The prices of the several components in Example 3 are so related, that the question may be answered at once, without employing the pen: we know that 6*s*. tea and 2*s*. tea, mixed in equal quantities, any whatever, will make 4*s*. tea; and so likewise will equal quantities of 5*s*. and 3*s*. tea. Hence a suitable compound will be obtained, by taking 20 lbs. of the 6*s*. tea, and any *equal* weights whatever of the teas at 5*s*. and 3*s*., and then mixing all with the given 20 lbs. at 2*s*. The two sets of components above, are those only which the RULE determines. (See the *Remarks* at page 121.)

PROBLEM 4.

When the quantity of each of two or more of the ingredients is given, to find how much of each of the other ingredients will be required to make the mixture of all worth a proposed intermediate price, per pound, gallon, &c.

RULE.—First find the price per lb., gallon, &c., of the mixture formed by combining those ingredients *only* of which the quantities are given (Prob. 1, p. 114).

Then, regarding this mixture as so many lbs., gallons, &c., of a single ingredient, at a given price per lb., gallon, &c., proceed as in last problem.

Examples.

1. 3 lbs. of tea at 2*s.* per lb., and 6 lbs. at 2*s.* 6*d.*, are to be mixed with two other sorts, at 3*s.*, and 4*s.*, respectively, so that the mixture may be worth 3*s.* 6*d.* per lb., how much of these other sorts must be used for the purpose?

First: 3 lbs. at 2*s.* = 6*s.*
and 6 lbs. „ $2\frac{1}{2}$*s.* = 15*s.*

9) 21*s.* ($2\frac{1}{3}$*s.* = 2*s.* 4*d.*

We have therefore to mix 9 lbs. at 2*s.* 4*d.* with other teas at 3*s.* and 4*s.*, in such quantities as to produce a mixture worth 3*s.* 6*d.* per lb. The proper quantities are to be found by the former Rule, thus :—

	28	6		
42	36	6	which × 9 and ÷ 6 = 9 at 36*d.*	= 324*d.*
	48	14 + 6 = 20	„ = 30 „ 48	= 1440
			Also 9 „ 28	= 252
			48)	2016 (42*d.*

Hence, if to the given quantities there be added 9 lbs. at 3*s.*, and 30 lbs. at 4*s.*, making in the whole 48 lbs., the mixture, as here shown, will be worth 42*d.* per lb., as it was required to be.

2. A grócer desires to mix 4 lbs. of coffee, at 1*s.* 6*d.* per lb., and 8 lbs., at 1*s.* 10*d.*, with such a quantity of other coffees, at 15*d.* and 16*d.* per lb., respectively, as will make the mixture worth 17*d.* per lb. How much of these other coffees must he use? *Ans.* $14\frac{2}{3}$ lbs. of each.

3. A retailer of spirits mixes 5 gallons of spirits, at 9*s.* 6*d.* per gallon, with 7 gallons, at 10*s.* 6*d.*; and he then desires to add a sufficient quantity, at 13*s.* per gallon, as will make the whole mixture worth 12*s.* per gallon: how much of the latter must he add? *Ans.* 23 gallons.

Problem 5.

When the price per gallon, pound, &c., of each ingredient in the compound, is given, to find the several quantities which may be taken, in order to form a mixture of assigned measure or weight, at an assigned price per gallon, pound, &c.

Rule.—Take the difference between each given price and the assigned intermediate price, as before: then—

As the *sum* of the differences is to either one of those differences, so is the whole compound to the quantity of that

particular ingredient against the price of which the difference thus employed is written.

Examples.

1. A druggist desires to mix ingredients at 12*d.*, 10*d.*, 6*d.*, and 4*d.* per lb., so as to make a composition of 144 lbs. worth 8*d.* per lb.: what weight of each may he take?

	12	4	48 at 12*d.*	= 576*d.*	
8	10	2	24 „ 10	= 240	12 : 4 : : 144 lbs. : 48 lbs.
	6	2	24 „ 6	= 144	12 : 2 : : 144 : 24
	4	4	48 „ 4	= 192	
		12	144)	1152 (8*d.* Proof.	

By linking differently, it will be found that by mixing 24 lbs. at 12*d.*, 48 lbs. at 10*d.*, 48 lbs. at 6*d.*, and 24 lbs. at 4*d.*, the desired composition will also be formed. (See the *Remarks* below.)

2. A grocer desires to mix together currants at 11*d.*, 9*d.*, 6*d.*, and 4*d.* per lb., so as to make a mixture of 240 lbs. worth 8*d.* per lb. How many lbs. of each sort may he use?
   Ans. 96 lbs. at 11*d.*, 48 at 9*d.*, 24 at 6*d.*, and 72 at 4*d.*

3. Sweet wines at 5*s.*, 6*s.*, 8*s.*, and 9*s.* per gallon, are to be mixed so as to make 87 gallons worth 7*s.* per gallon; how much of each sort will suffice for the purpose?
   Ans. 14½ gals. at 5*s.*, 29 at 6*s.*, 29 at 8*s.*, and 14½ at 9*s.*

4. What quantities of the several kinds of coffee, at 15*d.*, 17*d.*, 18*d.*, and 22*d.* per lb., may be mixed together in order to make a composition of 40 lbs., worth 20 pence per lb.?
   Ans. 5 lbs. at 15*d.*, 5 at 17*d.*, 5 at 18*d.*, and 25 at 22*d.*

5. Drugs at 8*s.*, 5*s.*, and 4*s.* per lb. are to be so compounded as to make a mixture of 42 lbs. worth 7*s.* per lb: required the weight suitable for this purpose of each of the three ingredients?
   Ans. 30 lbs. at 8*s.*, 6 lbs. at 5*s.*, and 6 lbs. at 4*s.*

General Remarks on the foregoing Problems.

In the foregoing problems and examples, we have comprehended all the varieties of cases which we believe can occur in the actual practice of dealers in compounds. The Rules given for determining the proper proportions in which the several distinct ingredients should be mixed together, conduct us sometimes to only one set, and sometimes to two or more sets of answers; the number of sets furnished by these rules being always limited. But whenever more than two ingredients, each at a given price, are to be so compounded as to produce a mixture at a proposed intermediate price per

gallon, per lb., &c., the sets of *possible* proportions are really innumerable; and by means of algebraic formulæ, we can assign as many of them as we please. We mention this lest the reader should infer, from the language used in the preceding questions, that the answers recorded are the only answers that can be given. We may easily show, without the aid of Algebra, that the answers to some of these questions are innumerable; an instance has indeed been already given in the Note at p. 119:—Take as another instance the worked example at p. 121, in which 144 lbs. at 8*d*. per lb. is to be made up by mixing together four several ingredients at 12*d*., 10*d*., 6*d*., and 4*d*. per lb., respectively. Each of the four *equal* parts of 144 is 36; and if we write against the highest and the lowest of the given prices any (the same) number *less* than the number 36, and then against each of the intermediate prices a number as much *greater* than 36, the four numbers will also express weights of the several component ingredients against the prices of which they are written, suitable for the proposed compound. Thus, writing 30 against the 12*d*. and also against the 4*d*., the intermediate numbers will each be 42; and we shall have

		d.	d.				d.	d.
30	at	12 =	360;	or writing	24	at	12 =	288
42	„	10 =	420	„	48	„	10 =	480
42	„	6 =	252	„	48	„	6 =	288
30	„	4 =	120	„	24	„	4 =	96
144)			1152 (8*d*.		144)			1152 (8*d*.

And the result would have been the same if any other number below 36 had been chosen, whether whole or fractional; and if this chosen number had been written against the two *intermediate* prices, instead of against the two extreme prices, the result would have been still the same.

Again: the answer to every such example may be verified in a manner different from that which we have adopted (pp. 115-18). In arranging the prices of the several component ingredients in column, we have recommended a small space to be interposed between those of these prices which exceed the proposed mean price, or worth of the compound, and those of them which fall short of this mean price. If the dealer

estimate his gain on the component articles on the one side of this interval, and his loss on those on the other side, by selling *all* of them at the stated intermediate price, the gain and loss must be equal, if the operation be correct; that is, the loss on one side must just balance the gain on the other. Thus, taking the example here adduced: the loss on the sale of the 48 at 8*d.* is 48 times 4*d.*, that is, 16*s.*; and the loss on the sale of the 24 at 8*d.* is 4*s.*; the entire loss being £1. But the gain on the sale of the next 24 at 8*d.* is 4*s.*; and the gain on the sale of the following 48 at 8*d.* is 16*s.*; the whole gain being £1, which balances the loss: and similarly in all other like cases.

[The reader may turn to the examples, at the pages referred to above, and verify the results in this way. He ought here, however, to be reminded that any verification of the results of an arithmetical operation which has been accurately worked out in accordance with a prescribed *Rule*, is quite superfluous whenever that Rule is previously demonstrated to be true, *generally*; that is, in every individual case. The Rules in the foregoing article have not been proved to be thus universally true: we have thought it better to give such proof in a supplementary Note, as it is necessary, for the purpose, to use the symbols of Algebra; although, as only the very first principles of that science are brought into operation, we think that no reader of ordinary intelligence will feel any difficulty in following out the reasoning.

Suppose *two* different ingredients, of given prices per lb., or per gallon, &c., are to be compounded in such proportions that the mixture may fairly bear an assigned intermediate price per lb., per gallon, &c. Let us represent each of these three distinct prices by a letter:—the respective prices (per lb., &c.) of the two ingredients, by A and C, and the intermediate price,—the price of the mixture, by B: these three letters will then symbolize *given* values, or numbers. Further: let the *unknown* number of lbs., or gallons, &c., of the commodity at price A, be represented by x: and the unknown number at price C, by y: then, obviously the condition to be satisfied is that A times x + C times y shall be equal to B times x + B times y; that is, there must be the equality

$$Ax + Cy = Bx + By$$

and therefore, the equality $(A - B)\,x = (B - C)\,y$. And to bring this equality about, it is plain that it is enough that the *known* quantity $B - C$ should replace the hitherto *unknown* quantity x, and that the known quantity $A - B$ should replace the hitherto unknown quantity y; for then we should have

$$(A - B)\,(B - C) = (B - C)\,(A - B)$$

in which we see that the desired equality *is* brought about, it being universally true that $A - B$ multiplied by $B - C$ must be equal to $B - C$ multiplied by $A - B$; and this is just what the above form declares.

Hence, arranging as in the margin, the Rule for two ingredients is proved to be true generally; the values of x and y being—

$$B \begin{array}{ll} A & x \\ C & y \end{array}$$

$x =$ the difference between B and C, and $y =$ the difference between A and B. Now from this proof of the RULE, under Prob. 2, for *two* ingredients, the truth of it for any number of ingredients immediately follows; for the several quantities are always linked together in *pairs*; and, as here shown, the required condition is always fulfilled for *each pair*; and therefore, it is necessarily fulfilled for the sum of *all* the pairs.

In Problem 3, the *quantity*, as well as the price per lb., per gallon, &c., is *also* fixed or assigned: thus, in Ex. 2 (p. 118), it is stipulated that there be just "*three* gallons at 4*s*. per gallon" in the mixture. Were it not for this limitation, the work would have been as here annexed; and there would then have been, as we here see, 30 gallons in the mixture which would, just as correctly, have borne the contemplated price, namely, 5*s*. 4*d*. per gallon. But, instead of 8 gallons at 4*s*., it is stipulated that there should be only 3; that is, only $\frac{3}{8}$ths of this quantity: *consequently*, there must be only $\frac{3}{8}$ths of each of the other quantities here determined in the margin, in order that the due *proportions* may be preserved: and similarly in all such cases. And hence the truth of the RULE.

		gal.		*d.*
64	48	8	at	48
	60	2	„	60
	66	4	„	66
	72	16	„	72
		30		

The reader who satisfies himself in this way of the truth of the two Rules under Problems 2 and 3, cannot have any doubts about the Rules under Problems 4 and 5.]

Calculations useful to Goldsmiths, Silversmiths, and Chemists.

[The calculations in the following article are all to be performed by Troy weight.]

Problem 1.

The price of a grain being given, to find the price of an ounce; and, conversely, the price of an ounce being given, to find the price of a grain.

Rule.—The price per grain, in *halfpence*, will be the price per ounce in *pounds* (£). And the price, per ounce, in *pounds* (£), will be the price of a grain in *halfpence*.

For since there are 24 × 20 grains, that is, 480 grains in an ounce, the price of an ounce, at a halfpenny a grain, is 480 halfpence, or 240 pence; that is £1; so that there must be as many *pounds* (£) in the price of an ounce, as there are *halfpence* in the price of a grain. And conversely, there must be as many halfpence in the price of a grain, as there are pounds (£) in the price of an ounce.

Examples.

1. What is the value of an ounce of gold, at the rate of 2*d.* per grain?
   Ans. £4.
2. What is the value of an ounce of gold, at the rate of 1¾*d.* per grain?
   Ans. £3 + £½ = £3 10*s.*
3. The Mint price of gold is £3 17*s.* 10½*d.* per ounce; what is that per grain?

The shillings and pence here are to be expressed in parts of a pound, and *decimal* parts are the most convenient. A Table of such decimal parts is given towards the end of the book. By referring to this Table, we find that 17*s.* 10½*d.*, expressed in decimals of £1, is £·89375; hence, £3 17*s.* 10½*d.* = £3·89375. Therefore the value of a grain of Mint gold is 3·89375 halfpence. Converting the decimal part of this value into farthings, by multiplying it by 2, the value of the grain is 3 halfpence farthing *plus* the decimal ·7875 of a farthing, which is $\frac{79}{100}$ths, very nearly; and this again is nearly $\frac{8}{10}$ths or $\frac{4}{5}$ths of a farthing: the answer is therefore 1¾*d.* + $\frac{4}{5}$*f.* *nearly.*

[The subject of Decimals will be found sufficiently explained in the Appendix.]

Problem 2.

The price of a pennyweight being given, to find the price of a pound.

Rule.—As many pence as there are in the price of 1 dwt. (or one-fourth as many farthings), so many pounds (£) will there be in the price of 1 lb. troy.

For there are 240 dwts. in 1 lb. troy, and as many pence in £1, so that each penny in the price of 1 dwt. amounts to £1 in the price of 1 lb.

Examples.

1. At 4*d.* per dwt., what will 80 lbs. cost? $80 \times 4 = £320$ *Ans.*
2. At $2\frac{1}{4}d.$ per dwt., what will 14 lbs. cost?
   $14 \times 2\frac{1}{4} = £31\frac{1}{2} = £31\ 10s.$ *Ans.*
3. At $1\frac{3}{4}d.$ per dwt., what will 9 lbs. cost? $£1\ 15s. \times 9 = £15\ 15s.$ *Ans.*
4. At $\frac{1}{4}d.$ per dwt., what will a lb. cost? *Ans.* 5*s.*
5. If 1 dwt. of silver cost $3\frac{1}{4}d.$, what will 1 lb. cost?
   Ans. $£3\frac{1}{4} = £3\ 5s.$

Problem 3.

The price of an ounce being given, to find the price of a pound.

Rule.—As many pence as there are in the price of 1 oz., so many shillings will there be in the price of 1 lb. troy. For there are 12 oz. in 1 lb. troy, and as many pence in 1*s.*

Examples.

1. At $4\frac{1}{2}d.$ per oz., what is the price of 7 lbs.?
   $4\frac{1}{2} \times 7 = 31\frac{1}{2}s. = £1\ 11s.\ 6d.$ *Ans.*
2. At $2\frac{3}{4}d.$ per oz., what is the price of 11 lbs.?
   $2s.\ 9d. \times 11 = £1\ 10s.\ 3d.$ *Ans.*
3. If an ounce of gold cost £3 17*s.* $10\frac{1}{2}d.$, what is the price of 1 lb.?
   $£3\ 17s.\ 10\frac{1}{2}d. = 934\frac{1}{2}d.$; and $934\frac{1}{2}s. = £46\ 14s.\ 6d.$ *Ans.*

Expressed as the fraction of a pound (£), 14*s.* 6*d.* is $£\frac{14\frac{1}{2}}{20} = £\frac{29}{40}$; hence, 1 lb. of Mint gold is coined into $46\frac{29}{40}$ sovereigns; or rather (multiplying by 40), 40 lbs. of gold is coined into 1869 sovereigns, as already observed at p. 13.

Problem 4.

The price of an ounce being given, to find the price of any number of pounds, ounces, pennyweights, and grains.

Rule—1. Reduce the pounds to ounces, taking in the

ounces given. Consider every ounce as £1, every pennyweight as 1*s.*, and every grain as one halfpenny.

2. Then, whatever part the price per oz. is of £1, that same part of the sum thus obtained will be the answer.

Examples.

1. At 5*s.* 6*d.* per oz., what is the value of 10 lbs. 6 dwts. 14 grs.?

$$\begin{array}{ccc} \text{lbs.} & \text{dwts.} & \text{grs.} \\ 10 & 6 & 14 \end{array} = \begin{array}{ccc} \text{oz.} & \text{dwts.} & \text{grs.} \\ 120 & 6 & 14. \end{array} \quad \text{Hence, by the Rule:}$$

$$\left.\begin{array}{l} 5s. = \tfrac{1}{4} \text{ of a £} \\ 6d. = \tfrac{1}{10} \text{ of } 5s. \end{array}\right\} \quad \begin{array}{rrrr} & \text{£} & s. & d. \\ 4) & 120 & 6 & 7 = \text{value at £1 per oz.} \\ \hline 10) & 30 & 1 & 7\tfrac{3}{4} \\ & 3 & 0 & 1\tfrac{39}{40} \\ \hline & \textit{Ans.}\ \text{£}33 & 1s. & 9\tfrac{29}{40}d. \\ \hline \end{array}$$

The reason of the above Rule is this: £1 per ounce troy is one halfpenny per grain: for 24 grains × 20 = number of grains in 1 lb:; and 24 halfpence × 20 = number of halfpence in £1; so that by reckoning the grains as so many halfpence, or half the number as so many pence, the dwts. (24 grains) as so many shillings, and the ounces (20 dwts.) as so many £'s, we express correctly the value of the given weight, *at the rate of* £1 *per oz.* If therefore the rate per ounce be only a fractional part of £1, the value of the weight can be only the same fractional part of the value it would have if the rate were a whole £ per ounce.

2. What is the price of a piece of plate, weighing 3 lbs. 7 oz. 14 dwts. 12 gr., at the rate of 7*s.* 6*d.* per oz.?

$$\begin{array}{cccc} \text{lbs.} & \text{oz.} & \text{dwts.} & \text{grs.} \\ 3 & 7 & 14 & 12 \end{array} = \begin{array}{ccc} \text{oz.} & \text{dwts.} & \text{grs.} \\ 43 & 14 & 12. \end{array} \quad \text{Hence, by the Rule:}$$

$$\left.\begin{array}{l} 5s. = \tfrac{1}{4} \text{ of £1} \\ 2s.\,6d. = \tfrac{1}{2} \text{ of } 5s. \end{array}\right\} \quad \begin{array}{rrrr} & \text{£} & s. & d. \\ 4) & 43 & 14 & 6 = \text{value at £1 per oz.} \\ \hline 2) & 10 & 18 & 7\tfrac{1}{2} \\ & 5 & 9 & 3\tfrac{3}{4} \\ \hline & \text{£}16 & 7s. & 11\tfrac{1}{4}d. = \text{value at } 7s.\ 6d. \text{ per oz.} \\ \hline \end{array}$$

3. Required the price of 4 lbs. 5 oz. 9 dwts. 10 grs. of plate, at 6*s.* 8*d.* per oz.? *Ans.* £17 16*s.* 5½*d.* + ⅔*f.*

4. A chased gold vase weighs 1 lb. 3 oz. 4 dwts. 18 grs.; required its value at £5 17*s.* 6*d.* per oz.? *Ans.* £89 10*s.* 4⅞*d.*

[In this example, 5 times the value, at £1 per oz., is to be added to the value at 17*s.* 6*d.* per oz. This latter value is most readily computed by subtracting from the value at

£1, its ⅛th part; because 17*s.* 6*d.* differs from £1, by the ⅛th part of £1.]

Calculations for Articles sold by Avoirdupois Weight.

The following Table for readily ascertaining the price of a hundred weight, from knowing the price of a pound, will often be found useful. It will serve too for the *ton* as well as for the cwt. (see p. 135).

Table.

For the price of 1 *cwt., at from ¼d. to 3s. 6d. per lb., avoirdupois.*

Price Per lb.	Per cwt.			Price Per lb.	Per cwt.			Price Per lb.	Per cwt.			Price Per lb.	Per cwt.		
d.	£	*s.*	*d.*	*d.*	£	*s.*	*d.*	*d.*	£	*s.*	*d.*	*d.*	£	*s.*	*d.*
¼ is	0	2	4	6 is	2	16	0	11¾ is	5	9	8	20 is	9	6	8
½ ,,	0	4	8	6¼ ,,	2	18	4	12 ,,	5	12	0	21 ,,	9	16	0
¾ ,,	0	7	0	6½ ,,	3	0	8	12¼ ,,	5	14	4	22 ,,	10	5	4
1 ,,	0	9	4	6¾ ,,	3	3	0	12½ ,,	5	16	8	23 ,,	10	14	8
1¼ ,,	0	11	8	7 ,,	3	5	4	12¾ ,,	5	19	0	24 ,,	11	4	0
1½ ,,	0	14	0	7¼ ,,	3	7	8	13 ,,	6	1	4	25 ,,	11	13	4
1¾ ,,	0	16	4	7½ ,,	3	10	0	13¼ ,,	6	3	8	26 ,,	12	2	8
2 ,,	0	18	8	7¾ ,,	3	12	4	13½ ,,	6	6	0	27 ,,	12	12	0
2¼ ,,	1	1	0	8 ,,	3	14	8	13¾ ,,	6	8	4	28 ,,	13	1	4
2½ ,,	1	3	4	8¼ ,,	3	17	0	14 ,,	6	10	8	29 ,,	13	10	8
2¾ ,,	1	5	8	8½ ,,	3	19	4	14¼ ,,	6	13	0	30 ,,	14	0	0
3 ,,	1	8	0	8¾ ,,	4	1	8	14½ ,,	6	15	4	31 ,,	14	9	4
3¼ ,,	1	10	4	9 ,,	4	4	0	14¾ ,,	6	17	8	32 ,,	14	18	8
3½ ,,	1	12	8	9¼ ,,	4	6	4	15 ,,	7	0	0	33 ,,	15	8	0
3¾ ,,	1	15	0	9½ ,,	4	8	8	15½ ,,	7	4	8	34 ,,	15	17	4
4 ,,	1	17	4	9¾ ,,	4	11	0	16 ,,	7	9	4	35 ,,	16	6	8
4¼ ,,	1	19	8	10 ,,	4	13	4	16½ ,,	7	14	0	36 ,,	16	16	0
4½ ,,	2	2	0	10¼ ,,	4	15	8	17 ,,	7	18	8	37 ,,	17	5	4
4¾ ,,	2	4	4	10½ ,,	4	18	0	17½ ,,	8	3	4	38 ,,	17	14	8
5 ,,	2	6	8	10¾ ,,	5	0	4	18 ,,	8	8	0	39 ,,	18	4	0
5¼ ,,	2	9	0	11 ,,	5	2	8	18½ ,,	8	12	8	40 ,,	18	13	4
5½ ,,	2	11	4	11¼ ,,	5	5	0	19 ,,	8	17	4	41 ,,	19	2	8
5¾ ,,	2	13	8	11½ ,,	5	7	4	19½ ,,	9	2	0	42 ,,	19	12	0

Problem 1.

The price of a dram being given, to find the cost of any number of pounds.

Rule—1. Multiply the number of farthings in the given

price of the dram by 16, and the product by the number of lbs.

2. Double the last, or unit's figure of the result for *shillings*, the remaining figures denoting *pounds;* and divide the sum, thus expressed, by 6 for the answer.

EXAMPLES.

1. What will 8 lbs. cost, at the rate of $3\frac{1}{4}d.$ a dram?

Here the price of a dram is 13 farthings; hence by the Rule, $13 \times 16 \times 8 = 1664$; and doubling the 4 for shillings, we have £166 8*s.* ÷ 6 = £27 14*s.* 8*d.* *Ans.*

This short and convenient Rule rests mainly on the following principle, namely: If any number of *pounds* (£) be divided by a number terminating in a 0, the quotient will be the same as if we replace the final figure in the dividend by double that number of *shillings*, and then suppress the 0 in the divisor; as for instance—

$$\frac{£734}{30} = \frac{£73\ \ 8s.}{3};\ \frac{£734}{70} = \frac{£73\ \ 8s.}{7};\ \frac{£2587}{60} = \frac{£258\ \ 14s.}{6};$$

and so on; for the second form of expression, in every such case, is restored back again to the first form by multiplying numerator and denominator by 10. The final figure in the dividend, which is so many *pounds*, being regarded as *shillings*, and doubled, and then multiplied by 10, is thus taken 20 times; so that the shillings are reconverted into the pounds which they replaced.

This being understood, and knowing that the price of a lb., in *farthings*, is 16 × 16 × price of a dram in farthings, we must divide this product by 4 × 12 × 20 to bring the price of 1 lb. into *pounds* (£). But

$$\frac{\text{Price of dr.} \times 16 \times 16}{4 \times 12 \times 20} = \frac{\text{Price of dr.} \times 16^{*}}{60}$$

* This more simple form of the fraction is not the result of performing the multiplications indicated in the first form, and then reducing the fraction to one of lower terms. This kind of work is altogether dispensed with, and the simplified form written down at once, by merely eliminating the factors common to numerator and denominator in the first form. We know that 16 in the numerator is 4 × 4, and that

This expresses the number of pounds (£) in the price of a lb., the factor, "Price of dr.," standing for the number of *farthings* in the price of a dram. The unabbreviated *Rule* would therefore be to multiply this number of farthings by 16, and to divide the product by 60; in order to get the price, in pounds (£), of a single lb.; but by the foregoing principle, the Rule becomes abridged to that given above.

2. At $4\frac{1}{4}d.$ the dram, what will 68 lbs. cost? *Ans.* £326 8*s.*
3. At $\frac{1}{4}d.$ the dram, what will 1 lb. cost? *Ans.* 5*s.* 4*d.*
4. Required the cost of 7 lbs., at $3\frac{3}{4}d.$ per dram? *Ans.* £28.

NOTE.—It is worth while to observe here, that all examples coming under the present problem, may also be readily worked by help of the fact, that at $\frac{1}{4}d.$ the dram, the cost of 1 lb. is 5*s.* 4*d.* Thus taking Ex. 1, the cost of 1 lb., at 13 farthings per dram, is 5*s.* 4*d.* × 13 = £3 9*s.* 4*d.*; and therefore the cost of 8 lbs. is £27 14*s.* 8*d.*

PROBLEM 2. (CONVERSE OF PROB. 1.)

The price of 1 lb. being given, to find the price of a dram.

RULE.—Divide the number of pence in the price of 1 lb. by 64; the quotient will be the number of farthings in the price of a dram. The reason is obvious from the NOTE above, seeing that 5*s.* 4*d.* is 64 pence.

EXAMPLES.

1. If 1 lb. cost £3 9*s.* 4*d.*, what will a dram cost?
   69*s.* 4*d.* = 832*d.*; and 832 ÷ 64 = 13*f.* = $3\frac{1}{4}d.$
2. What is the price of a dram when 1 lb. costs £4? *Ans.* $3\frac{3}{4}d.$
3. If 1 lb. cost £2 15*s.* 8*d.*, what will a dram cost? *Ans.* $2\frac{1}{2}d.$, $\frac{7}{16}f.$

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4 × 12 in the denominator is 4 × 4 × 3 (the 12 being 4 × 3); dismissing then these 4's, there remain but 16 in the numerator, and 3 × 20 or 60 in the denominator.

This note is scarcely necessary to readers having a moderate acquaintance with fractions; but it may be useful for others to be distinctly apprised, once for all, that when, as above, the numerator and denominator of a fraction are expressed in the form of numbers multiplied together, those factors of these numbers which are common to both numerator and denominator,—and which are usually discoverable upon mere inspection—may always be suppressed, and only the *other* factors retained. (See pp. 53–56, on *Vulgar Fractions.*)

PROBLEM 3.

The price of an ounce being given, to find the price of 1 *lb.*

RULE.—Regard the given price, in farthings, as so many shillings, and divide these shillings by 3.

For taking the farthings for shillings, is the same as multiplying by 48, or by 16 and 3: dividing then the product by 3, we get the price of an ounce, multiplied by 16; that is, the price of 16 oz., or 1 lb. avoirdupois.

EXAMPLES.

1. What will 1 lb. avoirdupois come to, at $7\frac{1}{2}d.$ per oz.?
   $7\frac{1}{2}d. = 30$ farthings; therefore the price of 1 lb. is $30s. \div 3 = 10s.$
2. What will 1 lb. cost, at $10\frac{3}{4}d.$ per oz.? *Ans.* 14*s.* 4*d.*
3. What will 7 lb. come to, at $2\frac{1}{4}d.$ per oz.? *Ans.* £1 1*s.* 0*d.*
4. What will 13 lb. come to, at $3\frac{3}{4}d.$ per oz.? *Ans.* £3 5*s.* 0*d.*

PROBLEM 4. (CONVERSE OF PROB. 3.)

The price of 1 *lb. being given, to find the price per ounce.*

RULE.—Regard the given price, in shillings, as so many farthings, and multiply these farthings by 3.

The truth of this Rule is plain from that of the former Rule.

EXAMPLES.

1. At 6*s.* per lb., what will an ounce cost? $6f. \times 3 = 18f. = 4\frac{1}{2}d.$ *Ans.*
2. At 9*s.* per lb., the price of 1 oz. is $6\frac{3}{4}d.$ *Ans.*
3. At 4*s.* 9*d.* per lb., the price of 1 oz. is $3\frac{1}{2}d. + \frac{1}{4}f.$ *Ans.*

For 4*s.* $9d. = 4\frac{3}{4}s.$; and this number of *farthings*, multiplied by 3, gives for product $12\ \frac{9}{4}f. = 14\frac{1}{4}f. = 3\frac{1}{2}d. + \frac{1}{4}f.$

PROBLEM 5.

The price of 1 *oz. being given, to find the price of* 1 *stone, of* 1 *qr., and of* 1 *cwt.*

RULE I.—Find the price of 1 lb., by Prob. 3, and thence the price of 14 lb. (or a stone). Multiply this by 2, for the price of a quarter, and by 8, for the price of a cwt. The reason is obvious.

RULE II.—Multiply the pence per oz. by the number of

cwts.; and whatever part of £1 the product is, take that part of £1792: the result will be the price of 1 cwt. in £ *s.* *d.*

The number 1792 is the product of 112 by 16; so that £1792 is the value of 1 cwt., at £1 per oz.; and therefore, whatever fraction of £1 the price of 1 oz. may be, the same fraction of £1792 must be the price of 1 cwt.; or whatever fraction of £1 the price of any number of ounces may be, the same fraction of £1792 must be the price of as many cwts.

Examples.

1. What will 6 cwts. come to, at 5*d.* per oz.? First by Rule I.

By Prob. 3, $20s. \div 3 = 6s.\ 8d.$ = price of 1 lb.
$6s.\ 8d. \times 14 = 93s.\ 4d.$ = price of 1 stone.
$93s.\ 4d. \times 8 = 746s.\ 8d.$ = price of 1 cwt.;

and 6 times this, namely, $4480s. = £224$ = price of 6 cwt.

Again: by Rule II.

$5d. \times 6 = 30d. = 2s.\ 6d. = £\frac{1}{8}$; and
$1792 \div 8 = £224$, *Ans.*

We thus arrive at the price of the cwts. more expeditiously than by Rule I.: but if the price, when multiplied by the given number of cwts., be not a convenient fraction of £1, the first Rule may prove the more expeditious, as in the example next following.

2. At $2\frac{1}{4}d.$ per oz., what is the cost of 1 cwt.?

The fraction that $2\frac{1}{4}d.$, or 9 farthings, is of £1, is

$\frac{9}{4 \times 12 \times 20}$, or (expunging the factor 3 from the 9 and the 12),

$$\frac{3}{4 \times 4 \times 20} = \frac{3}{320}: \text{ hence, by Rule II.,}$$

$$1792 \times \frac{3}{320} = 5376 \div 320 = 16\tfrac{256}{320} = 16\tfrac{4}{5};$$

and $£16\frac{4}{5} = £16\ 16s.$, the *Ans.*

This is much more troublesome than the following operation by Rule I. (See also Rule III. next page, and Ex. 8.)

$9s. \div 3 = 3s.$; and $3s. \times 14 = 42s.$; and $£2\ 2s. \times 8 = £16\ 16s.$

3. What will 10 cwts. come to, at 4*d.* per oz.? *Ans.* £298 13*s.* 4*d.*
4. What will 8 cwts. come to, at 3*d.* per oz.? *Ans.* £179 4*s.*
5. What will 5 cwts. come to, at $4\frac{1}{2}d.$ per oz.? *Ans.* £168.

RULE III.—Multiply 2*s*. 4*d*. by the number of pence in the price of 1 lb.; the product will be the price of a quarter. Multiply 9*s*. 4*d*. by the number of pence in the price of 1 lb., and the product will be the price of 1 cwt. Thus:—

6. At 4*d*. per lb., what is the price of a quarter, or 28 lbs.?

2*s*. 4*d*. × 4 = 9*s*. 4*d*. *Ans.*

It is obvious that this is correct, because 2*s*. 4*d*. is the price of 28 lbs., at 1 penny per lb.; and 9*s*. 4*d*. is the price of 4 quarters, or 1 cwt., at 1 penny per lb.

7. What is the price of 1 cwt., at 5*d*. per oz., or 6*s*. 8*d*. per lb.? (Ex. 1.)

6*s*. 8*d*. = 80*d*.; and 9*s*. 4*d*. × 80 = 746*s*. 8*d*. = £37 6*s*. 8*d*. *Ans.*

8. What is the value of 1 cwt., at $2\frac{1}{4}d$. per oz., or 3*s*. per lb.? (Ex. 2.)

9*s*. 4*d*. × 36 = 336*s*. = £16 16*s*. *Ans.*

It is thus seen that this third Rule will sometimes be more expeditious than either of the other Rules, when we commence the operation with the price of a lb.

9. What is the cost of 1 cwt. of beef, at $7\frac{1}{4}d$. per lb.? *Ans.* £3 7*s*. 8*d*.

Since 9*s*. 4*d*., and 2*s*. 4*d*., are each divisible by 4, without fractions, the price per cwt., or per quarter, at any number of pence and farthings per lb., may always be found with but little trouble by Rule III., as in the margin. But it may be ascertained by inspection of the Table at p. 128, even though the given price per lb. exceed the limit of that Table, that is, although the price be beyond 3*s*. 6*d*. Thus: in order to make the Table available for Ex. 7, above, we have only to divide 6*s*. 8*d*. into parts, each of which comes within the tabular limits—into two *equal* parts for instance, 3*s*. 4*d*. and 3*s*. 4*d*. For this price per lb., the Table gives (against 40 pence) £18 13*s*. 4*d*., the double of which is £37 6*s*. 8*d*., as determined by actual calculation. Or we may take any other two suitable parts of 6*s*. 8*d*., as 3*s*. 6*d*. and 3*s*. 2*d*.; that is, 42*d*. and 38*d*., against which in the Table stand £19 12*s*. and £17 14*s*. 8*d*., the sum of which is £37 6*s*. 8*d*., as before. And if the price per lb. *exceed* the double of 3*s*. 6*d*., or 7*s*., we may cut up the price into *three* parts; and so on, taking out the corresponding prices per cwt., and adding them together. If instead of 1 cwt., the price of a quarter only be required, we must divide the price of the cwt. by 4; and, if it be a stone, by 8; or, if we please, and it be equally convenient to do so, we may divide the price of the lb. by 4 or 8, and enter the Table with the quotient: thus, taking Ex. 6, and entering the Table with 1*d*., we find against it 9*s*. 4*d*., the price of a quarter.

4)	9*s*.	4*d*.
		7
	65	4
	2	4
Ans.	67*s*.	8*d*.

The converse of the preceding problem, namely, the price per stone, per quarter, or per cwt., being given, to find the price per lb., is readily enough worked by ordinary

arithmetic; that is, by dividing the given price by 14, or by 28, or by 112, as the case may be. But here, too, the Table will give the price per lb. to the nearest farthing. Thus, suppose we wished to know the price per lb. at £14 12*s.* 8*d.* per cwt. Referring to the Table, we find the price at £14 9*s.* 4*d.* to be 31*d.* per lb.; but £14 9*s.* 4*d.* is 3*s.* 4*d.* short of the price stated; and the nearest to 3*s.* 4*d.* is 2*s.* 4*d.* and 4*s.* 8*d.*, against which stand $\frac{1}{4}$*d.* and $\frac{1}{2}$*d.*; so that the price per lb. is between 31$\frac{1}{4}$*d.* and 31$\frac{1}{2}$*d.*; either sum being correct to the nearest farthing. And so in other cases; and if the price per quarter, or per stone, were given, we should enter the Table with 4 times, or 8 times this price, against which would be found the price per lb. to the nearest farthing, if not exactly.

Problem 6.

The price of 1 *lb. being given, to find the price of a ton.*

Rule I.—Find the price of 1 cwt. by Rule III., p. 133; regard the *shillings* in this price as so many *pounds*, and the pence as fractional parts; and the answer in pounds will be expressed. Or—

Rule II.—Multiply the number of farthings in the price of the lb. by 7, and divide the product by 3, for the answer in pounds.

The truth of the first of these two Rules will be seen by considering that by taking the shillings in the price of 1 cwt. as so many pounds, we must get the price of 20 cwt. And the truth of the second Rule follows from the fact that by replacing the price in farthings by so many pounds, we virtually multiply that price by $4 \times 12 \times 20$; therefore multiplying the number of farthings (taken as pounds) by 7, and then dividing by 3, the price per lb. is multiplied by $4 \times 12 \times 20 \times 7 \div 3 = 4 \times 4 \times 20 \times 7 = 28 \times 4 \times 20$; and 28 lbs. = 1 qr., 4 qrs. = 1 cwt., and 20 cwt. = 1 ton.

We thus see that by the contrivance of multiplying by 7 and dividing by 3, the price per lb. is virtually multiplied by 28, 4, and 20; thus giving for product the price of a ton.

EXAMPLES.

1. If 1 lb. cost 7*d.*, what will a ton cost?

By RULE I.

9*s.* 4*d.* × 7 = 65*s.* 4*d.* = price of 1 cwt.
£65$\frac{1}{3}$ = £65 6*s.* 8*d.* = price of 1 ton.

By RULE II.

£28 × 7 ÷ 3 = £196 ÷ 3 = £65 6*s.* 8*d.* *Ans.*

2. If 1 lb. cost 1$\frac{3}{4}$*d.*, what will a ton cost?

By RULE II. £7 × 7 ÷ 3 = £49 ÷ 3 = £16 6*s.* 8*d.* *Ans.*

3. If 1 lb. cost 10*d.*, what will a ton cost?

By RULE I.

9*s.* 4*d.* × 10 = 93*s.* 4*d.*
£93$\frac{1}{3}$ = £93 6*s.* 8*d.* *Ans.*

By RULE II.

£40 × 7 ÷ 3 = £280 ÷ 3 = £93 6*s.* 8*d.* *Ans.*

[As by Rule II. the divisor is 3, the fraction of £1, which gives the shillings and pence, must always be either £$\frac{1}{3}$, or £$\frac{2}{3}$; that is, either 6*s.* 8*d.* or 13*s.* 4*d.*]

4. If 1 lb. cost 4$\frac{3}{4}$*d.*, what will a ton cost?

By RULE II., £19 × 7 ÷ 3 = £133 ÷ 3 = £44 6*s.* 8*d.* *Ans.*

5. If 1 lb. cost 2$\frac{3}{4}$*d.*, what will a ton cost? *Ans.* £25 13*s.* 4*d.*
6. If 1 lb. cost 4$\frac{1}{4}$*d.*, what is a ton worth? *Ans.* £39 13*s.* 4*d.*

NOTE.—The Table at page 128, for finding, by inspection, the price of 1 cwt., from that of 1 lb. being given, may be advantageously employed for also finding the price of a ton; for a ton being 20 cwt., we have only to regard every shilling, in the price there, of 1 cwt., as £1, and every 4*d.* as 6*s.* 8*d.* Thus, taking Example 4, above, and referring to the Table, we find against 4$\frac{3}{4}$*d.*, 44*s.* 4*d.*, which, for a *ton*, we should read as £44 6*s.* 8*d.* Again, taking Ex. 5, the Table gives 25*s.* 8*d.*, to be read as £25 13*s.* 4*d.*; and similarly in other cases.

PROBLEM 7. (CONVERSE OF PROB. 6.)

The price of a ton being given, to find the price of 1 *lb.*

RULE.—Multiply the price of the ton, expressed in £'s and fractions of a £, by 3, and then divide by 7: the quotient will be the number of farthings in the price of 1 lb.

EXAMPLES.

1. If a ton of iron cost £16 6*s.* 8*d.*, what is the price per lb.?
   £16 6*s.* 8*d.* = £16$\frac{1}{3}$, and 16$\frac{1}{3}$ × 3 ÷ 7 = 49 ÷ 7 = 7 farthings.
2. If a ton cost £39 13*s.* 4*d.*, what is the price per lb?
   £39 13*s.* 4*d.* = £39$\frac{2}{3}$, and 39$\frac{2}{3}$ × 3 ÷ 7 = 119 ÷ 7 = 17*f.* = 4$\frac{1}{4}$*d.*
3. If a ton cost £84, what will 1 lb. cost?
   84 ÷ 7 = 12, and 12 × 3 = 36*f.* = 9*d.*
4. If a ton cost £36 10*s.*, what will 1 lb. cost?
   £36 10*s.* = £36$\frac{1}{2}$, and 36$\frac{1}{2}$ × 3 ÷ 7 = 109$\frac{1}{2}$ ÷ 7 = 15$\frac{9}{14}$ farthings; that is, 3$\frac{3}{4}$*d.* and the $\frac{9}{14}$th of a farthing.

It will be observed that in each of the examples which precede this last, the given price is such, that when it is multiplied by 3, the product is an integral number of pounds, —without shillings or pence. In *this* example the product involves a fraction, namely, $\frac{1}{2}$. Whenever such is the case, the answer to the question will always be a certain number of farthings and some fraction of a farthing besides. 1 lb. therefore could not be purchased at its exact value in existing coin; consequently, in order that the seller of a single lb. may not sustain loss, the fraction must be rejected, and an additional farthing added. In the case before us, less than a stone, or 14 lbs., could not be sold at its exact value: but since 15 $\frac{9}{14}$ × 14 = 219, the exact price of a stone would be 219 farthings, or 4*s.* 6$\frac{3}{4}$*d.* And even though 3 times the price of the ton be an integral number of £'s, yet a fraction of a farthing must form part of the exact price of 1 lb., if the product by the 3 be not exactly divisible by 7.

5. If a ton cost £42, what is the price per lb.? *Ans.* 4$\frac{1}{2}$*d.*
6. If a ton cost £23 6*s.* 8*d.*, how much is that per lb.? *Ans.* 2$\frac{1}{2}$*d.*
7. If a ton cost £43 15*s.*, what will 1 lb. cost? *Ans.* 4$\frac{1}{2}$*d.* + $\frac{3}{4}$*f.*

In order to prove the truth of the foregoing Rule, we need only observe that the price of the ton being expressed in £'s, if we multiply the price by 20 × 12 × 4, we shall get the price in farthings; and from this, to get the price of a lb., we must divide it by 20 × 112; or which is the same thing, we must divide 12 × 4 times the price in £'s by 112, the common factor 20 being suppressed in both multiplier and divisor. (See foot-note, p. 129.) But

$$\frac{12 \times 4}{112} = \frac{3 \times 16}{7 \times 16} = \frac{3}{7};$$

the factor 16 in both numerator and denominator being

suppressed; so that we have only to multiply the price of the ton, in £'s, by 3, and then divide by 7; which is the Rule.

PROBLEM 8.

The price of 1 cwt. being given, to find the price of any number of tons.

RULE.—Multiply the number of pence in the price of the cwt. by the number of tons, and divide the product by 12: the quotient will be the number of £'s in the price of the tons. [Or 20 times the price of 1 cwt. is the price of a ton.]

For by multiplying the given price, in pence, by 20, we get the price, in pence, of a ton; and by dividing this number of pence by 12 and 20, we get the number of *pounds* in the price of a ton, and thence the price, in pounds, of any number of tons. But 20 being here both a multiplier and a divisor may be expunged, and hence the Rule.

EXAMPLES.

1. 6 tons, at 2*d*. per cwt. $= 2 \times 6 \div 12 = 1$: therefore £1 is the price of the 6 tons. Or 2*d*. per cwt. is 40*d*. or 3*s*. 4*d*. per ton; therefore the price of 6 tons is £1.
2. 8 tons, at 3*d*. per cwt. $= 3 \times 8 \div 12 = 2$; therefore the price is £2.
3. 24 tons, at $7\frac{1}{2}d$. per cwt. $= 7\frac{1}{2} \times 24 \div 12 =$ £15.
4. 36 tons, at $13\frac{1}{2}d$. per cwt., is $13\frac{1}{2} \times 36 \div 12 =$ £$40\frac{1}{2}$ = £40 10*s*.
5. Required the price of 60 tons, at $19\frac{1}{2}d$. per cwt.? *Ans*. £97 10*s*.

NOTE.—If either of the two numbers to be multiplied together be divisible by 12, the division should be executed *before* the multiplication: thus, in Examples 3, 4, and 5, the use of the pen is scarcely necessary, seeing that 24, 36, and 60, by division by 12, give the small multipliers 2, 3, and 5; and that twice $7\frac{1}{2}$, 3 times $13\frac{1}{2}$, and 5 times $19\frac{1}{2}$, give products which no pen-work is needed to determine.

PROBLEM 9. (CONVERSE OF PROB. 8.)

The price of a ton being given, to find the price of 1 cwt.

RULE.—Express the price of the ton in pounds (£) and fractions of a £; then the price of a cwt. will be that number of *shillings*. This is obvious, because the price of 1 cwt. must be the twentieth part of the price of a ton, and the twentieth part of any number of pounds is that same number of shillings.

EXAMPLES.

1. If a ton cost 3*s.* 4*d.*, what will 1 cwt. cost?
   3*s.* 4*d.* $= £\frac{1}{6}$; and $\frac{1}{6}s. = 2d.$ *Ans.*
2. If a ton cost 14*s.* 2*d.*, what will 1 cwt. cost?

What fraction of one pound 14*s.* 2*d.* is, is not readily seen: we know however that 6*s.* 8*d.* $= £\frac{1}{3}$; and that 6*s.* 8*d.* + 7*s.* 6*d.* = 14*s.* 2*d.*; also that 7*s.* 6*d.* $= £\frac{3}{8}$. Consequently, 14*s.* 2*d.* $= £\frac{1}{3} + £\frac{3}{8}$: hence the price of 1 cwt. is $\frac{1}{3}s. + \frac{3}{8}s. = 4d. + 4\frac{1}{2}d. = 8\frac{1}{2}d.$ *Ans.*

3. If a ton cost 12*s.* 8*d.*, what will 1 cwt. cost?

$12s.\ 8d. = 10s. + 2s.\ 6d. + 2d. = £\frac{1}{2} + £\frac{1}{8} + 2d.$, therefore,
$\frac{1}{2}s. + \frac{1}{8}s. + \frac{2}{20}d. = 6d. + 1\frac{1}{2}d. + \frac{1}{10}d. = 7\frac{1}{2}d. + \frac{2}{5}$ farthing. *Ans.*

We here divide the odd twopence by 20, because the 20th part of the entire sum, in the price of 20 cwt., must be taken to get the price of 1 cwt. And in imitation of this manner of computing must every case be treated in which overplus pence remain after the fractions of £1 are subducted from the given price of the ton.

4. If a ton cost 12*s.* 6*d.*, what will 1 cwt. cost? *Ans.* $7\frac{1}{2}d.$
5. If a ton cost £1 12*s.* 6*d.*, what will 1 cwt. cost? *Ans.* 1*s.* $7\frac{1}{2}d.$
6. If a ton cost £1 2*s.* 6*d.*, what will 1 cwt. cost? *Ans.* 1*s.* $1\frac{1}{2}d.$
7. If a ton cost 17*s.* 10*d.*, what is the price of 1 cwt.? *Ans.* $10\frac{1}{2}d. + \frac{4}{5}f.$

PROBLEM 10.

The price of 1 cwt. being given, to calculate the cost of any number of cwts., qrs., and lbs.

RULE.—Write down first as many pounds (£) as there are cwts.; then, for every quarter, put 5*s.*, and for every lb., write $2\frac{1}{7}d.$ We shall thus have the value of the proposed weight at £1 per cwt. Take parts for the shillings and pence, in the given price per cwt., and the answer will be obtained. [If the price per cwt. be *less* than £1, see RULE p. 97.]

For at £1 per cwt., the value of 1 qr. is the fourth part of £1, that is, 5*s.*; and the value of 1 lb. is the 28th part of this, namely, $60d. \div 28 = 2\frac{1}{7}d.$; so that whatever fractional parts of £1 the actual price of the cwt. may be or may include, the same parts of these several amounts must be taken to obtain the correct answer to the question.

EXAMPLES.

1. What is the price of 13 cwts. 2 qrs. 14 lbs., at £1 6*s*. 8*d*. per cwt.?

	£	*s*.	*d*.
The weight is 13 cwts. $2\frac{1}{2}$ qrs., and the price at £1 is	13	12	6
and at 6*s*. 8*d*., or £$\frac{1}{3}$, it is	4	10	10
Hence the whole price is	£18	3*s*.	4*d*. *Ans*.

2. What will 19 cwts. 3 qrs. 19 lbs. cost, at £4 15*s*. 6*d*. per cwt.?

4*s*. 0*d*. = £$\frac{1}{5}$
4*s*. 0*d*. = £$\frac{1}{5}$
5*s*. 0*d*. = £$\frac{1}{4}$
2*s*. 6*d*. = £$\frac{1}{8}$ = $\frac{1}{2}$ of £$\frac{1}{4}$

15*s*. 6*d*.

£	*s*.	*d*.	
19	15	0	
	3	$4\frac{5}{7}$	= $2\frac{1}{7}$*d*. × 19
19	18	$4\frac{5}{7}$	= Price at £1
		4	
79	13	$6\frac{6}{7}$	= Price at £4
3	19	$8\frac{1}{7}$	= $\frac{1}{5}$th price at £1
3	19	$8\frac{1}{7}$	= ditto
4	19	$7\frac{5}{28}$	= $\frac{1}{4}$th „
2	9	$9\frac{33}{56}$	= $\frac{1}{8}$th „

Ans. £95 2*s*. $3\frac{51}{56}$*d*. = £95 2*s*. 4*d*. *very nearly*.

[There are, of course, various ways of cutting up the shillings and pence into fractions of £1: the computer must be left to his own judgment and sagacity as to the choice of the most convenient subdivision. The subdivision here might have been 10*s*. + 5*s*. + 6*d*. = 15*s*. 6*d*.]

3. What is the price of 27 cwt. 2 qrs., at £2 14*s*. 6*d*. per cwt.?
Ans. £74 18*s*. 9*d*.
4. What is the price of 42 cwt. 1 qr. 20 lbs., at £1 13*s*. 4*d*. per cwt.?
Ans. £70 14*s*. $3\frac{3}{7}$*d*.
5. What will 85 cwt. 1 qr. 10 lbs. come to, at £2 17*s*. 6*d*. per cwt.?
Ans. £245 7*s*. $0\frac{3}{28}$*d*.

It is scarcely necessary to say that the fractions of a penny, retained in these answers, would be disregarded in actual practice: they are given here, as in several other of the results in this book, chiefly that the reader may put to the test his ability to compute with fractions.

Calculations respecting Land, Timber, &c.

Problem 1.

The price of a perch being given, to find the price of an acre.

Rule.—Regard the price of the perch, in pence, as so many pounds, and from that number of pounds subtract a third part; the remainder will be the price of an acre.

For, by regarding the price, in pence, as so many pounds, we, in effect, multiply that price by 20 and by 12; that is, by 20×12 or 240. Now the number of perches in an acre is 40×4 (Table, p. 23), so that if the price of a perch be multiplied by 40×4 (that is, by 160), the result will be the price of an acre. But two-thirds of 20×12 *is* 40×4 (twice the 20 multiplied by a third of the 12); consequently two-thirds of the price, in pence, of a perch, regarding those pence as so many pounds, must be the price of an acre, and two-thirds of anything is the whole *minus* one-third; hence the Rule.

Examples.

1. What is the price of an acre, at the rate of $15\frac{3}{4}$*d.* per perch?
   £15 15*s.* $\div$ 3 = £5 5*s.*, therefore the price is £10 10*s.* per acre.
2. What is the price of an acre, at the rate of 2*s.* $7\frac{1}{2}$*d.* per perch?
   £31 10*s.* $\div$ 3 = £10 10*s.*, therefore the price is £21. *Ans.*
3. What is the price of an acre, at 1*s.* $8\frac{1}{4}$*d.* per perch? *Ans.* £13 10*s.*
4. What is the price of an acre, at 3*s.* $10\frac{3}{4}$*d.* per perch?
   Ans. £31 3*s.* 4*d.*

Problem 2.

The price of a perch being given, to find the price of a rood.

Rule.—One-sixth as many pounds as the perch costs pence will be the price of a rood.

For we have seen (last Prob.) that $\frac{2}{3}$rds of this number of pounds is the price of an acre; so that, dividing the $\frac{2}{3}$rds by 4, we must get the price of a rood. But $\frac{2}{3} \div 4 = \frac{2}{12}$, and $\frac{2}{12} = \frac{1}{6}$; hence the Rule.

Examples.

1. If a perch cost 6*d.*, what will a rood cost? £6 $\div$ 6 = £1, *Ans.*
2. If a perch cost 2*s.* $7\frac{1}{2}$*d.*, what will a rood cost?
   £31 10*s.* $\div$ 6 = £5 5*s.*, *Ans.*

Note.—This problem may obviously be made to supersede the former

one, for we have only to multiply the price of a rood by 4 to get the price of an acre; thus, 4 times the sum which is the answer to this last question is the answer to question 2 of last problem.

3. What is the price of a rood, if a perch cost 15¾*d.*? *Ans.* £2 12*s.* 6*d.*
4. What is the price of a rood, if a perch cost 1*s.* 8¼*d.*? *Ans.* £3 7*s.* 6*d.*

Problem 3. (Converse of Probs. 1 and 2.)

The price of an acre, or of a rood, being given, to find the price of a perch.

Rule.—*For the acre.* To the price of the acre, in pounds, add its half; the result will be the number of pence in the price of a perch.

For the rood. Six times the number of pounds in the price of a rood will be the number of pence in the price of a perch.

Examples.

1. At £10 10*s.* per acre, what is the price of a perch?
£10 10*s.* + £5 5*s.* = £15 15*s.*; therefore, 15¾*d.* = price of a perch.

It has already been shown (Prob. 1) that *two-thirds* of the number of pence in the price of a perch is the number of pounds in the price of an acre; consequently the *whole* number of pence in the price of a perch must be equal to the number of pounds in the price of an acre, and half that number of pounds besides; because the *whole* of anything is ⅔rds of it and half as much more, and hence the truth of the Rule.

2. If a rood of land cost £2 12*s.* 6*d.*, what will a perch cost?
£2 12*s.* 6*d.* = £2½ + £⅛; and 6 times this is £15 + £¾.
Hence, by the Rule, 15¾*d.* is the price of a perch.

If it had been an *acre*, instead of a rood, we should have multiplied the number of pounds in the price by 1½; but as it is, we ought to multiply by 4 times this, since the perch is 4 times as dear as it would be if the assigned price had been per acre instead of per rood. Hence we must multiply by 1½ × 4 = 6, as the Rule directs.

3. At £13 10*s.* per acre, what is the price of a perch? *Ans.* 1*s.* 8¼*d.*
4. At £31 3*s.* 4*d.* per acre, what is the price of a perch?
Ans. 3*s.* 10¾*d.*
5. If a rood cost £5 5*s.*, what is the price of a perch? *Ans.* 2*s.* 7½*d.*
6. If a rood cost £3 7*s.* 6*d.*, what will a perch cost? *Ans.* 1*s.* 8¼*d.*

Problem 4.

The price of a square yard of land being given, to find the price of an acre.

Rule I.—Multiply the number of pence, in the price per yard, by 20, and consider the result as so many pounds, to which add as many times 3*s.* 4*d.* as there are pence in the given price; the sum will be the price per acre.

The price of an acre may, of course, be found by multiplying the price of a square yard by the number 4840, because there are 4840 square yards in an acre; or, which is the same thing, by multiplying 4840 pence by the number of pence in the price of a square yard. Now, by regarding these pence as so many pounds, we virtually multiply them by 240; and by again multiplying by 20, we multiply, on the whole, by 4800, which is 40 short of the number 4840; hence, to the product thus obtained, we must add 40 times the number of pence in the price of the yard in order to get the full price of an acre; or, which is the same thing, we must add 40 pence multiplied by the number of pence in the price of the yard. But 40 pence is 3*s.* 4*d.*; and hence the Rule.

Examples.

1. If a square yard be worth 2*d.*, what is the value of an acre?
   £2 × 20 = £40; which, with twice 3*s.* 4*d.*, gives £40 6*s.* 8*d.* *Ans.*
2. If a square yard be worth $3\frac{1}{4}$*d.*, what is an acre worth?
   £$3\frac{1}{4}$ × 20 = £65; and $3\frac{1}{4}$ times 3*s.* 4*d.* is 10*s.* 10*d.*: therefore £65 10*s.* 10*d.* is the value of an acre.

Note.—It is easy to see that the Rule may be varied a little in its expression. It may be as follows: £20 3*s.* 4*d.* being 4840*d.*, and £5 0*s.* 10*d.* = 4840*f.*

Rule II.—Multiply £20 3*s.* 4*d.* by the number of *pence* in the price of a square yard: the product will be the price of an acre. Or, multiply £5 0*s.* 10*d.* by the number of *farthings* in the price of the square yard; since a farthing per sq. yard is £5 0*s.* 10*d.* per acre. Or, lastly: take the number of pence in the price of a sq. yd. as so many *pounds* (£), and multiply that number of pounds by $20\frac{1}{6}$. Thus, in Ex. 2, above, the operation by this last Rule will be that here annexed; remembering that £$3\frac{1}{4}$ = £3 5*s.*

	£	s.	d.
6)	3	5	0
			20
	65	0	0
		10	10
	£65	10*s.*	10*d.*

3. If a square yard cost $1\frac{3}{4}d.$, what will an acre cost? *Ans.* £35 5*s.* 10*d.*
4. If a square yard cost $2\frac{1}{4}d.$, what will an acre cost? *Ans.* £45 7*s.* 6*d.*
5. What is the price of an acre, at the rate of $2\frac{1}{2}d.$ per square yard? *Ans.* £50 8*s.* 4*d.*

PROBLEM 5. (CONVERSE OF PROB. 4.)

The price of an acre being given, to find the price of a square yard.

RULE.—Divide the price of the acre, expressed in pounds and fractions of a pound, by $£20\frac{1}{6}$: the quotient will be the number of pence in the price of a square yard.

For in the preceding operations, in order to get the price of an acre from that of a yard, we multiplied £20 by the number of pence in the price of the yard, and *also* 3*s.* 4*d.* by that number of pence; that is to say, we multiplied £20 3*s.* 4*d.* by the number of pence in the yard, in order to get the price per acre. Hence, in the reverse operation, we must *divide* the price of the acre, in pounds, by $£20\frac{1}{6}$, this being the equivalent of £20 3*s.* 4*d.*

EXAMPLES.

1. If an acre of land be worth £65 10*s.* 10*d.*, what is the worth of a square yard?

   £65 10*s.* 10*d.* (see Table, p. 57) $= £65\frac{1}{2} + £\frac{1}{24}$, or $£\frac{131}{2} + £\frac{1}{24}$.

   Dividing each by $20\frac{1}{6}$, that is, multiplying by $\frac{6}{121}$ (p. 56), the fractions are $\frac{393}{121} + \frac{1}{484} = \frac{1572+1}{484} = \frac{1573}{484} = 3\frac{1}{4}d.$

NOTE.—The converse of a problem is usually more troublesome to work than the direct problem itself. It is likely that, in the present converse problem, some might find it easier to proceed according to the Rule of common arithmetic; namely, to reduce the £65 10*s.* 10*d.* to pence, and then to divide by 4840.

2. If an acre be worth £45 7*s.* 6*d.*, what is a square yard worth? *Ans.* $2\frac{1}{4}d.$
3. If an acre be worth £50 8*s.* 4*d.*, what is a square yard worth? *Ans.* $2\frac{1}{2}d.$

PROBLEM 6.

To reduce Irish acres to English, and English to Irish.

RULE.—Multiply the number of *Irish* acres by 196, and divide by 121: the result will be the number of English acres.

Multiply the number of *English* acres by 121, and divide by 196: the result will be the number of Irish acres. For an Irish acre contains 7840 yards, and an English acre 4840; and these numbers are in the ratio of 196 to 121.

EXAMPLES.

1. How many English acres are there in 484 Irish acres?
   $484 \times 196 = 94864$; and this $\div 121$ (or by 11 and 11) gives 784; or more simply, $4 \times 196 = 784$, the *Ans.*
2. How many Irish acres are there in 784 English acres?
   $784 \times 121 \div 196 = 484$; or $4 \times 121 = 484$ Irish acres, the *Ans.*
3. How many English acres are there in $134\frac{1}{2}$ Irish acres?
   Ans. $217\frac{7}{8}$ *very nearly.*
4. How many Irish acres are there in $273\frac{1}{4}$ English acres?
   Ans. $168\frac{9}{13}$ *very nearly.*

PROBLEM 7.

To reduce Irish miles to English, and English to Irish.

The English mile contains 1760 yards, and the Irish mile 2240 yards, which two numbers are to one another as 11 is to 14, or as 1 to $1\frac{3}{11}$; so that an Irish mile is $1\frac{3}{11}$ times as long as an English mile; or 11 Irish miles make 14 English. Hence this—

RULE.—Increase the number of *Irish* miles by the $\frac{3}{11}$ part of that number: the result will be the number of English miles.

Diminish the number of *English* miles by the $\frac{3}{14}$ part of that number; the result will be the number of Irish miles. Or, which is the same thing, multiply the number of Irish miles by 14, and divide the product by 11; the quotient will be the number of English miles.

Multiply the number of English miles by 11, and divide by 14 (or by 7 and 2); the quotient will be the number of Irish miles.

NOTE.—When the number of English miles is *even*, we may take half that number only, using for divisor 7 instead of 14, as in Ex. 2, next page.

EXAMPLES.

1. How many English miles are there in 27 Irish miles?
   $27 + \frac{81}{11} = 27 + 7\frac{4}{11} = 34\frac{4}{11}$, the number of English miles.
   Or thus; $27 \times 14 = 378$; and this $\div$ 11 gives $34\frac{4}{11}$.

2. How many Irish miles are there in 40 English miles?

$40 - \frac{120}{14} = 40 - 8\frac{4}{7} = 31\frac{3}{7}$, the number of Irish miles.

Or thus (see NOTE above), $20 \times 11 = 220$, which $\div$ 7 gives $31\frac{3}{7}$.

3. How many English miles are there in 75 Irish miles? *Ans.* $95\frac{5}{11}$.
4. How many Irish miles are there in 95 English miles? *Ans.* $74\frac{9}{14}$.

The foregoing calculations, respecting Land, must suffice for the present small volume. It would, of course, be out of place here to enter upon the general subject of *surveying*. We shall now therefore proceed to some useful particulars connected with the measurement of TIMBER.

TIMBER TRADE.

I. Superficial measure.

When timber is sold in planks,—or in logs for the purpose of being cut into planks,—its price is estimated by *superficial* measurement; that is, by the number of square feet contained under the length and breadth of the plank; the price per square foot differing, of course, for different thicknesses of plank.

PROBLEM 1.

To find the number of square feet in a board or plank.

RULE.—Multiply the length by the breadth; the product will be the number of square feet, when the plank is of the usual form; that is, a rectangle.

But if the plank be tapering, add the two widths at the ends together, and take half the sum for the mean breadth, by which multiply the length: the result will be the number of square feet.

[We have here spoken of multiplying length by breadth, as it is the universal form of expression in the trade: but, in fact, only abstract numbers, and not the concrete quantities, feet and inches, are really worked with. See the remarks on this subject at p. 36.]

EXAMPLES.

1. How many square feet are there in a plank 16 feet 6 inches long, and 14 inches broad? (See "Duodecimals," p. 41.)

ft.	in.
16	6
1	2
16	6
2	9
19 Ft.	3 Pts.

Ans. $19\frac{3}{12}$ sq. ft. $= 19\frac{1}{4}$ sq. ft.

or: 16 ft. 6 in. = 198 in.; and $198 \times 14 = 2772$.
144) 2772 ($19\frac{36}{144} = 19\frac{1}{4}$ sq. ft. *Ans.*

NOTE.—It would be a mistake to call the area of the plank 19 sq. ft. 3 *inches:* the 3 does not denote square inches, but so many 12ths of a square foot; that is, it denotes 36 square inches; so that, expressed in feet and inches, the area of the plank is 19 sq. ft. 36 sq. inches. Instead of calling the 3, in the answer above, 3 *inches*, we should call it 3 *parts* of a square foot; *one* "part" being a strip a foot long and an inch broad, and containing therefore 12 square inches. (See "Duodecimals," p. 41.)

2. How many square feet are there in a plank $20\frac{3}{4}$ feet long, and $12\frac{1}{2}$ inches broad?

ft.	in.	
20	9	
1	$0\frac{1}{2}$	
20	9	
	10	$4\frac{1}{2}$
21 Ft.	7 Pts.	$4\frac{1}{2}$ In.

Otherwise: 249 inches in $20\frac{3}{4}$ feet.

249
$12\frac{1}{2}$
2988
$124\frac{1}{2}$
144) $3112\frac{1}{2}$ (21 Ft. $88\frac{1}{2}$ In.

By multiplying the 7 Pts. by 12, to bring them into square inches, the result of the first of these operations is 21 sq. ft. $88\frac{1}{2}$ sq. in., agreeing with the result of the second operation.

3. How many square feet are there in a plank 15 feet 6 inches long, and 10 inches broad? *Ans.* 12 sq. ft. 132 sq. in.
4. What is the area of a plank 16 feet 5 inches long, and $10\frac{1}{2}$ inches broad? *Ans.* 14 sq. ft. $52\frac{1}{2}$ sq. in.
5. How many square feet are there in a plank 12 feet long, and $8\frac{1}{2}$ inches broad? *Ans.* 8 sq. ft. 72 sq. in. = $8\frac{1}{2}$ sq. ft.

PROBLEM 2.

To find how many square feet of inch-thick plank can be cut out of a log of given length, breadth, and thickness.

RULE.—Multiply the number of *inches* in the breadth by

the number in the thickness, and the product by the number of *feet* in the length of the log: $\frac{1}{12}$th the result will be the number of square feet of inch-thick plank in the log.

There are obviously as many inch-thick planks as there are inches in the thickness of the log; and the area of each plank, *in feet*, is found by multiplying the length in feet by the breadth *in feet*; that is, by $\frac{1}{12}$th the number of *inches* in the breadth: hence, to find the area of *all* the planks, we must multiply together the length in feet, the breadth in inches, and the thickness in inches, and then take $\frac{1}{12}$th of the product; which is the Rule.

Examples.

1. How many square feet of inch-thick plank can be cut out of a piece of timber 16 feet long, 9 inches broad, and 4 inches thick?

We see here at a glance, that of the last two dimensions to be multiplied together, one is divisible by 3 and the other by 4; so that, expunging these factors of 9 and 4 (which is the same as dividing their product by 12), we have then only to multiply together 16 and 3, which gives 48 for the number of square feet of plank: this number being only the $\frac{1}{12}$th part of what would result from multiplying 16, 9, and 4 together.

2. How many square feet of inch-thick plank are there in a piece of timber 21 feet long, 18 inches broad, and $3\frac{1}{2}$ inches thick?

Expunging the factor 6 from the multiplier 18, and also from the divisor 12, the former is reduced to 3 and the latter to 2; and the numbers to be multiplied together are 21, 3, $3\frac{1}{2}$; and $21 \times 3 \times 3\frac{1}{2} \div 2 = 110\frac{1}{4}$ sq. ft. = 110 sq. ft. 36 sq. in.

3. In a mahogany log 33 inches broad, 19 inches thick, and $23\frac{1}{2}$ feet long, how many square feet of inch-thick plank are there?

Here 33 is divisible by 3, a factor also of the divisor 12; so that expunging this factor from each, the work is $11 \times 19 \times 23\frac{1}{2} \div 4 = 1227\frac{7}{8}$ sq. feet.

4. How many square feet of inch-thick plank are there in 5 pieces, each 15 feet long, 8 inches broad, and 3 inches thick?
   Ans. 150 sq. ft.

5. In a mahogany log $25\frac{1}{2}$ inches broad, 16 inches thick, and $15\frac{1}{2}$ feet long, how many square feet of inch-thick plank are there?
   Ans. 527 sq. ft.

6. In six planks of mahogany, each 18 feet long, $8\frac{1}{2}$ inches broad, and $1\frac{1}{4}$ inch thick, how many inch-thick feet are there? *Ans.* $95\frac{5}{8}$.

Note.—If the planks to be cut out of a log be of any other thickness than an inch, as $\frac{7}{8}$, $\frac{3}{4}$, $\frac{3}{2}$ (or $1\frac{1}{2}$) &c., we shall merely have to multiply the result reached as above by, $\frac{8}{7}$, $\frac{1}{3}$, $\frac{2}{3}$, &c., as the case may be; the number

of superficial feet being *inversely* proportional to the thickness of the plank; for instance, if we wished to know how many square feet of $\frac{7}{8}$ inch plank there are in the mahogany log in Ex. 5, we should first compute for inch-thick planks, and then multiply the number of these, namely, 527, by $\frac{8}{7}$; that is, we should multiply this number by 8, and divide the result by 7; or, which is a little better, we should add to the number $\frac{1}{7}$ of itself, because $\frac{8}{7} = 1 + \frac{1}{7}$, as in the margin; from which we see that the log contains $602\frac{2}{7}$ sq. ft. of $\frac{7}{8}$ inch plank. If the thickness had been $1\frac{1}{2}$ inch, we should have got the number of square feet by adding to 527 its half; and if the thickness had been $\frac{3}{4}$ inch, we should have got the correct result by subtracting from 527 its fourth part.

$$
\begin{array}{r}
7)\ 527 \\
75\frac{2}{7} \\
\hline
602\frac{2}{7} \\
\hline
\end{array}
$$

We may as well add here, though the circumstance is little likely to be overlooked, that whatever be the proposed thickness of the planks, unless the breadth of the log contain this thickness an exact number of times, there will be waste, or at least the final plank will have less thickness; its correct measurement, however, in accordance with this reduced thickness, is included in the result of the calculation. For instance, in cutting inch-thick planks out of a log, if the last plank should be only a quarter of an inch thick, the calculation would give, for this thin slice, the value of only one quarter of a complete inch-thick plank.

II. Solid Measure.

Problem 1.

To find the cubic contents of a squared log of timber.

[A log of timber is said to be *squared* when its outer irregularities have been removed, and it is reduced to a piece with four flat sides.]

Rule I.—*When the four sides are rectangles.* Multiply the three dimensions,—length, breadth, and thickness (or rather the *numbers* expressing these) together: the product will be cubic contents.

Examples.

1. A log of timber is $26\frac{1}{2}$ feet long, $18\frac{1}{2}$ inches broad, and $14\frac{1}{2}$ inches thick: how many cubic feet are there in it?

	ft.	in.	
	26	6	
$18\frac{1}{2}$ in. =	1	6	6
	26	6	
	13	3	
	1	1	3
	40	10	3

	40	10	3		
$14\frac{1}{2}$ in. =	1	2	6		
	40	10	3		
	6	9	8	6	
	1	8	5	1	6
Ans.	49 Ft.	4	4	7	6

Or thus: $26\frac{1}{2}$ ft. = 318 inches
Multiplied by $18\frac{1}{2}$

= 5883
Multiplied by $14\frac{1}{2}$

1728) $85303\frac{1}{2}$ (49 Ft.

Remainder $631\frac{1}{2}$ Inches.

In the first of these operations, the entire process is worked out in full: in the second, the details of the two multiplications and of the final division are suppressed, in order to save room. The former mode of working, by duodecimals, or *cross multiplication*, we consider to be that which should, in general, be preferred: the successive steps of the work may, by help of the *Pence Table*, be very rapidly executed (see p. 44); and the final result is always in accordance with the duodecimal scale of notation, the several terms, proceeding from left to right, regularly descending by *twelfths*: thus, the result 49 ft. 4 4 7 6, above, indicates that the cubic content of the log is 49 cubic feet, 4 twelfths of 1 cubic foot, 4 twelfths of one of *these* twelfths, 7 twelfths of one of the last mentioned portions; and, finally, 6 twelfths of one of the preceding portions. We have not here given any distinctive *names* to these several denominations, nor have we thought it necessary to introduce marks or dashes over the figures; the computer may do so or not, as he pleases, as we have done at p. 45: the gaps which separate the different denominations suffice to keep them distinct; and it is enough to know that the unit of any term is, in value, one-twelfth of the unit in the immediately preceding term: the first 4, after the 49 Ft., is $\frac{4}{12}$ths of a cubic foot; the next 4 is $\frac{4}{144}$ of a cubic foot; the 7 is $\frac{7}{1728}$ of a cubic foot; and the 6 is $\frac{6}{1728 \times 12}$ or $\frac{6}{20736}$ of a cubic foot. It is thus plain that the *third* term after the 49 denotes 7 cubic *inches*, the the term following denoting 6 *twelfths* of a cubic inch, that is, $\frac{1}{2}$ a cubic inch.*

* The computer, however, will perhaps prefer to distinguish these several denominations by dashes marked with the pen, writing the foregoing result thus, 49 Ft. 4' 4'' $7\frac{1}{2}$ In. We have marked them in this way in the answers to some of the questions which follow. If to the cubic contents of the log in Ex. 1 above, as furnished by the com-

The strict accuracy with which this result is given is not absolutely necessary in the practical measurement of timber; yet it is important that the computer should know the exact amount of error committed by rejecting one or more of the final denominations in any result worked out, as in the above instance, with scrupulous accuracy. In order to do this, whatever be the dimensions of the log with which he is concerned, let him imagine a second log, of which each of the ends is just a square foot: then every unit in the first denomination of his result, after cubic feet, will represent a slice off the end of this second log, an inch thick: in the foregoing Example, the 4 represents a solid slice 4 inches thick. Every unit in the *next* term represents a marginal portion of the inch-thick slice, an inch wide; that is, a stick of wood a foot long and an inch square at the ends. Again; every unit in the *next* term denotes an inch cut off the end of this stick of wood, and is therefore 1 cubic inch; and each unit in the term next following denotes a twelfth of a cubic inch: the 6, in the case above, being, as before remarked, $\frac{1}{2}$ of a cubic inch. For all practical purposes, fractions of a cubic inch, even in such costly wood as Spanish mahogany, may be disregarded, at least, whenever the fraction is below $\frac{1}{2}$: when it is (as above) exactly $\frac{1}{2}$, as also when it exceeds $\frac{1}{2}$, it may be taken as an additional whole inch. If it be desired to express the duodecimal result in cubic feet and inches only,—without the intermediate denominations—we shall have merely to multiply the first term, after *Feet*, by 12, taking in the next following term; then to multiply the product by 12, taking in the next term; that is, the *Inches:* thus, in the case worked above, multiplying the 4 by 12, and taking in the 4 next to it, we have 52; and then multiplying this by 12, and taking in the $7\frac{1}{2}$ *inches*, we have $631\frac{1}{2}$ Inches, agreeing with the result in the second operation; so that, in Feet and Inches, the cubic content of the log is 49 Ft. $631\frac{1}{2}$ In.

We have deemed it to be instructive to make these remarks because, in most books upon duodecimal calculations, more especially in reference to "Solid Measure," there is much

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puted result, such marks were attached, there would be *three* dashes over the 7, and *four* over the 6. These accumulated dashes would have an unsightly appearance *in print*, and would occasion a good deal of extra trouble to the compositor.

perplexity and confusion, *twelfths* of a cubic foot being not unfrequently denominated *Inches;* whereas, in truth, every such so-called *inch* contains 144 true cubic inches.

2. How many cubic feet are there in a log of timber 12 ft. 9 in. long, 2 ft. 10 in. broad, and 2 ft. 1 in. thick?
   Ans. 75 Ft. 3' 1'' 6 In., or 75 Ft. 450 In.
3. How many cubic feet are there in a log of which the length, breadth, and thickness are respectively 23½ ft., 33 in., and 19 in.?
   Ans. 96 Ft. 1' 5'' 6 In.

[The single dash here marks 12ths of a Foot, and the ouble dash 144ths.]

RULE II.—*When the log regularly tapers from one end to the other.* Take the breadth and thickness at the middle of the log: these measurements will be the mean breadth and thickness; and these and the length multiplied together, as in Rule I., will give the cubic contents, *nearly.*

NOTE.—The dimensions at the middle of the log, that is, the *mean* breadth and thickness, may be found by taking half the sum of the breadths at the two ends for the mean breadth, and half the sum of the two thicknesses for the mean thickness.

4. Required the cubic contents of a log of which the length is 18 feet, and the mean breadth and thickness 18 inches and 10 inches?

This Example may be worked the more readily by proceeding agreeably to the second method of solving Ex. 1 (p. 149), but without taking the trouble of reducing the feet, in the length of the log, to inches; since *here* there is no fraction of a foot to be taken account of. If we thus omit to multiply this dimension by 12, we must expunge the factor 12 from the divisor 1728; that is, we must divide only by 144. Moreover, the given dimensions here happen to be such that all the work which is absolutely necessary may be executed mentally, without putting pen to paper: for the divisor 144 is readily seen to be 8 times 18, so that if the factor 18 be suppressed in the multiplication, the divisor, instead of 144, will be only 8: but the other two factors are 18 and 10; that is, 9×2, and 5×2, the product of which is 45×4; and expunging this 4, as also the factor 4 in the 8, the divisor of the 45 is reduced to the number 2; and therefore the quotient is 22½ Ft., the required cubic contents. And in this way, by expunging factors common to *either* multiplier and to the divisor, may the work, where such

common factors exist, always be shortened. The following is an additional example.

5. Required the cubic contents of a log of which the length is 16 feet, and the mean breadth and thickness each 14 inches?
   Ans. $21\frac{7}{9}$ Ft.
6. Required the cubic contents of a log 18 feet long, the ends of which are squares, each side of one end being 18 inches, and each side of the other end 12 inches?
   Ans. $28\frac{1}{8}$ Ft.

[It was stated in the Rule, that the cubic contents of a regularly tapering log, calculated in the foregoing manner, are not determined with strict mathematical accuracy, but only *nearly*. It is however the rule always employed in the timber trade, though the results it gives are usually somewhat short of the exact results; a sort of compensation being thus made for the departure of the shape of the log from that uniformity in breadth and thickness which it is desirable that a log of timber should have. The strictly correct rule for a tapering log, of which the ends, though unequal, are similar figures, is as follows:—

To the areas of the two ends add the square root of their product: multiply the sum by the length of the log, and take one-third of the product for the cubic contents.

Thus, taking Ex. 6, above, we have—

For the areas of the two ends, $18^2 + 12^2 = 324 + 144 = 468$; and for the sq. root of their product, $\sqrt{(324 \times 144)} = 18 \times 12 = 216$. Then, by this Rule, $(468 + 216) \times 18 \div 3 = 684 \times 6 = 4104$; and this divided by 144 gives $28\frac{1}{2}$ Feet; while the answer above is only $28\frac{1}{8}$ Ft., the difference being $\frac{3}{8}$ of a cubic foot, which may be regarded as an allowance for the inequality in the two ends of the log.]

PROBLEM 2.

To find the cubic contents of a log of round or unsquared timber.

When a tree is lopped of its branches, and but roughly dressed, it is called *round timber;* and the circumference of it at any part is called the *girt* (or girth) at that part. When the tree tapers regularly, the girt in the midde, or half the sum of the girts at the two ends, is the *mean girt:* but it is often more satisfactory to girt the tree in several places, and to divide the sum of the several girts by the number of them

for the mean girt. One-fourth of the mean girt is called the *quarter-girt* of the tree; and the Rule for the cubic contents is as follows.

RULE.—Multiply the number of *inches* in the quarter-girt by itself, and the product by the number of *feet* in the length: the result, divided by 144, will give the number of cubic feet in the tree.

EXAMPLES.

1. A tree is 24 feet long, its girt at the thicker end 14 feet, and at the thinner end 2 feet: what are its cubic contents?

   16 ft. $\div$ 2 = 8 ft. = 96 in., the mean girt, therefore 24 in. = the quarter-girt; then by the Rule, $24 \times 24 \times 24 \div 144 = 2 \times 2 \times 24 = 96$ Ft.

In *all* cases where multiplications and divisions are concerned, the computer should be on the look out for factors common to multipliers and divisors, and suppress them *before* entering upon the numerical work: thus, in the present example, an experienced calculator would not write down $24 \times 24 \times 24 \div 144$ at all; but, dismissing the factors 12, 12, which he would see at a glance to be common to the multipliers 24, 24, and to the divisor 144, would write merely the factors left.]

2. A tree is $20\frac{1}{2}$ feet long, and its quarter-girt is $10\frac{1}{4}$ inches: required its cubic contents?

$$
\begin{array}{rl}
 & 10\frac{1}{4} \\
 & 10\frac{1}{4} \\
\hline
 & 100 \\
 & 2\frac{1}{2} \\
 & 2\frac{1}{2} \\
\hline
 & 105 \\
 & 20\frac{1}{2} \\
\hline
 & 2100 \\
 & 52\frac{1}{2} \\
\hline
144) & 2152\frac{1}{2} \\
\hline
 & 14 \text{ Ft.} \\
\hline
12) & 136\frac{1}{2} \text{ Rem.} \\
\hline
 & 11\frac{3}{8}
\end{array}
$$

The operations to be performed are thus indicated, viz., $10\frac{1}{4} \times 10\frac{1}{4} \times 20\frac{1}{2} \div 144$, and these we shall execute as in the margin, disregarding the multiplication of $\frac{1}{4}$ by $\frac{1}{4}$; since, as we shall presently see, the fraction arising from this multiplication is of no practical consequence. The result of the operation shows that the contents of the tree is 14 cubic feet and $11\frac{3}{8}$ *twelfths* of a cubic foot besides.

Let us now estimate the amount of error in this conclusion, arising from our neglect of the $\frac{1}{4} \times \frac{1}{4}$, or $\frac{1}{16}$, which, in strictness, ought to have entered the first product, 105, the correct product being $105\frac{1}{16}$. The omission of this fraction causes an error of $\frac{1}{16} \times 20\frac{1}{2}$, that is, of $\frac{41}{32}$, in the *next* product, which, *accurately*, is $2152\frac{1}{2} + \frac{41}{32}$; so that, in

addition to the 14 Ft. $+ \frac{136\frac{1}{2}}{144}$ Ft., we ought to have had $\frac{41}{32 \times 144}$ Ft. besides. Now this fraction is obviously the sum of the two fractions $\frac{32}{32 \times 144}$ and $\frac{9}{32 \times 144}$; that is, it is equal to $\frac{1}{144} + \frac{9}{32 \times 144}$, and therefore *less* than $\frac{1}{144} + \frac{1}{3}$ of $\frac{1}{144}$. Hence, referring to our standard log, or log of reference, adverted to at p. 150, and which is a log 1 foot in breadth and in thickness throughout, it follows that the sacrifice of timber, in the case before us, is *less* than a slice off the end of that log, $\frac{1}{12}$ of an inch *plus* $\frac{1}{3}$ of $\frac{1}{12}$ in thickness; that is to say, a slice scarcely the thickness of a shilling, which, of course, in a log of rough timber, is of no moment whatever.

We have entered into these considerations here, in order that the timber-measurer may see how unnecessary it is, in multiplying the quarter-girt by itself, that such fractions of a square inch should be computed, and retained in the product. He will perceive, from the above, that in general, if this product be determined to the nearest inch only—in *excess* of the exact product, if the fraction be equal to or greater than $\frac{1}{2}$, and in *defect* of the exact product, if the fraction be less than $\frac{1}{2}$, no appreciable error in the cubic contents of the log will be committed.

Moreover, there is an additional reason for disregarding these niceties in calculating the cubic measurement of rough timber. The quarter-girt Rule, given above, however strictly followed, does not *itself* give the exact cubic measurement, nor is it intended to do so: the *true* result is about one-fourth more than the result arrived at by the Rule, as may be proved from geometrical considerations; but it is the custom of the trade to make this allowance of one-fourth for the waste (waste as *timber*), in squaring the rough log. The *greatest* squared log that can be cut out of any portion of a rough round tree, is that of which the breadth and thickness are equal; that is, the cross-section must throughout be a *square;* for any other four-sided shape, the timber, in the squared log, will be less; and this fact it is well should be

generally known: the allowance for waste presumes that the squaring is thus economically executed. This circumstance—of the trade-rule giving the cubic measurement of a rough log one-fourth less than the number of cubic feet in it actually is,—satisfactorily explains why it is that, in Tables of the bulks of different materials which go to a *load*, we find

"40 cubic feet of round, or rough, timber = 1 load."
and "50 feet of squared timber = 1 load."

The 40 cubic feet of round or rough timber, as calculated per Rule, is in reality 40 feet *plus* one-fourth of 40 feet; that is, it is 50 feet. (See TABLE, below.)

3. A piece of round timber is 9 feet 6 inches long, and its quarter-girt 42 inches: how many cubic feet are there in it? *Ans.* 116$\frac{3}{8}$ Ft.
4. The average girt of a round log is 6 feet 3 inches, and its length is 12 feet: how many cubic feet are there in it? *Ans.* 29$\frac{1}{4}$ Ft.

NOTE.—If the tree taper very irregularly, it is best to divide it into several lengths, and to find the cubic contents of each portion separately.

TIMBER TABLE.

40 cubic feet of round timber make 1 load.
50 „ squared timber „ 1 load.

600 square feet of inch-thick plank		„	1 load.
400 „	$1\frac{1}{2}$ in. „	„	
300 „	2 in. „	„	
200 „	3 in. „	„	
150 „	4 in. „	„	

Average Weight of a Cubic Foot, in lbs. Avoirdupois.

Name of Wood.	Weight in lbs.	Name of Wood.	Weight in lbs.
Cork	15	Cherry-tree . . .	44·68
Poplar	23·94	Teak	46·56
Larch	34	Maple and Riga Fir .	46·87
Elm	34·75	Ash and Dantzic Oak	47·5
Honduras Mahogany .	35	Dutch Yew . . .	49·25
Willow	36·56	Apple-tree	49·56
Cedar	37·25	Alder	50
Pitch Pine	41·25	Spanish Yew . . .	50·44
Pear-tree	41·31	Spanish Mahogany .	53·25
Walnut	41·94	Canada Oak . . .	54·5
Mar Forest Fir . .	43·37	French Box . . .	57
Elder-tree	43·44	English Oak . . .	60·62
Beech	43·5	Ditto 60 years old . .	73·12

[We see by this Table how inaccurate it is to call, indiscriminately, a load of wood (50 cubic feet) "a ton."]

From the foregoing Table, the number of cubic feet in a misshapen log, or block of wood, may be ascertained from knowing the weight of the mass; and conversely, the cubic contents being known, the weight may be found, as in the two problems following.

PROBLEM 1.

The weight being given, to find the cubic contents of a piece of timber.

RULE.—Divide the number of lbs. in the given weight by the number of lbs. in a cubic foot, as given in the Table; and the quotient will be the number of cubic feet in the piece.

EXAMPLES.

1. How many cubic feet are there in a ton of Honduras Mahogany? By the Table, a cubic foot weighs 35 lbs., and 2240 is the number of lbs. in a ton: hence by the Rule, $2240 \div 35 = 64$, the number of cubic feet in a ton. If the wood had been English Oak, the work would have been $2240 \div 60·62 = 36·95$, the number of cubic feet in a ton of English Oak.
2. How many cubic feet are there in 7 cwt. 12 lbs. of Spanish Mahogany? 7 cwt. 12 lbs. = 796 lbs.; then, by the Rule, $796 \div 53·25 = 14·94$, the number of cubic feet. *Ans.*
3. How many cubic feet are there in a ton of Beech? *Ans.* 51·49 cubic feet.
4. How many cubic feet are there in a ton of Riga Fir? *Ans.* 47·76 cubic feet.
5. How many cubic feet are there in a mass of English Oak weighing 11 cwt? *Ans.* 20·32 cubic feet.

[It will of course be understood that the above are the average, or medium results, in each case; for the same weight of different specimens may vary in bulk; and conversely.]

PROBLEM 2. (CONVERSE OF PROB. 1.)

The bulk of a piece of timber being given, to find its weight.

RULE.—Multiply the cubic contents of the piece by the corresponding tabular number; and the product will be the number of lbs. weight.

EXAMPLES.

1. What is the weight of a log of Larch 14 feet long, $2\frac{1}{2}$ feet broad, and $1\frac{1}{4}$ feet thick?

   $2\frac{1}{2} \times 14 = 35$; and $35 \times 1{\cdot}25 = 43{\cdot}75$ Ft., the cubic contents.
   Then, $43{\cdot}75 \times 34 = 1487{\cdot}5$ lbs. = 13 cwt. 1 qr. $3\frac{1}{2}$ lbs., *Ans.*

2. What is the weight of a log of Honduras Mahogany, of which the contents are 64 cubic feet? *Ans.* 1 ton.

3. What is the weight of a piece of Spanish Mahogany, of which the contents are 14·94 cubic feet? *Ans.* 7 cwt. 12 lbs.

4. What is the weight of a log of Pitch Pine, 24 feet long, 3 feet broad, and $2\frac{1}{2}$ feet thick? *Ans.* 3 tons 6 cwt. 23 lbs.

[From the numbers in the preceding Table may easily be deduced the *Specific Gravity* of any of the woods there registered. By the specific gravity of any substance, is meant the *ratio* of the weight of any bulk of it, to the weight of an equal bulk of distilled water, when at the temperature of 60° of Fahrenheit's thermometer. At this temperature, a cubic foot of the water weighs 1000 ounces avoirdupois, that is, $62\frac{1}{2}$ lbs. If therefore any one of the numbers in the Table be divided by $62\frac{1}{2}$, the quotient will be the ratio alluded to; that is, the specific gravity of the wood which that number stands against. For example, take larch: then $\frac{34}{62\frac{1}{2}} = \frac{68}{125}$ $= {\cdot}544$,—the specific gravity of larch.]

CALCULATIONS USEFUL IN THE WORKS OF ARTIFICERS.

By the term "Artificers" is to be understood those classes of workmen whose workmanship, as well as the materials on which they operate, is estimated by measurement. Such are Carpenters, Joiners, Masons, &c.

I.—CARPENTERS' AND JOINERS' WORK.

This kind of work, as called into requisition in a building, comprehends flooring, partitioning, wainscoting, roofing, &c. The workmanship is generally estimated in square measure, though cornices, mouldings, and such like ornamental work, are usually measured by the lineal foot. Flooring and partitioning, and other boarding of much extent, are measured

by the *square* of 100 superficial feet; that is, a square of boarding, whether floor, or wainscot, or partition, &c., is 100 superficial feet.

A SQUARE OF BOARDING REQUIRES,

With boards 10 *feet long,*

24 boards, 5 inches broad.	15 boards, 8 inches broad.
20 ,, 6 ,, ,,	12 ,, 10 ,, ,,

If the boards be 7 inches broad, there must be 17 of them, and a slip 1 inch broad off the whole length of another board besides, to make up the *square;* the measure of this additional piece being 120 square inches, or $\frac{5}{6}$ square feet. If the boards be 9 inches broad, there must be 13 of them, with an additional slip 3 inches broad besides; the measure of this slip being $2\frac{1}{2}$ square feet.

With boards 12 *feet long,*

20 boards, 5 in. broad.	12 boards, 8 in. broad + 4 sq. ft.
16 ,, 6 ,, + 4 sq. ft.	11 ,, 9 ,, + 1 sq. ft.
14 ,, 7 ,, + 2 sq. ft.	10 ,, 10 ,,

*** The additional square feet here is the measure of a slip so many *inches* broad and 12 feet long.

The reader may readily satisfy himself that the number of boards specified above is, in each case, such as to make up exactly 100 superficial feet; thus, the number of ten-foot boards, laid side by side, must make an entire breadth of boarding of 10 feet, or 120 inches, in order that the whole surface may measure 100 square feet; and it is easy to see that this is the breadth reached by using the number of boards stated in the Table. When the breadth of board is 7 inches or 9 inches, a fraction of a board, or a slip off the whole length of it, is necessary to complete the square; in the former case, the slip must be 1 inch broad, in the latter case, 3 inches, because $17 \times 7 = 119$ only, instead of 120, and $13 \times 9 = 117$ only, instead of 120. A slip an inch broad, off a board 10 feet or 120 inches long, measures, of course, 120 square inches.

With boards 12 feet long, the entire breadth, to complete 100 square feet, must be 100 inches, that is, $8\frac{1}{3}$ feet; and

every inch of breadth, cut off the entire length of such a board, is a slip measuring 144 square inches, or 1 square foot: the correctness of the statements in the Table are thus obvious. But if the boards are rough, and unprepared for *wrought* flooring, then an allowance must be made for the loss of breadth in planing the edges; thus, while twelve and a half 12-feet boards, 8 inches broad, suffice for a square of rough flooring, thirteen are allowed for a square of wrought flooring; and for rough boards, 9 inches broad, 11½ are allowed for a square of wrought flooring; while of 12-feet battens, 7 inches broad, fifteen are considered necessary, if the edges are to be planed. The narrower the board the greater, of course, is the waste in planing.

[There are what are called "Standard Measurements" for Deals and Battens, these latter being only a narrower kind of deals. The London and Dublin standard deal is 12 feet long, 9 inches broad, and 3 inches thick; while the batten (of whatever length) is 7 inches broad and 3 inches thick. When the breadth and thickness—one or both—differs from these dimensions, the piece is called simply a *board*, or a *plank*; but when a three-inch deal, or batten, is sawn into thinner boards for flooring, partitioning, &c., each board is called a *leaf*.

The standard for deals differs in different countries; the principal are as follows:—

Standard Deals.

	Length.	Breadth.	Thickness.	
Petersburg . . .	12 ft.	11 in.	1½ in.	= 16½ sq. ft., 1 in. thick.
Christiana . . .	11	9	1¼	= $10\frac{5}{16}$,, ,,
London and Dublin	12	9	3	= 27 ,, ,,
Quebec [100] . .	12	11	2½	= 27½ ,, ,,

In the Timber-market, 120 deals go to the hundred, except the Quebec 100; and, consequently, in a Petersburg Hundred there are 1980 superficial feet, 1 inch thick, or 165 cubic feet; in a Christiana Hundred, 1237½ superficial feet, 1 inch thick, or 103⅛ cubic feet; in a London and Dublin Hundred, 3240 superficial feet, 1 inch thick, or 270 cubic feet; and in a Quebec Hundred, 2750 superficial feet, 1 inch thick, or 229⅛ cubic feet.]

EXAMPLES.

1. A piece of boarding measures 96 ft. 3 in. by 21 ft. 3 in.: how many squares are there in it?

From the annexed work, it appears that there are 2045 sq. ft., 3 twelfths of a sq. ft., and 9 sq. in.; that is to say, 2045 Ft. 45 In.: so that, dividing the number of square feet by 100, the answer is 20 Squares, 45 Ft., 45 In.

ft.	in.	
96	3	
21	3	
2021	3	
24	0	9
20,45	3	9

2. If a floor be 57 ft. 3 in. by 28 ft. 6 in., how many squares are there in it? [See the work below.]

As the 7, in the second place, represents so many twelfths of a square foot, and the 6, in the next place, so many square inches, the answer is 16 Squares, 31 Ft., 90 In. = 16$\frac{8}{25}$ sq. nearly.

ft.	in.	
57	3	
28	6	
1603	0	
28	7	6
16,31	7	6

[In all calculations of this kind care must be taken to avoid the not uncommon error of regarding the *second* term in the result as so many square *inches*, instead of so many *twelfths* of a square foot. Each of these twelfths is, of course, 12 inches, not 1 inch; it is the *third* term that denotes square inches: the 7 above represents 84 square inches.]

3. A partition measures 91 ft. 9 in. by 11 ft. 3 in.: how many squares are there in it? *Ans.* 10 Squares. 32 Ft., 27 In.
4. A wainscoted room is 16 ft. 3 in. high, and 137 ft. 6 in. in compass: how many squares are there in it?
   Ans. 22 Squares, 34 Ft., 54 In.
5. A piece of boarding measures 36 ft. 4 in. by 12 ft. 3 in.: what did it cost (materials and workmanship), at the rate of £6 15*s.* per square? *Ans.* £30 0*s.* 10¼*d.*

It may be well that we should exhibit the work of this example at length.

ft.	in.	
36	4	
12	3	
436	0	
9	1	0
4,45	1 = 4 Sq. 45 Ft. 12 In.	

	£	s.	d.	
	6	15	0	
			4	
Value of 4 Sqs. =	27	0	0	
,, 25 Ft. =	1	13	9	= ¼ of £6 15*s.*
,, 20 Ft. =	1	7	0	= ⅕ ,,
	£30	0*s.*	9*d.*	

[45 Ft. = 25 Ft. + 20 Ft. = ¼ of 100 + ⅕ of 100.]

The 12 Inches are still to be valued; that is, the twelfth part of 1 Ft. The value of 1 Ft. being the 20th part of 27*s.*, we have to divide this

by 20 and 12, or to take the 20th part of 27 *pence*, which may be regarded as $1\frac{1}{2}d.$; so that the answer to the question is £30 0*s.* $10\frac{1}{2}d.$ The odd halfpenny would, of course, be disregarded. But the price of a square of boarding, or flooring, &c., being given, the price of any number of squares, and parts of a square, may be readily calculated by the following Rule.

Problem 1.

The price of a square being given, to find the price of any number of squares and fractions of a square.

Rule.—Regard each Square as £5, and therefore each Ft. as 1*s.*, and each twelfth of 1 Ft. as 1*d.* We shall thus have the price of the whole at the rate of £5 per square: then take parts for the difference between £5 and the given price per square.

Thus, returning to the Example above, we have :—

	£	s.	d.		£	s.
For 4 Sq. $45\frac{1}{12}$ Ft.	22	5	1 =	price at	5	0
$\frac{1}{5}$ of the above	4	9	0 =	,,	1	0
$\frac{1}{2}$ of last sum	2	4	6 =	,,	0	10
$\frac{1}{2}$,,	1	2	3 =	,,	0	5
	£30	0*s.*	10*d.*	,,	£6	15*s.*

We have here disregarded the fractions of 1*d.*; they are as follows: $\frac{1}{5}d. + \frac{1}{10}d. + \frac{1}{20}d. = \frac{4+2+1}{20}d. = \frac{7}{20}d.$, the same as before.

2. The dimensions of a floor are 53 ft. 6 in by 27 ft. 9 in.: required the cost, at £3 15*s.* per square? *Ans.* £95 15*s.* $11\frac{1}{2}d.$
3. If a floor be 57 ft. 3 in. by 28 ft. 6 in., what will be the cost of the boarding, at £1 13*s.* 4*d.* per square? *Ans.* £27 3*s.* $10\frac{1}{2}d.$
4. What will be the cost of a boarded floor, measuring 86 ft. 10 in. by 28 ft. 6 in., at the rate of 18*s.* per square? *Ans.* £37 16*s.*

Note.—In the foregoing examples we have considered the flooring to be the *boarded* flooring; but it may be proper here to remind the reader that, previously to the boarding, a *naked flooring*, as it is called, is constructed to support the boards. This is a framework, the component pieces of which are called *joists* and girders, which are partially inserted in the brickwork or masonry. The squares in a naked flooring are computed in exactly the same way as the squares in a boarded flooring,—the measurements of length and breadth being extended to the extreme limits of the timber, including the insertions in the walls, though these are out of sight. *Roofing*, also, is computed in like manner; the girt of the roof, measured with a string from the extreme

end of one rafter, over the ridge, to the extreme end of the opposite rafter, is taken for the breadth, and this is multiplied by the length, and the number of squares of roofing found, just as if these were the dimensions of a boarded floor.

We here, however, consider the roof to present but two sloping faces, terminating in a single ridge. But in many structures the roof presents four faces or slopes, the opposite pair being usually symmetrical. Every such face is measured and computed exactly as we measure and compute the surface of a tapering board or plank (p. 145), that is, the two parallel lengths are added together—the base of the slope and the upper parallel ridge—and half their sum is multiplied by the distance between them; this being the length of a string stretched from one to the other. Should either face terminate in a point, instead of in a ridge, then merely half the base of the triangle is to be multiplied by the distance of it from the vertex, or terminating point; for then there is no length of upper ridge to be added. The surfaces of all the faces being added together, the sum will be the surface-measure, in square feet, of the whole roof. If the faces terminate in a flat, the surface of this flat is of course to be added.

Problem 2.

The price per Hundred (120) *deals being given, to find the price per single deal.*

Rule.—Twice the number of £'s in the price of a Hundred will be the number of pence in the price per single deal.

Examples.

1. At £12 per Hundred, what is the price of a single deal?
   $12 \times 2 = 24$ pence; therefore the price is 2*s.*
2. At £14 5*s.* per Hundred, what is the price per deal?
   $14\frac{1}{4} \times 2 = 28\frac{1}{2}$ pence $=$ 2*s.* 4½*d.* *Ans.*

The reason of the Rule is pretty obvious: there are 120 twopences in £1; so that, at £1 per Hundred, each deal would cost 2*d.*; hence as many £'s as there are in the cost of a Hundred, so many twopences must there be in the cost of one.

3. At £15 5*s.* per Hundred, what is the price per deal? *Ans.* 2*s.* 6½*d.*
4. At £16 15*s.* per Hundred, what is the price per deal? *Ans.* 2*s.* 9½*d.*

Problem 3. (Converse of Prob. 2.)

The price of a single deal being given, to find the price of a Hundred (120).

Rule.—Half the number of pence in the price of one will be the number of £'s in the price of 120.

This Rule is an obvious inference from that of last problem.

Examples.

1. If a single deal cost 2*s.* 10*d.*, what is the price of 120?

   2*s.* 10*d.* = 34 pence; therefore the price of 120 is £17.

2. If a single deal cost 2*s.* 6½*d.*, what will 120 cost? *Ans.* £15 5*s.*
3. If a single deal cost 2*s.* 3*d.*, what will 120 cost? *Ans.* £13 10*s.*

Problem 4.

The price per running foot (foot of the length) of a standard deal being given, to find the price of a Hundred; and conversely.

Rule—1. Six times the number of pence in the price of a running foot will be the number of £'s in the price of 120 standard deals.

For as many pence as 1 foot costs, so many shillings will 12 feet, or a whole deal cost. But there are 120 deals; and $120 = 6 \times 20$: hence, if we multiply the before-mentioned number of shillings (that is, the given number of pence) by 6, the result must be the number of £'s in the cost of the whole 120.

2. *Conversely.* Divide the number of £'s, in the cost of 120 standard deals, by 6, and the result will be the number of pence per running foot.

These two Rules may be expressed a little differently: thus, for the first we may say,—Regard the pence in the given price as so many £'s, and multiply by 6: and for the second,—Regard the £'s in the given price as so many pence, and divide by 6. We shall here work by the Rules in this form.

Examples.

1. What is the price of 120 standard deals, at 3½*d.* per running foot; and what the price at 2¾*d.*?

   1st. £3½ × 6 = £21. *Ans.* 2nd. £2¾ × 6 = £2 15*s.* × 6 = £16 10*s.* *Ans.*

2. At £16 per Hundred, what is the price of a running foot; and what the price at £13 10*s.*?

   1st. 16*d.* ÷ 6 = 2⅔*d.* *Ans.* 2nd. 13½*d.* ÷ 6 = 27*d.* ÷ 12 = 2¼*d.* *Ans.*

3. Required the price of 120 standard deals, at 2¼*d.* per running foot? *Ans.* £13 10*s.*
4. If 120 standard deals cost £16 15*s.*, what is the cost per running foot? *Ans.* 2⅚*d.*

Note.—A 12-foot board, whether a standard deal or not, always

contains as many superficial feet as there are inches in its breadth ; for the surface, in sq. feet, is 12 ft. multiplied by the 12th part of the number of inches in the breadth; and 12 times the 12th part of any number is that number itself. The value of any portion of breadth cut off, and running the whole length of such a board, is therefore easily found; thus,—As the whole breadth is to the breadth of the slip cut off, so is the price of the board to the value of the part taken away. For example: From a 12-foot deal, 9 in. broad, a slip, 2 in. broad, is cut off; what is the value of it, at 2*s.* 6*d.* for the deal?

The value is $\frac{2}{9}$ of 2*s.* 6*d.* = 5*s.* ÷ 9 = 6$\frac{2}{3}$*d.*

Again:—If 2*s.* 6*d.* be charged for a 12-foot batten, 7 inches broad, cut off a board of the same length, 11 inches broad, what is the entire board valued at? The value is $\frac{11}{7}$ of 2*s.* 6*d.* = 27*s.* 6*d.* ÷ 7 = 3*s.* 11$\frac{1}{7}$*d.**

II.—Bricklayers' Work.

Brickwork is usually measured by either the square yard, or the square pole, or rod, or perch. As the linear English rod, or perch, is 5½ yards, or 16½ feet, the square rod or perch contains 30¼ square yards, or 272¼ square feet.

The surface of the brickwork being measured in this way, the unit or standard of thickness is fixed at a brick and a half, in addition to the mortar; so that if there be more or less than a brick and a half in the thickness of a wall, the number of square yards or rods is computed accordingly. Thus, if the wall be 2 bricks thick, its thickness is 1$\frac{1}{3}$ of the standard thickness; if 3 bricks thick, twice the standard thickness; if 1 brick thick, $\frac{2}{3}$ of the standard thickness, and so on.

Problem 1.

The length and height of a wall being given, and the number of half bricks in its thickness, to find how many yards or rods of brickwork it contains.

Rule—1. Multiply the length and height together, for the number of square feet in the surface of the wall.

2. Multiply this number of square feet by the number of half bricks in the thickness, and divide the product by 3: the quotient will be the number of square feet a brick and a

* For much practical information respecting the purchasing and measurement of Timber and Deals, see "The Timber Importer's and Timber Merchant's Guide," by Richard E. Grandy: published by Lockwood and Co.

half thick. Divide again by 9, to bring these feet to square yards, or by $272\frac{1}{4}$ to bring them to square rods or perches:

EXAMPLES.

1. How many sq. rods of brickwork are there in a wall, of which the length is 57 ft. 3 in., and the height 24 ft. 6 in ; the wall being $2\frac{1}{2}$ bricks thick?

	ft.	in.		
	57	3		
	24	6		
	1374	0		
	28	7	6	
	1402	7	6	= area of surface of wall.
			5	= half-bricks in thickness.
3)	7013	1	6	
$272\frac{1}{4}$)	2337	8	6	(8 sq. rods, $17\frac{2}{3}$ sq. yards; the *Ans.*
	2178			
9)	159			
	$17\frac{6}{9} = 17\frac{2}{3}$ sq. yards.			

The $8\frac{1}{2}$ twelfths of a sq. foot have not been divided by the 9, and so converted into a fraction of a yard, as the result would be insignificant; but instead of entirely rejecting the $8\frac{1}{2}'$ as here, the small error would have been still less if we had replaced it by an additional foot, making the dividend 2338, and therefore the overplus sq. yards $17\frac{7}{9}$; which errs less in excess than $17\frac{6}{9}$ errs in defect; the former measure being only $3\frac{1}{2}'$ in excess, while the latter is $8\frac{1}{2}'$ in defect.

$57\frac{1}{4}$
$24\frac{1}{2}$
234
114
$28\frac{5}{8}$
$1402\frac{5}{8}$
5
3) $7013\frac{1}{8}$
2338

The calculation may be performed a little more expeditiously, without duodecimals, in the manner here annexed. The number 2338 is the measure of the brickwork to the nearest sq. foot; and the measure in sq. rods is obtained by dividing this number by $272\frac{1}{4}$, as above. Whenever the odd inches in the linear measurement amount to half or a quarter of a foot, the superficial contents may often be most readily computed in this way.

2. How many square yards of brickwork are there in a wall 75 ft. long, and 15 ft. 9 in. high, the wall being 3 bricks thick?
Ans. 262 Yds. $4\frac{1}{2}$ Ft.

[When the thickness is 3 bricks, instead of multiplying by the number of half-bricks (6), and dividing by 3, we need merely multiply by 2; and when the thickness is $4\frac{1}{2}$ bricks, we have only to multiply by 3.]

3. A garden, 160 feet broad, contains exactly one acre: what would be the expense of inclosing it with a brick wall $10\frac{1}{2}$ ft. high and $2\frac{1}{2}$ bricks thick, allowing the work to be at the rate of 5*s.* $7\frac{1}{2}$*d*

per sq. yard, of standard thickness; deductions being made for two doors, each 6 ft. 9 in. by 4 ft., and for a gateway, the height of the wall, and 11 ft. wide? *Ans.* £463 18*s.* 10¾*d.*

It has been found that 272 feet of superficial area of brickwork, a brick and a half thick, requires 4500 bricks, making due allowance for waste: hence the number of bricks necessary for every square foot of this surface will be found by dividing 4500 by 272. Now

$$4500 \div 272 = 16 \cdot 544117...$$

Consequently two-thirds of this will be the number of bricks required for each square foot of the work, when it is only 1 brick thick; four-thirds, when it is 2 bricks thick; five-thirds when it is 2½ bricks thick; and so on; and in this way is the following Table constructed.

TABLE

Showing the number of bricks required for walls of different thicknesses.

Super-ficial area in sq. ft.	Number of bricks required.				
	1 brick.	1½ bricks.	2 bricks.	2½ bricks.	3 bricks.
1	11·029...	16·544...	22·059	27·574	33·088
2	22·059	33·088...	44·118	55·147...	66·176...
3	33·088...	49·632...	66·176...	82.721	99·265
4	44·118	66·176...	88·235...	110·294...	132·353
5	55·147...	82 721	110·294...	137·868	165·441
6	66·176...	99·265	132·353	165·441...	198·529...
7	77·206	115·809	154·412	193·015	231·618
8	88·235...	132·353	176·471	220·589	264·706
9	99·265	148·897...	198·529...	248·162	297·794...

The practical use of this Table will be sufficiently seen from the following example.

4. Required the number of bricks necessary to build a wall 2½ bricks thick, the superficial area of the face of the wall being 2346 feet?

Number of bricks for 2000 sq. ft. = 1000 times the number for 2 sq. ft.	= 55147
Number of bricks for 300 sq. ft. = 100 times the number for 3 sq. ft.	= 8272·1
Number of bricks for 40 sq. ft. = 10 times the number for 4 sq. ft.	= 1102·94
Number of bricks for 6 sq. ft. = 6 times the number for 1 sq. ft.	= 165·44
The number of bricks required	= 64687·48

Hence 64688 bricks will be necessary.

NOTE.—Brick and mortar work is frequently estimated not by the square rod or perch, which is a square surface of brickwork $16\frac{1}{2}$ feet each way and $1\frac{1}{2}$ bricks thick, but by what is called the "standard perch," which is a mass of brickwork, in surface $16\frac{1}{2}$ feet by 1 foot, and (bricks and mortar) 14 inches thick. The cubic contents of the standard perch is therefore $16\frac{1}{2}$ ft. $\times 1 \times 1\frac{1}{6} = 19\frac{1}{4}$ cubic feet, and consequently by this number must the cubic contents of the brickwork be divided to get the number of standard perches in it. If however the work be a brick and a half thick, that is (including the mortar joint), 14 inches, then it will be sufficient to compute the *surface* only of the brickwork, and to divide the result by $16\frac{1}{2}$, instead of by $19\frac{1}{4}$; because in finding the *cubic* contents we should multiply by 14 inches, or $1\frac{1}{6}$ ft., and then afterwards divide by $16\frac{1}{2} \times 1\frac{1}{6}$: it is better therefore to omit the $1\frac{1}{6}$ both as multiplier and divisor. We thus have the following Rule to find the number of "standard perches" in a piece of brickwork.

PROBLEM 2.

To find the number of standard perches in a piece of brickwork.

RULE.—1. *When the work is* 14 *inches thick.* Find the number of square feet in the surface, and divide that number by $16\frac{1}{2}$: the result will be the number of standard perches.

2. *When the work is of any other thickness.* Find the number of cubic feet in the entire mass, and divide by $19\frac{1}{4}$: the result will be the number of standard perches.

EXAMPLES.

1. How many standard perches of brickwork are there in a wall 40 ft. long, $10\frac{1}{2}$ ft. high, and 14 in. thick?

$$\begin{array}{r l} & 10\frac{1}{2} \\ & \underline{40} \\ 16\frac{1}{2}) & 420 \text{ sq. ft. of surface.} \\ & 25\frac{15}{33} = 25\frac{5}{11} \text{ standard perches. } \textit{Ans.} \end{array}$$

2. How many standard perches of brickwork are there in the front wall of a house 25 ft. wide, 30 ft. high, and 18 in. thick; the following openings (or "opes," as workmen call them) being deducted,—namely,—

2 windows	5 ft. by $3\frac{1}{2}$ ft.	=	35 sq. ft.
2 "	6 ft. by 4 ft.	=	48 "
2 "	$4\frac{1}{2}$ ft. by 3 ft.	=	27 "
1 door	7 ft. by $3\frac{1}{2}$ ft.	=	$24\frac{1}{2}$ "

Total deduction for "opes" = $134\frac{1}{2}$ sq. ft.

One of the three dimensions to be multiplied together, namely, the $1\frac{1}{2}$ ft. of thickness, we shall replace by the number 6, which is 4 times $1\frac{1}{2}$. By so doing we shall simplify the work, replacing the divisor $19\frac{1}{4}$ by 77, which is 4 times $19\frac{1}{4}$: the calculation will then stand as follows:—

$$
\begin{array}{rl}
25 \times 30 = 750 & \text{sq. ft. of wall} \\
\text{Deduct } 134\frac{1}{2} & \text{for "opes."} \\
\hline
615\frac{1}{2} & \text{sq. ft. of actual brickwork.} \\
6 = 4 & \text{times } 1\frac{1}{2}. \\
\hline
4 \text{ times } 19\frac{1}{4} = 77)\ 3693 & (47\tfrac{74}{77} \text{ standard perches.} \\
308 & \\
\hline
613 & \\
539 & \\
\hline
74 &
\end{array}
$$

It thus appears that there are 48 standard perches of brickwork, very nearly.

It should be mentioned that the mass of brickwork in an Irish perch differs from that in a standard perch. The linear measure of the Irish perch is 7 yards, instead of $5\frac{1}{2}$ yards; and the Irish perch of brickwork is 21 ft. $\times 1 \times 1\frac{1}{6} = 24\frac{1}{2}$ cubic ft., per Irish perch.

3. How many Irish perches of brickwork are there in a wall 60 feet long, 8 ft. 4 in. high, and 18 in. thick?

[To avoid fractions, we shall multiply the thickness ($1\frac{1}{2}$ ft.) by 2, and also the divisor $24\frac{1}{2}$ by 2.]

$8\frac{1}{3} \times 60 = 500$; and 3 times this is 1500; then 49) 1500 ($30\frac{30}{49}$, so that the wall contains $30\frac{30}{49}$ Irish perches; that is, $30\frac{5}{8}$ Irish perches very nearly.

4. A piece of brickwork is 66 ft. long, 20 ft. 6 in. high, and 28 in. thick: required 1st, the number of standard perches in it; and 2nd, the number of Irish perches?

Ans. 164 standard, and $128\frac{6}{7}$ Irish perches.

NOTE.—A standard perch of brickwork is to an Irish perch as 11 is to 14; and in this proportion is the *number* of Irish to the number of standard perches in any piece of brickwork.

[Although a *standard*, whether of measurement or of anything else, ought to be fixed and definite, yet builders recognize *two* standard perches of brickwork; namely, the standard perch for 14-inch work, and the standard perch for 9-inch work; the superficial dimensions, that is, the area of

the face of the brickwork, being the same in both, $16\frac{1}{2}$ sq. feet. The standard perch of 9-inch work contains

$$16\tfrac{1}{2} \times \tfrac{9}{12} = 16\tfrac{1}{2} \times \tfrac{3}{4} = 12\tfrac{3}{8} \text{ cubic feet};$$

so that allowing 9 inches for the standard thickness, the number of cubic feet in the wall must be divided by $12\frac{3}{8}$ to get the number of standard perches, of that thickness. It is easy to see that 14 in.: 9 in.:: $19\frac{1}{4}$: $12\frac{3}{8}$; as it ought to be.]

III.—Masons' Work.

The calculations required for the measurement of masonry are so much like those for bricklayer's work, that but little in addition to what has been explained in the preceding article need be said here. As in brickwork, so in masonry, the material and workmanship are estimated either by the superficial foot, the cubic foot or yard, or the standard perch. This last measure however differs from that employed in brickwork: the standard perch for masonry being $16\frac{1}{2}$ ft. $\times$ 1 ft. $\times$ $1\frac{1}{2}$ ft. = $24\frac{3}{4}$ cubic feet; and for the Irish perch, 21 ft. $\times$ 1 ft. $\times$ $1\frac{1}{2}$ ft. = $31\frac{1}{2}$ cubic feet.

About 16 cubic feet of Portland stone weighs one ton; about 17 of Bath stone; $13\frac{1}{2}$ of Granite; and, at a medium, 13 of Marble.

As the cubic contents of every rectangular mass, of whatever material, are computed in one uniform manner, namely by multiplying the three dimensions,—length, breadth and thickness—together, special rules for the purpose in the case of stone would be unnecessary: the method of proceeding will sufficiently appear from the subjoined examples. We may observe here, however, that in computing the superficial workmanship the dimensions are taken by pressing the measuring tape or string in and around every place which the tool touches. It is by superficial measure that chimney-pieces, pavements, and slabs are estimated.

Examples.

1. How many cubic feet are there in a stone block which is 6 ft. 8 in. long, 5 ft. 6 in. broad, and 4 ft. 4 in. thick?

We see that if we were to multiply the first and third of these dimensions, each by 3, we should get for results feet only without odd inches: the second dimension will also be free from odd inches by multiplying it by 2. We shall therefore for simplicity of work use these multiples of the given dimensions, and then compensate for the changes by dividing the final product by $3 \times 3 \times 2 = 18$: the work will then be as here annexed. The result shows that the cubic contents of the block are 159 cubic feet, within $\frac{1}{9}$th of a cubic foot.

$$
\begin{array}{r}
\text{feet.} \\
11 \\
20 \\
\hline
220 \\
13 \\
\hline
2)\,2860 \\
\hline
9)\,1430 \\
\hline
158\frac{8}{9} \text{ cubic ft.} \\
\hline
\end{array}
$$

By using suitable multiples of the given dimensions, computations of this kind may often be abridged in a similar manner; but even here, the greatest possible reduction of work has not been effected: for since the second of the given dimensions becomes freed from odd inches, by whatever *even* number we multiply it, and because 3 times the first dimension *is* an even number of feet, namely, 20, our purpose will be accomplished by multiplying the other two dimensions, each by 3, leaving the second dimension as it is, and then dividing the result by 9. In this way the work stands as in the margin. Expedients for abridging the computation, in any such case, are not to be sought for by any long study or examination of the given dimensions, but only when they suggest themselves intuitively upon a slight inspection of those dimensions.

$$
\begin{array}{r}
\text{ft.} \\
5\frac{1}{2} \\
20 \\
\hline
110 \\
13 \\
\hline
9)\,1430 \\
\hline
158\frac{8}{9} \\
\hline
\end{array}
$$

2. How many standard perches of stone-masonry are there in a wall 50 ft. long, 10 ft. 3 in. high, and 1 ft. 4 in. thick?

Using these dimensions as they are, we shall have to divide the product of them by $24\frac{3}{4}$; but by multiplying the second dimension by 4, and this divisor also by 4, we get rid, at once, of the fraction in this latter, and also of the odd inches in 10 ft. 3 in., that is, the second dimension becomes 41 ft., and the divisor 99, and the work is reduced to that in the margin. It is readily seen that the fraction $\frac{60\frac{1}{3}}{99}$ is $\frac{181}{297} = \frac{2}{3}$ nearly: hence there are $27\frac{2}{3}$ standard perches in the wall, nearly. Instead of multiplying by $1\frac{1}{3}$, as here, we might have used the number 4, which is 3 times as great; and then have divided by 297 instead of by 99.

The computer will find it useful, for the purpose of shortening his figure-work, always to recollect that he is at liberty to take the double, treble, &c., of any dimension, provided he take the half, third, &c., of one of the other dimensions; or, leaving both these unchanged, provided he also take the double, treble,

$$
\begin{array}{r}
\text{feet.} \\
41 \\
50 \\
\hline
2050 \\
1\frac{1}{3} \\
\hline
2050 \\
683\frac{1}{3} \\
\hline
99)\,2733\frac{1}{3}\,(27\frac{181}{297} \\
198 \\
\hline
753 \\
693 \\
\hline
60\frac{1}{3} \\
\hline
\end{array}
$$

&c., of the divisor, he may also take the half, third, &c., of either dimension, provided he take the same half, third, &c., of the divisor. The following is an illustrative example.

3. How many cubic yards of stone-masonry are there in a wall 114 ft. long, 10 ft. high, and 18 in. thick?

Here we foresee that we shall have to divide the product of these dimensions by 27,—the number of cubic feet in a yard: we shall therefore first double the 18 in., regarding it as 3 feet, and then halve the 10 feet, regarding it as 5 ft. We shall then omit the multiplication by the 3 feet altogether, and use for divisor only the third part of 27, namely, 9; as in the annexed calculation, showing the cubic contents of the masonry to be $63\frac{1}{3}$ Yards. We may observe that the above changes in the given dimensions are suggested by simply glancing at them.

	feet.
	114
	5
9)	570
	$63\frac{1}{3}$ Yds.

4. In a chimney-piece the lengths of the mantel and slab are each 4 ft. 6 in., the breadth of both together, 3 ft. 2 in.; the length of each jamb 4 ft. 4 in., and the breadth of both together, 1 ft. 9 in. Required the superficial contents of the whole?

ft.	in.		ft.	in.	
4	6		4	4	
3	2		1	9	
13	6		4	4	
	9		3	3	
14 Ft.	3'	added to	7 Ft.	7'	$= 21\frac{5}{6}$ sq. ft. *Ans.*

IV.—Plasterers,' Painters,' and Paviors' Work.

Plasterers' work is estimated either by the square yard, or by the *square* of 100 square feet. Cornices and mouldings, put on after the common plastering is finished, are generally estimated by the linear foot.

Painters compute their work by the square yard, and every part is measured which the brush passes over, the measuring tape being pressed into all cavities and closely applied to all mouldings.

Paviors' work is usually estimated by the square yard.

EXAMPLES.

1. The compass of a room is 69 ft. 4 in., and the height of it 10 ft. 3 in.: what will be the expense of plastering the walls, at 9*d.* per square yard?

The calculation annexed shows that the number of square feet of plastering is 710 sq. ft. and $\frac{8}{12}$ of a sq. ft; that is, it is 710 $\frac{2}{3}$ sq. ft. To bring this into sq. yds. we should have to divide it by 9; but since, at 9*d.* per sq. yd., we should have afterwards to multiply by 9, we may dispense with both operations, and regard the result just arrived at as 710$\frac{2}{3}$ *pence*, which is 59*s.* 2$\frac{2}{3}$*d.* = £2 19*s.* 2$\frac{2}{3}$*d.*

ft.	in.
69	4
10	3
693	4
17	4
710 Ft.	8'

2. A room whose height, girt over the mouldings, is 16 ft. 6 in., is 97 ft. 9 in. in compass: how many sq. yds. of painters' work will it require?

We shall here take twice the first dimension and 4 times the second, and then divide the resulting sq. ft. by $9 \times 8 = 72$. The number of sq. yds. is thus found to be 179 $\frac{15}{72}$ = 179 $\frac{5}{24}$, or 179 $\frac{1}{5}$ Yards *full.*

feet.
391
33
1173
1173
72) 12903 (179 $\frac{5}{24}$
72
570
504
663
648
15

3. Required the quantity of plastering and ceiling in a room 14 ft. 5 in. by 13 ft. 2 in., the height, up to the under side of the cornice, being 9 ft. 3 in. The cornice girts 8$\frac{1}{2}$ in., and projects 5 in. from the wall, at the upper part, next the ceiling. The deductions for apertures are as follows:—one door, 7 ft. by 4 ft., two windows, each 5 ft. by 3$\frac{1}{2}$ ft., and a fireplace, 5$\frac{1}{2}$ ft. by 4 ft.

1st. *For the walls.*

		ft.	in.		
	Length + breadth	27	7		
	Height	9	3		
		248	3		
Deductions.		6	10	9	
Two windows	$5 \times 7 = 35$				
One door	$7 \times 4 = 28$	255	1	9	
Fireplace	$5\frac{1}{2} \times 4 = 22$			2	
	85	510	3	6	
	—	85	0	0	(*Subtract.*)
		9) 425	3	6	

Sq. yds. of plastering 47 Yds. 2$\frac{1}{4}$ Ft. 6 In.

[The 255 Ft. 1 9 are multiplied by 2, because this is the measure of only *one* side-wall and *one* end-wall.]

2nd. *For the ceiling.*

	ft.	in.	
14 ft. 5 in. minus 10 in. =	13	7	
13 ft. 2 in. minus 10 in. =	12	4	
	163	0	
	4	6	4
	9) 167	6	4
Sq. yds. of ceiling	18 Yds.	5½ Ft.	4 In.

3rd. *For the cornice.*

	ft.	in.			ft.	in.	
	27	7			27	7	
	0	8½		Or thus:	1	5	
	19	5½	11½		27	7	
			2		11	5	11
Sq. ft. of cornice	39	0	11		39	0	11

Hence the superficial measure of the four lengths of cornice is 39 sq. ft. 11 sq. in. In the second method of computing this we have taken twice the 8½ inches, in anticipation of the final multiplication by 2, and have thus rendered the computation a little easier.

Suppose the common plastering of the walls (*rendering*, as it is called) cost 10*d.* per sq. yd.; the ceiling, 1*s.* 8*d.*; and the cornices 1*s.* 4*d.* per sq. ft.; the account for the above would be as follows.

	£	*s.*	*d.*
Rendering 47¼ yds., at 10*d.*	1	19	4½
Ceiling 18 11/18 yds., " 20*d.*	1	11	0¼
Cornice 39 1/12 Ft., " 16*d.*	2	12	1¼
Total cost	£6	2*s.*	6*d.*

The foregoing worked examples sufficiently exhibit the methods of computation, whether the work be plastering, painting, or paving; so that the reader can have no difficulty in calculating the following for himself.

4. What will be the charge for plastering a partition 7 ft. 8 in. by 10 ft. 3 in., at 5*d.* per sq. yd., deducting for a door 6 ft. 3 in. by 2 ft. 10 in.? *Ans.* 2*s.* 9¾*d.*

5. What would be the cost of painting and graining four doors on both sides, of which the dimensions of each are 7 ft. $4\frac{1}{2}$ in. by 3 ft. $8\frac{3}{4}$ in.—the charge for the work being 1*s.* 8*d.* per sq. yd.? *Ans.* £2 0*s.* 9*d.*

6. What will be the expense of paving a rectangular court-yard, which measures 62 ft. 7 in. by 44 ft. 5 in., and through which there is to be laid a footpath along the whole length of it, $5\frac{1}{2}$ ft. wide: the footpath to be paved with flagstones, at 3*s.* per sq. yd., and the rest with pebbles, at 2*s.* 6*d.* per sq. yd.? *Ans.* £39 11*s.* $3\frac{1}{4}$*d.*

As the computation of the number of square feet, square yards, &c., in any surface, must be just the same, whether that surface be of plaster, paint, glass, or paper, &c., it would be superfluous, after what has now been done, to extend these articles to the special consideration of window-glazing or room-papering: it will be sufficient to merely mention the following particulars in reference to these.

Glazing.—The dimensions are taken in feet and inches, and the work is computed in square feet; the dimensions for a window are the entire length and width, the cross-bars, separating the panes, being included in the measurement; if the window be circular, or oval, the dimensions are taken just as if it were square, or rectangular; that is, the greatest length and breadth are regarded as the dimensions, for although the quantity of glass actually used, in glazing such a window, is less than the computation assigns to it, yet the extra waste and trouble justify the extra charge.

Room-papering is generally estimated by the *square* of 100 superficial feet. It is purchased, and laid on, in slips, commonly of 21 inches wide; a complete slip, or *piece*, is 12 yards long. The length of this paper, for 1 square yard. is $5\frac{1}{7}$ ft., because $5\frac{1}{7} \times 21 = 108$; and as the 21 here is the number of *inches* and not of *feet*, we must divide the 108 by 12, to get the number of square feet; the result being 9 sq. ft. = 1 sq. yd. And from thus knowing the length of paper for 9 square feet, we can readily find the length for 100 square feet (a *square*) thus; $9 : 100 : : 5\frac{1}{7}$ ft. : $57\frac{1}{7}$ ft. $= 19\frac{1}{21}$ yds.; so that one dozen (12 yards) and 7 yards of paper, 21 inches wide, will very nearly cover a *square;* the deficiency being only $\frac{1}{7}$ of a foot, or less than an inch and three-quarters length of paper.

In a similar manner it will be found that a dozen of paper, 21 inches wide, will cover exactly a space of 7 sq. yds.

It may be worth while to remind the reader here that, in actually working out the proportions just mentioned, and indeed, in working out any proportion, the first and third terms, as also the first and second, may always be multiplied or divided by any number we please, without the result being affected; and that by taking advantage of this circumstance, any fraction in the stating may always be removed: thus, multiplying the first and third terms, in the stating above, by 7, it is converted into the more convenient form:

$$63 : 100 :: 36 \text{ ft.} : 57\tfrac{1}{7} \text{ ft.} = 19\tfrac{1}{21} \text{ yds.}$$

So likewise in the second proportion alluded to, for finding the space which a single dozen of paper will cover, the stating $5\frac{1}{7}$ ft. : 36 ft. (12 yds.) :: 1 Yd. : 7 Yds., becomes 36 : 36 × 7 :: 1 Yd. : 7 Yds. Or, rather, dividing by the 36, 1 : 7 :: 1 Yd. : 7 Yds.

And these changes, suggested by the above-mentioned general principle, may be at once made, without writing down the original numbers (as here) at all.

We may add, however, that the last result above (7 Yds.) may be readily deduced independently of proportion; for the number of square yards in a piece 12 yards long and 21 inches wide, is 21 times 12, divided first by 12, to bring the inches to feet, and then by 3, to bring these feet to yards; and since the division by 12 neutralizes the multiplication by 12, we have only to divide the 21 by 3, giving for result 7 Yds. And in reference to the first proportion, 63 and 36, by the principle before named, might be replaced by 7 and 4, the work then being simply that here annexed; and by thus rejecting factors, common to a multiplier and divisor, figure-work may always be saved.

$$\begin{array}{r} 7)\,400 \\ \hline 57\frac{1}{7} \\ \hline \end{array}$$

NOTE.—The reader will have perceived that throughout the division of the present work, which we here conclude, we have made no reference to the *Slide-rule*, an instrument which workmen frequently use in order to discover the superficial or cubic contents of wood, brickwork, masonry, &c. The omission has been designedly made for two reasons: the first is, the Slide-rule cannot be satisfactorily explained by written description, nor the method of using it by mere verbal directions: it

must be handled, and its different capabilities *shown*. The second reason is, the results furnished by the Slide-rule are, in most instances, only *approximative*, strict accuracy being in general unattainable by it. We have moreover been especially desirous to familiarize ordinary workmen with the way of computing superficial and solid measurements by duodecimals, or cross-multiplication, a subject which any person of average intelligence may acquire in as short a time as he must expend in learning the use of the *rule*, provided only that he knows common multiplication and the pence table; and the advantage of figures over the instrument is, that the results of the former are *accurate*. But the learner must be cautioned against mis-calling, and, in consequence, mis-apprehending, the different terms, or denominations, in his computed results; a caution the more necessary as so many books on the subjects discussed in the foregoing articles misapply these denominations. Thus, in superficial measure, they call the denomination immediately next to square feet *square inches*, whereas it is *twelfths of a square foot;* that is, each unit in this second term is not 1 square inch, but 12 square inches; and the third term, which really denotes so many square inches, these writers call "Parts," a designation vague and non-significant. Other writers, on the contrary, denominate the *second* term "Parts," and the third (which it is), square inches.

There is the like confusion and inaccuracy in stating the several denominations of cubic measure, and even greater confusion and inaccuracy. But all becomes perfectly intelligible and unambiguous by regarding each unit of any term in the completed result—whether that result express superficial or cubic measure—as a *twelfth* part of a unit of the immediately preceding term; and there is no necessity to give special *names* to any quantities, or measures, intermediate between feet and inches, or of lower values than inches: it is enough to know that one of these is a *twelfth* of one of the units of the term immediately preceding; thus, if it be superficial measure, each unit of the first term is 1 square foot; each unit of the second term is $\frac{1}{12}$th of 1 square foot; each unit of the third term $\frac{1}{12}$th of $\frac{1}{12}$th of 1 square foot; that is, $\frac{1}{144}$th, which is an *inch*. So with cubic measure; each unit of the first term is 1 cubic foot; each unit of the second term is $\frac{1}{12}$th of 1 cubic foot; each unit of the third term is $\frac{1}{12}$th of this twelfth, and each unit of the fourth term is $\frac{1}{12} \times \frac{1}{12} \times \frac{1}{12}$ of 1 cubic foot; that is, $\frac{1}{1728}$ of 1 cubic foot, which is 1 cubic *inch;* and so on.

Calculation of Wages.

Problem 1.

Knowing the daily wages, or pay, to find the yearly salary.

Rule I.—Regard the pence as so many pounds; add to this sum the half of it and also 5 days' wages.

EXAMPLES.

1. What will $15\frac{3}{4}d.$ per day amount to in a year?

	£	s.	d.
$£15\frac{3}{4}$ =	15	15	0
The half =	7	17	6
Pay for 5 days =	0	6	$6\frac{3}{4}$
	£23	19s.	$0\frac{3}{4}d.$ *Ans.*

By taking the pence for pounds, we virtually multiply the daily pay by 240; that is, we get the pay for 240 days; the half of this is the pay for 120 days; and therefore the two added together, with 5 days' pay besides, gives the pay for 365 days.

RULE II.—Multiply £1 10*s*. 5*d*. by the number of pence per day: the product will be the amount in 1 year; because £1 10*s*. 5*d*. is equal to 365 pence. [This is a very convenient Rule when the daily pay is free from odd farthings.]

2. What will 5*d*. per day amount to in a year?

By Rule I.

£	s.	d.
5	0	0
2	10	0
	2	1

Ans. £7 12*s*. 1*d*.

By Rule II.

£	s.	d.
1	10	5
		5

£7 12*s*. 1*d*.

3. What will $18\frac{1}{4}d.$ per day amount to in a year? *Ans.* £27 15*s*. $1\frac{1}{4}d.$
4. What will 16*d*. per day amount to in a year? *Ans.* £24 6*s*. 8*d*.
5. What will $3\frac{1}{2}d.$ per day amount to in a year? *Ans.* £5 6*s*. $5\frac{1}{2}d.$

NOTE.—When the pay is only for the ordinary working days, that is, when it is stopped for the 52 Sundays, for Good Friday, and for Christmas Day, only 311 days are paid for in a year. The amount, at 1*d*. a day, is therefore £1 5*s*. 11*d*.; and, at a farthing a day, it is 6*s*. $5\frac{3}{4}d.$ Consequently, the Rule will then be this: Multiply £1 5*s*. 11*d*. by the number of *pence* per day, and 6*s*. $5\frac{3}{4}d.$ by the number of additional farthings, and add the results; thus:—

6. What will be the amount of $3\frac{3}{4}d.$ per day for 311 days?

	£	s.	d.	
	1	5	11	
			4	
	5	3	8	
Subtract		6	$5\frac{3}{4}$	for 311 days at $\frac{1}{4}d.$
	£4	17*s*.	$2\frac{1}{4}d.$ *Ans.*	

Or thus:

£	s.	d.	
1	5	11	
		3	
3	17	9	
Add	19	$5\frac{1}{4}$	= 3 times 6s. $5\frac{3}{4}d.$
£4	17s.	$2\frac{1}{4}d.$ *Ans.*	

7. At 14*d.* per day, what will be the amount in 311 days? *Ans.* £18 2*s.* 10*d.*
8. If a workman's wages be 5*s.* 10*d.* per day, what is his yearly income? *Ans.* £90 14*s.* 2*d.*

Problem 2. (Converse of Prob. 1.)

Knowing the yearly salary, to find the pay per day.

Rule.—Regard the pounds in the salary as so many pence, and consider the shillings, when 10*s.* or more, as $\frac{1}{2}d.$, but when less than 10*s.*, reject them. Multiply these pence by 2, and divide the product by 3, observing to allow $\frac{1}{2}d.$ should there be 1 for remainder, and $\frac{3}{4}d.$ if the remainder be 2; the result will be the daily income *nearly*, if not to the nearest farthing.

Note.—The year is here the whole 365 days.

Examples.

1. If a servant's wages be £24 a year, what is his daily pay?
   $24d. \times 2 \div 3 = 8d. \times 2 = 16d.$, to the nearest farthing.
2. If the yearly wages be 26 guineas, what is the pay per day?
   $27d. \times 2 \div 3 = 9d. \times 2 = 18d.$, to the nearest farthing.
3. If the yearly wages be £23 15*s.*, what is the pay per day?
   $23\frac{1}{2}d. \times 2 \div 3 = 47d. \div 3 = 15\frac{3}{4}d.$, to the nearest farthing.
4. If the yearly income be £150, what is the amount per day?
   $150d. \times 2 \div 3 = 100d. = 8s.\ 4d.$ *nearly.*

The foregoing Rule is suggested by the following considerations. If the year consisted of only 360 days instead of 365, then, by dividing the salary by 360, we should get the daily pay. Now if the salary expressed in pounds be regarded as so many pence, it becomes virtually divided by $20 \times 12 = 240$; so that the salary is 240 times this number of *pence;* but multiplying anything by 240, and then dividing by 360, is the same as multiplying by 2, and then dividing by 3; and hence, by proceeding in this way, the 360th part of the salary will be accurately determined. And since the difference between the 360th part and the 365th part of a

comparatively small sum is but trifling—being only the $\frac{1}{360 \times 73}$ part of the whole, which is but a farthing in £28—it follows that, for an income not exceeding £27, the error cannot be so great as a farthing; and it is always in *excess.* If a few shillings—say 5*s.*—be subducted from the yearly income, this excess will be pretty nearly counterbalanced; for then there will be subducted from the daily pay $\frac{5}{365}$*s.*, about $\frac{2}{3}$ of a farthing. For an income much above the limit (£28) this deduction will be insufficient; and to avoid division into cases, and likewise to preclude a fractional remainder, from the divisor 3, the Rule recommends, generally, that if the shillings connected with the pounds, in the income, be fewer than 10*s.*, they should be rejected, as also the overplus above 10*s.* As the Rule is professedly only (in most cases) a close *approximation* to the strict truth, and as, from its simplicity, it is so easily worked *mentally,* it is retained in this edition. We merely caution the reader that it is not to be relied upon, within a halfpenny or penny of the truth, for incomes yielding several shillings per day. Thus, take the yearly income at £150; the Table below shows that, to the nearest farthing, the daily pay is 8*s.* 2¾*d.*; we have seen above (Ex. 4) that the Rule makes it 8*s.* 4*d.*, which is 1¼*d.* too much. On the whole, when the daily payment is to be found with greater exactness, we recommend recourse to the subjoined Table.

WAGES.

Table of salaries, etc., from £1 *to* £150 *per annum, reduced to so much per month, per week, per day.**

Y.	Pr. M.	Pr. W.	Pr. D.	Y.	Pr. M.	Pr. W.	Pr. D.	Y.	Pr. M.	Pr. W.	Pr. D.
£	*s. d.*	*s. d.*	*s. d.*	£	£ *s. d.*	*s. d.*	*s. d.*	£	£ *s. d.*	£ *s. d.*	*s. d.*
1	1 8	0 4¾	0 0¾	11	0 18 4	4 2¾	0 7¼	30	2 10 0	0 11 6½	1 7¾
2	3 4	0 9¼	0 1¼	12	1 0 0	4 7½	0 8	40	3 6 8	0 15 4½	2 2¼
3	5 0	1 1¾	0 2	13	1 1 8	5 0	0 8½	50	4 3 4	0 19 2¾	2 9
4	6 8	1 6½	0 2¾	14	1 3 4	5 4½	0 9¼	60	5 0 0	1 3 0¾	3 3½
5	8 4	1 11	0 3¼	15	1 5 0	5 9¼	0 10	70	5 16 8	1 6 11	3 10
6	10 0	2 3¾	0 4	16	1 6 8	6 1¾	0 10½	80	6 13 4	1 10 9¼	4 4½
7	11 8	2 8¼	0 4½	17	1 8 4	6 6½	0 11¼	90	7 10 0	1 14 7½	4 11¼
8	13 4	3 0¾	0 5¼	18	1 10 0	6 11	0 11¾	100	8 6 8	1 18 5½	5 5¾
9	15 0	3 5½	0 6	19	1 11 8	7 3½	1 0½	125	10 8 4	2 8 0	6 10½
10	16 8	3 10	0 6½	20	1 13 4	7 8¼	1 1¼	150	12 10 0	2 17 8¼	8 2¾

NOTE.—One farthing per day is 7*s.* 7¼*d.* per year.

* The above table is calculated to the nearest amount that either employer or employed can insist upon.

We shall now show the use of this Table by applying it to the Examples worked by the Rule (p. 178).

	Ex. 1. per day.		Ex. 2.		Ex. 3
For £20	1s. $1\frac{1}{4}d.$	For £20	1s. $1\frac{1}{4}d.$	For £20	1s. $1\frac{1}{4}d.$
„ £ 4	$2\frac{3}{4}$	„ £ 7	$4\frac{1}{2}$	„ £ 3	2
		6s. (Note)	$\frac{1}{4}$	15s. (Note)	$\frac{1}{2}$
„ £24	1s. 4d.		1s. 6d.		1s. $3\frac{3}{4}d.$

We thus see that for such small incomes as these, the Rule is sufficiently accurate; but, as already shown, for an income of £150 it errs in excess by $1\frac{1}{4}d.$

As another Example, let us take an income of £40.

By the Rule.	By the Table.
3) 80d.	For £40 2s. $2\frac{1}{4}d.$
$26\frac{3}{4}d.$ = 2s. $2\frac{3}{4}d.$	The difference here is $\frac{1}{2}d.$

If the income had been £100, the difference would have been one penny.

In concluding these remarks, it may be as well to observe that the reason why the Rule directs that when the remainder, from the division by 3, is 1, a farthing should be allowed for it, but that when it is 2, three farthings should be allowed, is because $\frac{1}{3}d. = \frac{4}{3}f. = 1f.$ *and* $\frac{1}{3}f.$; and $\frac{2}{3}d. = \frac{8}{3}f. = 3f.$ all *but* $\frac{1}{3}f.$

Problem 3.

The number of shillings in a week's earnings being given, to find the earnings per year.

Rule—Add together $2\frac{1}{2}$ times as many pounds as there are shillings, and twice the shillings themselves: the result will be the earnings for the year.

For 20 times the number of shillings make so many pounds, and these pounds are the earnings of 20 weeks; so that $2\frac{1}{2}$ times this sum must be the earnings of 50 weeks; and twice one week's earnings being added, the result must be the earnings in a year.

Examples.

1. A year's earnings, at 11s. a week, are £11 $\times\ 2\frac{1}{2}$ + 22s. = £28 12s.
2. A year's earnings, at 16s. a week, are £16 $\times\ 2\frac{1}{2}$ + 32s. = £41 12s.

3. A year's earnings, at 18*s.* 6*d.* a week, are £18½ × 2½ + 37*s.* = £37 + £9 5*s.* + £1 17*s.* = £48 2*s.* Or thus: Since 52*d.* = 4*s.* 4*d.*, we have only to add six times this, namely, £1 6*s.*, to the year's income at 18*s.* per week: so that the work may stand as follows, namely, £18 × 2½ + £1 16*s.* + £1 6*s.* = £45 + £3 2*s.* = £48 2*s.* And in this manner we may always proceed when there are odd pence in the week's wages.
4. How much is earned in a year, at the rate of 16*s.* 10*d.* per week? *Ans.* £43 15*s.* 4*d.*
5. If a family spend, on the average, 7*s.* 3½*d.* per week for bread, what is the expenditure per year? *Ans.* £18 19*s.* 2*d.*
6. What is the yearly income of a person who earns £2 17*s.* 8¼*d.* per week? *Ans.* £149 19*s.* 9*d.*

[See the Table, in which the weekly pay, for £150 a year, is stated to be the above sum, and allowably so; because the overplus 3*d.*, divided by 52, gives a fraction too small for representation as money.]

CALCULATION OF INTEREST.

PROBLEM 1.

To find the interest of any sum of money at five per cent. per annum.

RULE.—Regard the pounds as so many shillings, and allow at the rate of *threepence* for every additional 5*s.*; and, consequently, one farthing for every 5*d.* The interest, at 5 per cent., will then be expressed.

EXAMPLES.

1. The interest, for one year, on £75, at 5 per cent., is 75*s.* = £3 15*s.*
2. The interest, for one year, on £110, at 5 per cent., is 110*s.* = £5 10*s.*
3. The interest, for one year, on £69 15*s.*, at 5 per cent., is 69*s.* 9*d.* = £3 9*s.* 9*d.*
4. What is the interest on £587 16*s.* 4*d.*, for 7 years, at 5 per cent.?

[The interest of a sum for 7 years is, of course, the interest of 7 times that sum for one year.]

	£	s.	d.				d.	
	587	16	4					
			7	Interest for		15*s.* =	9	
2,0)	411,4	14	4	„	„	10*d.* =	½	Subtract.
	£205	14*s.*	8½*d.*	„	for 14*s.*	2*d.* =	8½*d.*	

Here £4114 are regarded as so many shillings, and are therefore divided by 20, to bring them into pounds. The interest for 14*s.* 2*d.* is then found separately, according to the Rule: the interest for the additional 2*d.*, being less than half a farthing, is disregarded. Of course the work for the 14*s.* 2*d.* may be readily executed mentally, and the resulting 8½*d.* annexed at once to the £205 14*s.*

5. What is the interest on £26 5*s.*, for 1 year, at 5 per cent.?
   Ans. £1 6*s.* 3*d.*
6. What is the interest on £47 10*s.*, for 1 year, at 5 per cent.?
   Ans. £2 7*s.* 6*d.*
7. What is the interest on £9826 13*s.* 8*d.*, for 1 year, at 5 per cent.?
   Ans. £491 6*s.* 8*d.*

Problem 2.

To find the interest at any rate per cent. per annum.

Rule I.—Find the interest at 5 per cent. by the last problem: one-fifth of this will be the interest at 1 per cent.; and this, multiplied by the given rate, will give the interest at that rate. But for certain rates the calculation may be abbreviated by taking *parts*, as in the third of the following Examples.

Examples.

1. What is the interest on £587 16*s.* 4*d.*, for 7 years, at 3 per cent., at 4 per cent., and at 6 per cent.?

	£	s.	d.	
	587	16	4	
			7 years	
2,0)	411,4	14	4	
5)	205	14	8½	, 5 per cent. (*See last page.*)
3 ×	41	2	11¼	, 1 per cent.
=	123	8	9¾	, 3 per cent.

£	s.	d.	
205	14	8½	, 5 per cent.
41	2	11¼	, 1 ”
164	11	9¼	, 4 ”
246	17	7¾	, 6 ”

In the calculation for 4 and 6 per cent., the interest at 1 per cent. is *subtracted* for the former rate, and *added* for the latter rate.

Note.—It is generally preferable to multiply by the number of years *first*, as above, since then there will never be any fraction of a penny to multiply.

2. What is the interest on £212 10*s.* 4*d.*, for $2\frac{3}{4}$ years, at $2\frac{1}{2}$ per cent.?

	£	*s.*	*d.*	
4)	212	10	4	
			3	years. [$2\frac{3}{4} = 3 - \frac{1}{4}$.]
	637	11	0	
Subtract	53	2	7	
2,0)	58,4	8	5	
2)	29	4	5	= Interest at 5 per cent.
Ans.	£14	12*s.*	$2\frac{1}{2}$*d.*	= " $2\frac{1}{2}$ "

3. What is the interest on £500, for 4 years, at £5 7*s.* 6*d.* per cent.?

Instead of multiplying the 500 by 4, and then dividing by 20, we shall divide simply by 5, rejecting the factor 4 from both multiplier and divisor.

	£		
	100	= Interest for 4 years, at	5 per cent.
20th part of £100	= 5	"	5*s.* "
$\frac{1}{2}$ of the above	= 2 10*s.*	"	2*s.* 6*d.* "
Ans.	£107 10*s.*	"	£5 7*s.* 6*d.* "

4. What is the interest on £896, for $2\frac{1}{2}$ years, at $3\frac{1}{4}$ per cent.?

	£	
	896	
	$2\frac{1}{2}$	years.
	1792	
	448	
2,0)	224,0	
5)	112 £	= Int. at 5 per cent.
	£22 8*s.*	" 1 "
	$3\frac{1}{4}$	the rate
	67 4	
For the $\frac{1}{4}$	5 12	
Ans.	£72 16*s.*	Int. at $3\frac{1}{4}$ "

Or thus:

	£
	896
	$2\frac{1}{2}$
	1792
	448
4)	2240
	$3\frac{1}{4}$
	6720
	560
	£72,80
	20
	16,00*s.*

The first of these methods admits of being considerably shortened, in consequence of the number of years being $2\frac{1}{2}$. If we take *double* this number, there will be first a multiplication by 5, and afterwards a division by 5,—two neutralizing operations. We may therefore suppress *both;* so that we shall get the interest, at 1 per cent., by simply

dividing £896 by the double of 20, seeing that $2\frac{1}{2}$ has been doubled; that is, the work for the 1 per cent will be merely this:

$$£896 \div 40 = £22\tfrac{2}{5} = £22\ 8s.$$

In the second method, instead of dividing by 20 and 5, and then multiplying by $3\frac{1}{4}$, we have first multiplied by $3\frac{1}{4}$, and have then divided by 100, which is the same as dividing by 20 and 5 in succession. This second method is in accordance with the following general Rule.

Rule II.—Multiply the principal by the number of years, the product by the rate, and divide the result by 100. Or multiply the principal by the *product* of the rate and number of years, and divide the result by 100.

Note.—1. If the product of the rate and number of years should be 10 (as in Ex. 2 following), then instead of multiplying by this 10, and afterwards dividing by 100, we should omit the multiplication altogether, and merely divide the principal by 10.

2. And if the product of the rate and number of years should be 20 (as in Ex. 4), then the multiplication by the 20 should, in like manner, be omitted, and the principal be divided by 5 only. In the former case, the factor 10 is suppressed in both multiplier and divisor, and in the latter case, the factor 20.

It should ever be present to the mind of the computer that, in any calculation whatever, factors common to a multiplier and a divisor may always be discarded, however far apart these two operations of multiplication and division may be, provided that no operations, *except* those of multiplication and division, intervene.

Examples.

1. What is the interest of £587 16*s*. 4*d*., at 6 per cent., for 7 years? (Ex. 1, p. 182.)

£	*s*.	*d*.	
587	16	4	
		7	years.
4114	14	4	
		6	, the rate.
246,88	6	0	
	20		
17,66			
12			
7,92			*Ans*. £246 17*s*. $7\frac{3}{4}$*d*.; or more nearly, £246 17*s*. 8*d*.
4			
3,68			

Whether this Rule or the former one be the more eligible, in any particular case, must be left to the computer to determine. It may be well for the reader to work the following examples by both Rules keeping in remembrance however the foregoing Note.

2. What is the interest on £765 12s. 7d., for 4 years, at $2\frac{1}{2}$ per cent.? *Ans.* £76 11s. 3d.
3. What is the interest on £78 12 10d., for $12\frac{1}{2}$ years, at 1 per cent.? *Ans.* £9 16s. $7\frac{1}{4}$d.
4. What is the interest on £325 7s. 6d., for $3\frac{1}{3}$ years, at 6 per cent.? *Ans.* £65 1s. 6d.
5. What is the interest on £193 12s., for 1 year, at £11 18s. 6d. per cent.? *Ans.* £22 2s. $4\frac{1}{2}$d.

The following short Table may be found useful on many occasions.

Table of the Interest on £1 *for* 1 *year, at from* 1 *to* 10 *per cent.*

Per cent.	Interest. s.	d.	Per cent.	Interest. s.	d.
1 . . .	0	$2\frac{2}{5}$	6 . . .	1	$2\frac{2}{5}$
2 . . .	0	$4\frac{4}{5}$	7 . . .	1	$4\frac{4}{5}$
3 . . .	0	$7\frac{1}{5}$	8 . . .	1	$7\frac{1}{5}$
4 . . .	0	$9\frac{3}{5}$	9 . . .	1	$9\frac{3}{5}$
5 . . .	1	0	10 . . .	2	0

If it be only borne in mind that the interest of £1 for 1 year, at 1 per cent., is $2\frac{2}{5}d.$; or without any effort of memory, if we reduce mentally $\frac{240}{100}d.$, that is, $\frac{12}{5}d.$, to $2\frac{2}{5}d.$, the interest for 1 year, at any other rate per cent., will be obtained by simply multiplying $2\frac{2}{5}d.$ by that rate: thus, $2\frac{2}{5}d. \times 3 = 7\frac{1}{5}d.$, the interest of £1, for 1 year, at 3 per cent.; also $2\frac{2}{5}d. \times 7 = 1s.\ 4\frac{4}{5}d.$, the interest of £1, for 1 year, at 7 per cent.; and so on.

By way of application, let us take Ex. 4, p. 183, the working of which will be as follows, the interest being calculated first for 3 per cent., and then that for $\frac{1}{5}$ per cent. added.

```
                                   896  No. of £'s.
                                    2½   „  years.
                                  ----
                                  1792
                                   448
                                  ----
                                  2240
     (Number from the Table.)       7⅕d.
                                 -----
                                 15680
                                   448
                                 -----
                             12) 16128d.
                                 -----
                            2,0) 134,4s.
Interest for 2½ years, at 3 per cent. £67  4s.
```

But we may work in a preferable manner, thus:—$2\frac{1}{2} \times 3 = 7\frac{1}{2}$; therefore the interest is the same as that for 1 year at $7\frac{1}{2}$ per cent.

The interest of £896, at	5	per cent.	= £44	16s.
"	$2\frac{1}{2}$	"	= 22	8
"	$7\frac{1}{2}$	"	= £67	4s.

And we thus see, in particular cases, that a general Rule may sometimes be superseded by a shorter method.

If one twelfth of this be added to it, the result will be £67 4s. + £5 12s. = £72 16s., the interest at $3\frac{1}{4}$ per cent., for $2\frac{1}{2}$ years.

Problem 3.

To find the interest on any number of pounds, at any rate per cent., for any number of months.

General Rule.—Regard the pounds as so many pence; multiply this number, the number of months, and the double of the rate together; a tenth part of the product will be the interest in *pence*.

[We recommend taking *twice* the rate, and dividing by 10, rather than taking the rate itself, and dividing by 5, for two reasons: first, by doubling the rate, we get rid of the fraction $\frac{1}{2}$, should it enter the rate: and, secondly, twice the rate can be written down quite as readily as the rate itself; and, easy as division by 5 may be, division by 10 is still easier; in fact division by 10 involves no actual figure-work at all. Of course, if seen to be the more convenient, we may divide either of the three factors to be multiplied together by 10 at the outset, instead of delaying the division till their product is obtained; and it always *will* be the more convenient to do this when either of the factors terminates with a cipher, as in Ex. 2, next page.]

Examples.

1. What is the interest on £36, for 3 months, at $2\frac{1}{2}$ per cent.?
   36*d.* × 3 × 5 = 540 : hence the interest is 54*d.* = 4*s.* 6*d.* *Ans.*
   Or thus : 3*s.* × 3 × 5 = 45*s.*; and 45*s.* ÷ 10 = 4*s.* 6*d.* *Ans.*

The reason of the foregoing Rule will appear from the following considerations.

$\frac{\text{Principal} \times \text{Rate}}{100}$ is the interest for 12 months; therefore

$\frac{1}{12}$ of this is the interest for 1 month: but $\frac{1}{12}$ is the same as $\frac{20}{240}$; so that instead of dividing the interest for 12 months by 12, we may divide it by 240, and then multiply by 20. Now by writing the *pounds* as so many *pence*, we *do*, virtually, divide by 240; and the multiplying by 20, and dividing by 100 is, in effect, the same as expunging the factor 20 in the divisor, 100, thus leaving for divisor only 5, and not multiplying at all; and this divisor, 5, is converted into 10 by multiplying it and the numerator by 2; that is, by taking twice the *Rate*. In this way the interest for 1 month (meaning by a month the 12th part of a year) is the number of pounds in the principal (taken as so many *pence*) multiplied by twice the rate, and the product divided by 10; and consequently, for any number of months, we have only to introduce that number as an additional multiplier. And this is the Rule.

2. What is the interest on £220, for 11 months, at 3 per cent.?
   Dividing by 10 *first*, 1*s*. 10*d*. $\times$ 6 = 11*s*.; and 11 times this is 121*s*. = £6 1*s*. *Ans*.
3. What is the interest on £245 13*s*. 4*d*., for 1 month, at 3 per cent.?
   13*s*. 4*d*. = £$\frac{2}{3}$; and 245$\frac{2}{3}$*d*. $\times$ 6 is 245 sixpences + 4*d*.; that is, it is 122*s*. 10*d*., the tenth part of which is 12*s*. 3$\frac{2}{5}$*d*., the interest required.
4. What is the interest on £144 15*s*., for 9 months, at 5 per cent.?
   15*s*. = £$\frac{3}{4}$; and 144$\frac{3}{4}$*d*. $\times$ 9 $\times$ 10 $\div$ 10 = 12*s*. 0$\frac{3}{4}$*d*. $\times$ 9 = £5 8*s*. 6$\frac{3}{4}$*d*. *Ans*.

[The 12*s*. 0$\frac{3}{4}$*d*. might be written at once from mere inspection of the principal.]

NOTE.—We see by this Example, that when the rate is 5 per cent., the multiplication by double the rate, and the division by 10, being operations which neutralize each other, may both be omitted: it is sufficient to multiply the number of pounds, taken as so many pence, by the number of months. It is proper to observe that when the shillings and pence, connected with the pounds in the principal, do not make a convenient fraction of £1, they may be subdivided into convenient fractions (none of them being more complicated than £$\frac{1}{16}$), without making any error greater than £$\frac{1}{32}$ in the principal; and this extreme error, even when the rate is so great as 10 per cent., and the months so many as 11, can cause an error in the interest equal only to

$$\tfrac{1}{32}d. \times 11 \times 20 \div 10 = \tfrac{1}{32}d. \times 11 \times 2 = \tfrac{22}{32}d., \text{ less than } \tfrac{3}{4}d.$$

For instance: suppose the principal in the last Example above had

been £144 16*s*. 7*d*. Then we might have subdivided the 16*s*. 7*d*. into 15*s*. + 1*s*. 3*d*. + 4*d*. = £$\frac{3}{4}$ + £$\frac{1}{16}$ + 4*d*., and have disregarded the 4*d*., as having no sensible influence on the resulting interest: the interest, in the Example above, with the foregoing addition of 1*s*. 7*d*. to the principal, would have to be increased by $\frac{1}{16}$*d*. $\times$ 9 = $\frac{9}{16}$*d*., or one halfpenny. But regarding the addition to be 1*s*. 8*d*., the increase of interest would be $\frac{1}{12}$*d*. $\times$ 9 = $\frac{3}{4}$*d*.

The general Rule by which the foregoing Examples have been worked, though sufficiently easy, becomes still further simplified for the rates 3 per cent., 5 per cent., and 6 per cent. In these cases it may be expressed in the three forms following.

1. *For* 3 *per cent.*—Regard the pounds in the principal as so many *shillings;* multiply these by the number of months, and divide by 20.

It is easy to see how this follows from the General Rule at p. 186. The double rate *here* is 6; and since the divisor is always 10, if we double the multiplier (6) and double also this 10, the *pence* in the Rule become virtually so many *shillings;* and instead of 10, the divisor becomes 20.

2. *For* 5 *per cent.*—Regard the pounds as so many *pence*, and multiply these by the number of months. (See Note, p. 187.)

3. *For* 6 *per cent.*—Regard the pounds as so many *shillings;* multiply these by the number of months, and divide the product by 10. [This is an obvious inference from the above Rule for 3 per cent.]

5. What is the interest on £87, for 5 months, at 3$\frac{3}{4}$ per cent.?
Ans. £1 7*s*. 2$\frac{1}{2}$*d*.
6. What is the interest on £110, for 9 months, at 5 per cent.?
Ans. £4 2*s*. 6*d*.
7. What is the interest on £90, for 8 months, at 6 per cent.?
Ans. £3 12*s*.
What is the interest on £619 9*s*. 6*d*., for 7 months, at 5$\frac{1}{2}$ per cent.?
Ans. £19 17*s*. 6*d*.

Problem 4.

To find the interest on any principal, at any rate per cent., for any number of days.

The ordinary and obvious Rule for this is—As 365 is to the proposed number of days, so is the interest for 1 year to the interest required. But, since twice 365 is 730, and since,

moreover, in finding the interest in the usual way, for 1 year, we have to divide by 100, after multiplying the principal by the rate, we may obviously proceed as follows.

RULE I.—Multiply the product of the principal and twice the rate by the number of days, and divide the result by 73000.

Now in dividing by so large a number as this, it is plain that a few shillings, more or less, in the dividend, cannot cause any appreciable difference in the result: the 73000th part of so much as 10*s.*, is only 7300th part of 1*s.*, which is less than the 608th part of a penny. We may therefore safely take the sum, to be divided, to the nearest *pound* only; increasing the number of pounds by a unit, when 10*s.* and upwards are connected with the pounds, and rejecting the overplus shillings altogether when they amount to less than 10*s.* By so doing, the error in the quotient can never be so great as the 608th part of a penny, as we have just seen.

EXAMPLES.

1. What is the interest on £325 7*s.*, for 89 days, at $4\frac{1}{2}$ per cent.?

£325 7*s.* × 9 = £2928 3*s.*; and this multiplied by 89 gives for product £260605 7*s.*; and rejecting the 7*s.*, as of no moment, the remainder of the work is that here annexed.

It thus appears that the interest required is £3 11*s.* $4\frac{3}{4}$*d.*

```
           £
73,000) 260,605 (3£
        219
        -------
         41605
            20
        -------
        832,100 (11s.
        803
        -------
         291
          12
        -------
        349,2 (4d.
        292
        -------
         572
           4
        -------
        228,8 (3f.
        219
        -------
           9
```

We have spoken above of the insignificant influence, upon the result, of the odd shillings connected with the pounds to be divided by the number 73000: it may be interesting and instructive to the reader to test for himself the trifling effect, upon the foregoing result, which would ensue from increasing the dividend here employed by so much as £120, thus converting it into £260725: he will find that the answer will differ from that arrived at above by less than one halfpenny.

But there is a more expeditious way of arriving at the quotient, in a case of this kind, than by actually dividing by 73000: it is as follows.

RULE II.—Conceive the dividend, that is, the product of the principal, the double rate, and the number of days, to be

divided by 100000; that is, cut off the last *five* figures of it, and then divide by 3: divide the quotient by 10, or, which is the same thing, push the preceding figures each one place further to the *right;* then divide again by 10, in a similar way: the results being in column, add all up; the sum will be the quotient expressed in pounds.

The work of the foregoing Example, by this Rule, is here annexed: the answer brought out is £3 11*s.* 4¾*d.*, as before; the neglect of the decimals, after the fifth place, not affecting even the farthings.

	£
3)	2·60605
	·86868
	8686
	868
	£3·57027
	20
	11·4054*s.**
	12
	4·8648*d.*
	4
	3·4592*f.*

It may be here noticed, however, that in calculations of Interest, the amount to the nearest penny is, in general, all that is demanded: in the present case the interest charged would be £3 11*s.* 5*d.*

We shall give another worked Example in illustration of the foregoing Rule.

2. What is the interest on £956 14*s.* 6*d.*, for 7 days, at 4½ per cent?

£	*s.*	*d.*	
956	14	6	
		9	double the rate.
8610	10	6	
		7	No. of days.
£60273	13*s.*	6*d.*	

3)	·60274
	·20091
	2009
	200
	£·82574
	20
	16·5148*s.*
	12
	6·1776*d.*

The answer is 16*s.* 6*d.* And similarly in all other cases in which the interest, for the specified number of days, does not exceed £10; or in which accuracy in the fifth place of figures, in the sum of the four numbers added together, is of no moment. We shall see hereafter, p. 192, why accuracy to the nearest farthing cannot be counted upon if the interest exceed £10. As far as this limit, as to the interest, Rule II. may always be safely depended upon; and therefore it cannot but be acceptable to persons engaged in computations of this kind in Savings Banks.

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* Final *ciphers*, in decimals, may always be suppressed as useless. Those readers of this book who may be but imperfectly acquainted with Decimals, are expected to consult the article on that subject in the APPENDIX, whenever the decimal calculations in the body of the work prove perplexing.

Note.—When the interest is 5 per cent., then since, in this case, the double of the rate is 10, we need multiply only by the number of days, and point off but *four* places of figures from the result, instead of five; and whenever the dividend happens to be so small as not to have so many figures as it is necessary to point off, we must prefix to it as many ciphers as will suffice to make up the required number of places.

3. What is the interest on £375, for 12 days, at $3\frac{1}{2}$ per cent.?
Ans. 8*s.* 6*d.*
4. What is the interest on £370, for 40 days, at 5 per cent.?
Ans. £2 0*s.* 6*d.*
5. What is the interest on £3204 14*s.*, for 37 days, at 5 per cent.?
Ans. £16 4*s.* 10*d.*
6. What is the interest on £950, for 80 days, at 7 per cent.?
Ans. £14 5*s.* 11*d.*

[In the foregoing Examples the interest is determined to nearest penny. It may be satisfactory to the reader to work Examples 5 and 6 by both Rules.]

It remains for us now to explain the principle upon which the foregoing easy and expeditious method of computing the interest for a specified number of days is founded.

In every operation of Division, we know that when the *complete* quotient is obtained, the product of this quotient and the divisor will give the dividend; that is to say, that there will always be the following equality, viz., Quotient × Divisor = Dividend. And further, that by whatever number we divide both divisor and dividend, before actually employing them as such, the *quotient* will remain just the same.

Suppose, in the case before us, that we in this way reduce the divisor 73000, more and more, by successively dividing it by 100000, 3, 10, and 10, as in the annexed operation; the division by the number 100000 reducing the whole number 73000 to the decimal ·73000, the superfluous ciphers here being retained merely for symmetry sake. Then provided we reduce the dividend in exactly the same way, we know, from the general principle stated above, that by employing each reduced divisor, in conjunction with the corresponding reduced dividend, the equality referred to always has place—the *quotient* continuing unaltered; so that this unaltered quotient, multiplied by any one of the varying divisors, will always produce the dividend in connection with that divisor. Consequently the fixed and constant quotient, multiplied by the *sum* of all the

$$\begin{array}{r} 3)\ \cdot 73000 \\ \cdot 24333\frac{1}{3} \\ \cdot 02433\frac{1}{3} \\ \cdot 00243\frac{1}{3} \\ \hline 1\cdot 0001 \\ \hline \end{array}$$

Note. $\frac{1}{3}$ is = ·333 . . . continued interminably.

varying divisors, must give a product equal to the *sum* of all the varying dividends; in other words, there will be the equality following, which, for the purpose of future reference, we shall mark [A].

Quotient $\times$ 1·0001 = Sum of all the several Dividends . . . . [A].

Now the multiplier 1·0001 differs from 1 by an amount so small that, in calculations such as those in which we are now engaged, the difference is inappreciable, and the number may with safety be replaced by 1 itself; in which case, the left-hand member of the foregoing equality becomes simply—Quotient = Sum of all the Dividends; and hence the quotient arising from the division by 73000, in accordance with Rule I., may be more expeditiously found by summing up all the reduced dividends arrived at conformably to the directions in Rule II.; in all those cases, that is, in which the replacing the multiplier 1·0001, as used above, by 1, can lead to no error of consequence; in other words, whenever the Interest (the *Quotient*) is not so large a sum as for the ten-thousandth part of it to be appreciable. [The 10000th part of the Quotient (or *Interest*) is Quotient $\times$ ·0001,—the amount which is rejected by the Rule.]

Now we know that £1 is equal to 960 farthings; consequently, £10 is equal to 9600 farthings; and this number, being less than 10000, the 10000th part of it is less than a farthing; it is, in fact, the decimal ·96*f*. We may infer therefore, from what is said above, that whenever the required interest is foreseen to be a sum not exceeding £10—and whether 10 times the divisor 73000 exceeds the dividend or not, may be ascertained at a glance,—Rule II. may be depended upon for the accurate determination of that interest, within a farthing; the error being a fraction of a farthing in *excess*.

NOTE.—The 10000th part of the "Sum of all the Dividends" is not, in strictness, the same as the 10000th part of the Interest, or Quotient:—it is the 10000th part of the Quotient, and the 10000th part of *that* part besides; as is evident from the equality marked [A] above.

This addition, however, is so utterly insignificant, that Rule II. very properly ignores its existence.

We may here remark, however, that in working Examples by this Rule, the decimals are not extended beyond *five* places, and therefore that appreciable error may be suspected to arise from *this* cause: let us see whether or not such can be the case. In the first division of the proposed dividend, namely, the division by 3, the greatest remainder that can arise is obviously 2, for which remainder the continuation of

the decimals would be the figures 666... Now even if the figures following the fifth place, in the two subsequent divisions (by 10), could be a row of 9's, the sum of the column of decimals, immediately beyond the fifth place, would amount only to the number 26; so that the error arising from rejecting these decimals would be more than compensated by increasing the fifth decimal by 3; and the value of this increase in the pounds would be only $960 \times \cdot 00003 = \cdot 0288$ *farthings*. In every case, the error from curtailing the decimals is, therefore, an error of a fraction of a farthing in *defect*. We have already seen that when the Interest is not more than £10, the error of Rule II., which arises from replacing $1\cdot 0001$ by 1, is likewise an error of only a fraction of a farthing, and that this error is in *excess*. Hence the Interest, as computed by Rule II., whenever that Interest does not exceed £10, is affected with an error which is merely the difference between two fractions of a farthing, and is therefore inappreciable.

CALCULATIONS RESPECTING COMMISSION, BROKERAGE, INSURANCE, &c.

PROBLEM.

To find the Commission or Brokerage upon any number of pounds, at a given rate per cent.

RULE I.—Regard the given number of pounds as so many shillings; multiply by the rate per cent., and divide by 5. Or,

RULE II.—Multiply the given number of pounds, taken as so many shillings, by twice the rate per cent., and divide by 10. Or,

RULE III.—Multiply the pounds, taken as so many shillings, by twice the rate per cent., and from the product cut off the unit's figure: the remaining figures express shillings, and the figure cut off denotes so many pence and *fifths* of a penny.

EXAMPLES.

1. What is the commission on £83, at 2 per cent.?

By Rule I.	By Rule II.	By Rule III.
83*s*.	83*s*.	83*s*.
2	4	4
5) 166	1,0) 33,2	33,2 = 33*s*. $2\frac{2}{5}$*d*.
Ans. 33*s*. $2\frac{2}{5}$*d*.	$33\frac{2}{10}$*s*. = 33*s*. $2\frac{2}{5}$*d*.	

2. What is the commission on £58, at $6\frac{1}{4}$ per cent.?

By Rule I.

58*s*.
$6\frac{1}{4}$
———
348
14 6
———
5) 362*s*. 6*d*.
———
Ans. 72*s*. 6*d*.

By Rule II.

58*s*.
$12\frac{1}{2}$
———
696
29
———
$72{,}5 = 72\frac{1}{2}$*s*. = 72*s*. 6*d*.

By Rule III.

58*s*.
$12\frac{1}{2}$
———
696
29
———
$72{,}5 = 72$*s*. $5\frac{5}{5}$*d*. = 72*s*. 6*d*.

3. What is the commission on £125, at $3\frac{3}{8}$ per cent.?

By Rule I.

125*s*.
$3\frac{3}{8}$
———
375
$375 \div 8 = 46$ $10\frac{1}{2}$
———
5) 421*s*. $10\frac{1}{2}$*d*.
———
Ans. 84*s*. $4\frac{1}{2}$*d*.

By Rule II.

125*s*.
$6\frac{3}{4}$
———
750
93 9 = 4) 375*s*. — 93*s*. 9*d*.
———
1,0) 84,3*s*. 9*d*.
———
Ans. 84*s*. $4\frac{1}{2}$*d*.

By Rule III.

125*s*.
$6\frac{3}{4}$
———
750
93*s*. 9*d*. = $93\frac{3}{4}$
———
$84{,}3\frac{3}{4} = 84$*s*. $3\frac{3}{4}$*d*. + $\frac{1}{5}$ of $3\frac{3}{4}$*d*., or $\frac{1}{5}$ of $\frac{15}{4}$*d*. = $\frac{3}{4}$*d*.; and 84*s*. $3\frac{3}{4}$*d*. + $\frac{3}{4}$*d*. = 84*s*. $4\frac{1}{2}$*d*. *Ans*.

[Whenever the given sum consists of pounds *only*, and has 5 or 0 in the unit's-place, we know that its fifth part will be a whole number; and in *this* case it will save figures, in working by Rule I., to execute the division by 5 *first*; and the same may be said, whatever be the unit's-figure of the pounds, if either 5*s*. or 10*s*. or 15*s*. be connected with those pounds. The work of Ex. 3 above may be thus shortened.]

The foregoing Rules are derived from the truth, that if the principal be expressed in pounds, the commission (or interest), expressed also in pounds, will be found by multi-

plying by the rate per cent., and dividing by 100. But if we take 20 times the number of pounds in the principal, the commission will, of course, be expressed in shillings; and the multiplying by 20 and then dividing by 100 is the same as not multiplying at all, and dividing by 5; so that we have only to regard the principal, in pounds, as so many shillings, and then to divide by 5, to get the commission in shillings; and this is Rule I. Rule II. is an obvious inference from it, for we merely double the multiplier (the rate), and double the divisor (5). And Rule III. is but a slight modification of this: in dividing by 10 we cut off the unit's-figure, convert the shillings thus cut off into pence, add these to whatever pence may follow, and *then* divide by 10; or, expressing the odd pence as a fraction of a shilling, we multiply all that is cut off by 12 and divide by 10; that is, we multiply by $\frac{12}{10} = \frac{6}{5} = 1\frac{1}{5}$; which suggests Rule III.

NOTE.—If the commission be 5 per cent., then whatever number of pounds and fractions of a pound there are in the principal, so many shillings and like fractions of a shilling are there in the commission; and the Rule at p. 181 may be employed.

Although in the foregoing Examples we have calculated the commission to the fraction of a farthing—as the Rules lead to results of the most perfect accuracy—yet, in the transactions of actual business, the Commission agent and the Broker usually charge an additional penny for every overplus fractional part of a penny. In the first Example above, the agent's charge would be 33*s*. 3*d*.; in the second, 72*s*. 6*d*., which it strictly is; and in the third it would be 84*s*. 5*d*.; so that, in working this third Example by Rule III., the supplementary operation for finding what $\frac{1}{5}$ of $3\frac{3}{4}d$. is, *exactly*, would, in practice, be omitted. A glance would show that the addition to the $3\frac{3}{4}d$. cut off would be greater than $\frac{1}{2}d$. and less than 1*d*.; so that the pence being between 4*d*. and 5*d*., the charge for commission would be 84*s*. 5*d*.

4. A stockbroker is employed to sell out £536 of Bank Stock: what will be his charge for brokerage, at 2*s*. 6*d*., that is, £$\frac{1}{8}$ per cent.?

By Rule I.	By Rule II.	By Rule III.
8) 536*s*.	4) 536*s*.	4) 536*s*.
5) 67	1,0) 13,4 = 13*s*. 5*d*. *Ans*.	13,4 = 13*s*. 5*d*. *Ans*.
13*s*. 5*d*. *Ans*.		

The result, by Rule I., is strictly 13*s*. $4\frac{4}{5}d$.; by Rule II. it is 13*s*. $4\frac{8}{10}d$.; and by Rule III., 13*s*. 4*d*. + $\frac{4}{5}d$., the results all agreeing, of course; but the $\frac{4}{5}d$. or $\frac{8}{10}d$. is charged as an additional penny.

5. What will be the charge for commission on £127 10s., at $3\frac{1}{2}$ per cent.? *Ans.* £4 9s. 3d.
6. What is the brokerage upon a money transaction for £385, at 2s. 6d. per cent.? *Ans.* 9s. 8d.
7. What sum must be paid for insuring a vessel and cargo, estimated at £2225, at $3\frac{1}{4}$ per cent.? *Ans.* £72 6s. 3d.

CALCULATIONS RESPECTING THE PURCHASE OF FREEHOLD PROPERTY.

PROBLEM 1.

Given the number of years' purchase (that is, the number of years' rent), to find the rate per cent. on the purchase-money.

RULE.—Divide £100 by the number of years' purchase; the quotient will be the rate per cent.

For the rent multiplied by the number of years' purchase is the purchase-money, or principal invested; and the rent itself is the interest received yearly; and as this principal is to £100, so must the interest on the principal (the rent) be to the interest on £100 (the rate per cent.); that is,

$$\text{Rent} \times \text{No. of years} : £100 :: \text{Rent} : \text{to Rate per cent.};$$

therefore, $\dfrac{\text{Rent} \times £100}{\text{Rent} \times \text{No. of years}} = \dfrac{£100}{\text{No. of years}} = \text{Rate per cent.};$*

which is the Rule.

EXAMPLES.

1. If 14 years' purchase be given for a freehold estate, what percentage does the purchaser receive per annum?
   £100 ÷ 14 = £7 2s. $10\frac{2}{7}$d. *Ans.*
2. If freehold property be purchased for 21 years' rental, how much per cent. will the purchaser receive for his investment?
   £100 ÷ 21 = £4 15s. $2\frac{6}{7}$d. *Ans.*

---

* That the reader may not suppose here that in the expression "Rent × £100," we are implying that money can be multiplied by money, it may be as well to state that what is really implied is that £100 is to be multiplied by the *number* of pounds in the Rent, or the Rent taken 100 times.

Problem 2.

To determine the rent, so that the purchase-money may yield a given rate per cent.

Rule.—Multiply the purchase-money by the given rate per cent., and then divide by 100.

For we have seen above that $\frac{\text{Rent} \times 100}{\text{Purchase-money}} = \text{Rate per cent.}$; therefore, Purchase-money × Rate ÷ 100 = Rent; which is the Rule.

Note.—It is obvious, in order that the investment may produce 5 per cent., that the price paid must be 20 years' purchase; and that a 20th part of the purchase-money must be the yearly rental, in order that 5 per cent. may be realized by the holder of the property.

Examples.

1. If a freehold estate be sold for £30,000, what must be the yearly rent, to allow the purchaser 4 per cent. per annum for his money?
   £300,00 × 4 = £1200. *Ans.*
2. If a freehold estate be sold for £11,000, what must be the yearly rent, to allow the purchaser 5 per cent. per annum?
   £110,00 × 5 = £550. *Ans.*

Problem 3.

The yearly rent and purchase-money being given, to find the rate per cent.

Rule.—100 times the rent divided by the purchase-money will give the rate per cent. [This is obvious from the fraction for "Rate" given above, Prob. 2.]

Examples.

1. If a freehold yearly rental of £550 be bought for £11,000, at what rate per cent. is the money invested?
   55000 ÷ 11000 = 5 per cent. *Ans.*
2. If an estate of £1200 a year is bought for £30,000, at what rate per cent. is the money invested? *Ans.* 4 per cent.

[This problem differs from Prob. 1 only in this, namely, that here the actual rental is given, and there only the

number of years' purchase: the number of years' purchase being assigned, the yearly rental need not be formally stated, since it is implied.]

We shall only add further on this subject, that since 100 divided by the number of years' purchase gives the rate per cent., it follows that 100 divided by the rate per cent. will give the number of years' purchase; thus, if only 3 per cent. is to be realized, the number of years' purchase for the estate must be $100 \div 3 = 33\frac{1}{3}$ years; if 4 per cent. is to be secured, the number of years' purchase must be $100 \div 4 = 25$ years; and so on. (See Ex. 1, Prob. 2.)

Discount, Stocks.

Discount is analogous to Commission; it is the percentage which the receiver of money allows to the payer for prompt payment; it is also the name given to the deduction which the Banker or Bill-broker makes upon Cashing a Bill or Promissory Note, which Bill or Note becomes due, or payable, only at a future specified time; and the calculation of the Discount on any sum is the same as the calculation of Interest on that sum.

The *present worth* of such a Bill is not the sum on which the discount is charged; it is that sum of money, paid down, which, when put out at the agreed-upon interest, for the specified time, will amount to just sufficient to pay the Bill when it becomes due: thus, if the Bill be for £105, payable in one year, interest being at 5 per cent. per annum, then, since £100 present money would amount in one year, at the proposed interest, to £105, the *present value* of the Bill is £100. But Bankers and Bill-discounters reasonably expect a profit, and therefore would charge, as *Discount*, the full interest of the £105, namely, £5 5*s*. And this would also be the discount, at 5 per cent., allowed to the purchaser of goods by the tradesman, for ready-money payment: in some kinds of purchases, however, $7\frac{1}{2}$, or even 10 per cent., is allowed. After what has been said in reference to Interest generally, a single illustrative example here will suffice; it being remembered that three days—called *days of grace*—are always added to the *specified* time of payment, so that the Bill is not really due till the third day after that time.

Examples.

1. A Bill for £77, drawn on the 8th of March, at 6 months, is discounted on the 3rd of June, at 5 per cent.: required the amount of discount?

The 6 months expire on September 8; therefore the Bill becomes due September 11. From June 3 to September 3 is 92 days (see Table, p. 26), and therefore to September 11 it is 100 days; and the interest (discount) on £77, at 5 per cent., for 100 days, is found, by the method employed at p. 189, to be £1 1*s.* 1¼*d.*; and therefore the discount charged would be £1 1*s.* 2*d.*

The following example belongs to a class of cases of frequent occurrence in Bill transactions.

2. A Bill for £500 was due February 2, 1870, but was allowed to remain at interest. £80 was paid March 9; £115 May 15; £25 June 1; and the balance, namely, £280, August 14: what interest was due at 5 per cent.?

	£	Days.		£
1870; Feb. 2, Due	500	× 35	= 17500	3) 7·1545 (See p. 190.)
Mar. 9, Paid	80			2·3848
	——			2384
	420	× 67	= 28140	238
May 15 ,,	115			———
	——			9·8015£
	305	× 17	= 5185	20
June 1 ,,	25			———
	——			16·03 *s.*
	280	× 74	= 20720	12
Aug. 14 ,,	280		———	———
	——		71545	·36 *d.*

The interest charged would be £9 16s. 1*d.*, which is the *Ans.*

The Rule at p. 189 directs us to point off *five* places of figures; whereas, above, we have pointed off only four from 71545: but it is to be remembered that this number is to be previously multiplied by 10,—twice the rate per cent.; so that the number to which the method referred to is to be applied is 715450, from which five figures being pointed off, we have 7·1545, as above, the 0 being omitted as non-significant.

The calculations concerned in the purchase of *Stock*, that is, property in the *Public Funds*, are very similar to those employed in the purchase of Freehold Estates. What, in reference to this latter kind of property, are called *Rents*, in

regard to the former kind are called *Dividends;* but in each case a permanent yearly or half-yearly income is purchased for a specified sum paid at once. Stock is not, in reality, *money;* it merely gives the purchaser or holder of it the claim to a certain yearly or half-yearly *dividend:* it is this right alone that he purchases, and which he can again sell, with but little trouble, whenever he pleases. Like most purchasable property, or income, the price fluctuates, the more perhaps in this kind of property than in any other kind, since the money invested in the purchase of the income goes to the Government, which, from commercial and political changes, may require, for the exigency of the occasion, more funds at one time than at another, and are therefore willing to sell the incomes, or dividends they grant, at a lower price. The following examples will serve to show the nature of the transactions here spoken of.

Examples.

1. A person invests £3500 in the $3\frac{1}{2}$ Per Cents. when the price of this stock is 98; that is, when he pays £98 for what is called £100 *stock:* what will his annual income from this investment be?

He purchases as many £100's *stock* as there are 98's in 3500, and for each of these £100's he is to receive £3 10*s.* per annum; therefore his yearly income will be

$$\pounds\frac{3500}{98} \times 3\tfrac{1}{2} = \pounds\frac{12250}{98} = \pounds125 \textit{ Ans.}$$

2. When the $3\frac{1}{2}$ Per Cents. are at 98, how much money must a person invest in that stock, in order to secure a yearly income of £150?

It is obvious that he must purchase as many £100's *stock* as there are £$3\frac{1}{2}$'s in £150; so that we have $150 \div 3\frac{1}{2}$, or $300 \div 7 = 42\frac{6}{7}$; and £98 × $42\frac{6}{7}$ = £4200, the sum to be invested. The purchaser will then possess or *hold* £$4285\frac{5}{7}$ stock in the $3\frac{1}{2}$ per cents.; since $42\frac{6}{7}$ times £100 amount to this sum, the £98 (the price) being the value of £100 *stock.*

3. When the $3\frac{1}{2}$ Per Cents. are at $99\frac{7}{8}$, how much money must be invested in them to produce an income of £140 per annum?
Ans. £3995.

4. When Bank Stock is at $131\frac{3}{4}$, the interest on it being at 5 per cent., how much money will purchase £575 10*s.* of it; and how much must be paid to the Stockbroker, who charges 2*s.* 6*d.* per cent. on the stock purchased?
Ans. Purchase-money, £758 4*s.* 5*d.*; Brokerage, 14*s.* 5*d.*

Profit and Loss.

Problem 1.

The prime cost and the selling price being given, to find the gain or loss per cent.

This problem is solved by a common Rule-of-Three operation. The difference between the cost and the selling price is the gain or loss; and the cost is to £100 as the profit or loss on the cost is to the profit or loss on £100, as is obvious; or without the formality of a Rule-of-Three stating, the Rule may be expressed thus:

Rule.—Divide 100 times the gain or loss by the prime cost; the quotient will be the number of pounds gain or loss per cent.

Examples.

1. If a horse is bought for £15, and then sold for £17 10*s*., what is the gain per cent.?

The gain on the £15 is £2 10*s*.; and $\frac{250}{15} = \frac{50}{3} = 16\frac{2}{3}$ per cent. *Ans.*

2. If cloth be bought at 6*s*. per yard, and sold at 7*s*., what is the gain per cent.?

The gain on the 6*s*. is 1*s*.; and $\frac{100}{6} = 16\frac{2}{3}$ per cent. *Ans.*

This profit, per cent., is the same as that in the former example: in both cases the gain on the cost is $\frac{1}{6}$ of that cost; so that the calculation in each case might stand thus: $\frac{1}{6} \times 100 = \frac{100}{6} = 16\frac{2}{3}$.

3. If linen be bought at 1*s*. per yard, and sold at 13½*d*., what is the gain per cent.? *Ans.* 12½ per cent.
4. If broadcloth be bought at £1 per yard, and sold at 13*s*. 4*d*., what is the *loss* per cent.? *Ans.* $33\frac{1}{3}$ per cent.
5. If, on the contrary, the cloth be bought at 13*s*. 4*d*. per yard, and sold at £1, what is the *gain* per cent.? *Ans.* 50 per cent.
6. If tea be bought at 2*s*. 9*d*. per lb., and sold at 3*s*. 4*d*., what is the gain per cent.? *Ans.* £21 4*s*. $2\frac{10}{11}$*d*. per cent.

Problem 2.

The prime cost being given, to find what the selling price must be, in order that an assigned rate per cent. may be obtained.

Rule.—Whatever part the proposed rate per cent. is of

100, that same part of the cost price will be the requisite profit; this, added to the cost price, is the selling price.

EXAMPLES.

1. A cargo of cotton is bought for £12345; what must it be sold for to yield a profit of 5 per cent.?

As 5 is the 20th part of 100, we proceed, per Rule, as here annexed; and we thus find the profit to be £617 5*s.*, and the selling-price to be £12962 5*s.* *Ans.*

	£	
20)	12345	
	617	5*s.*
	£12962	5*s.*

The reverse of this problem would be to find the prime cost from knowing the profit upon the whole, and the rate of profit per cent. In this case we should have to *multiply* the given profit by the number which here is the *divisor;* thus, in the instance of the profit being known to be £617 5*s.*, and 5 the rate per cent., we should have £617 5*s.* $\times$ 20 = £12345 = the prime cost. But we need not extend these examples. Any three of the terms in the general proportion—

Prime cost : £100 : : Gain on outlay : Gain per cent.—

being given, the fourth term may be found, and the figure-work economized, by discarding factors seen to be common to both multiplier and divisor; thus, in the example just considered, the proportion would be—

£100 : Prime cost : : Gain per cent. : Gain on outlay;

that is, £100 : £12345 : : 5 per cent. : £617 5*s.* Gain.

$$\text{Thus, } \frac{£12345 \times 5}{100} = \frac{£12345}{20} = £617\ 5s., \text{ as by the Rule.}$$

If the required profit had been at the rate of only 4 per cent., then we should have had—

$$\frac{£12345 \times 4}{100} = \frac{£12345}{25} = £493\ 16s.$$

But it is perhaps more expeditious, in both these, as well as in similar cases, to leave the divisor 100 unreduced, and to proceed thus. Multiplying in the first case by the 5, and in the second by the 4, and cutting off the last two figures of

each product, which is equivalent to dividing by 100, we have—

	£		£
1st.	617,25	2nd.	493,80
	20		20
	5,00s.; £617 5s.		16,00s.; £493 16s.

Proportional Parts.

Problem.

To divide a given quantity into parts which shall have the same relations to one another as any proposed numbers have to one another.

Rule I.—As the sum of the given numbers is to any one of them, so is the given quantity to be divided to the part of it corresponding to that number.

Examples.

1. It is required to divide £80 into three parts, that shall bear to one another the same relations as the numbers 2, 3, and 5.

1st. 10 : 2 : : £80 : £16. 2nd. 10 : 3 : : £80 : £24. 3rd. 10 : 5 : : £80 : £40. The required parts are therefore £16, £24, and £40, which together make up the whole £80. It is obvious that the Rule might be expressed a little differently, thus:—

Rule II.—Multiply the given quantity by each of the given numbers separately, and then divide each product by the sum of all the numbers.

2. A bankrupt owes £120 to A; £80 to B; and £75 to C: he possesses only £165: how is this sum to be equitably divided among his three creditors?

£		£		£	
120 ×	165 =	19800,	which ÷ 275 =	72,	the share of A.
80 ×	„ =	13200,	„ ÷ =	48,	„ B.
75 ×	„ =	12375,	„ ÷ =	45,	„ C.
275				165	

3. Three traders, A, B, and C, contribute the following sums to the business: A. £500; B, £650; and C, £700: the year's profits are £555: what is each partner's share?

Ans. A, £150; B, £195; C, £210.

4. A person bequeathed in his Will £140 to A; 100 guineas to B; 80 guineas to C; £70 to D; and £60 to E; but at his death left only £311 15*s*. How is this sum to be equitably divided, overplus fractions of a farthing being disregarded, because unpayable?

Ans. A, 95 1*s*. $8\frac{3}{4}$*d*.; B, £71 6*s*. $3\frac{1}{2}$*d*.; C, £57 1*s*. $0\frac{1}{2}$*d*.; D, £47 10*s*. $10\frac{1}{4}$*d*.; E, £40 15*s*. $0\frac{1}{4}$*d*.

The truth of the foregoing RULE scarcely requires any formal proof. Take the first Example: here we are told that the number 10 is divided into the three parts, 2, 3, and 5, and we are required to divide the number 80 in a similar way. It is plain that whether the proposed number be twice 10, or three times 10, or any number of times 10 whatever, the required component parts of it must be just as many times 2, 3, and 5; hence the given number (10) must be to the component part of it (2) as the proposed number (80) is to the corresponding component part of *it*; and so of each of the other component parts. And similarly in all other such cases.

THE CHAIN-RULE.

The Chain-Rule is a compendious method of computing Examples which, without it, would involve two or more distinct Rule-of-Three statings: the following Examples will sufficiently illustrate the mode of working by it.

EXAMPLES.

1. If 3 lbs. of tea cost as much as 8 lbs. of coffee, and 5 lbs. of coffee as much as 18 lbs. of sugar: how many pounds of sugar should be given in exchange for 20 lbs. of tea?

By the Rule-of-Three, we have:—

1st. 8 lbs. coffee : 5 lbs. coffee : : 3 lbs. tea : $\frac{3 \times 5}{8}$ lbs. tea.

2nd. $\frac{3 \times 5}{8}$ lbs. tea : 20 lbs. tea : : 18 lbs. sugar : $\frac{8 \times 18 \times 20}{3 \times 5}$ lbs. of sugar;

since *dividing* by $\frac{3 \times 5}{8}$ is the same as *multiplying* by $\frac{8}{3 \times 5}$.

Now, according to the Chain-Rule, the several quantities would be arranged in two columns, thus:—

3 lbs. tea = 8 lbs. coffee.
5 lbs. coffee = 18 lbs. sugar.
How many lbs. sugar = 20 lbs. tea?

Where it is to be observed that no two commodities of the same kind occur in the same column. Now by dividing the product of the numbers in the *complete* column, by the product of those in the column which the *answer*, if known, *would* complete, that answer is obtained: it is $\frac{8 \times 18 \times 20}{3 \times 5} = 192$; so that 192 lbs. of sugar is the answer. And it is plain that the result of the division here directed to be performed *must* give the true answer, because from the foregoing equalities it follows that we must have *also* the equality

$$3 \times 5 \times Ans. = 8 \times 18 \times 20, \text{ and therefore}$$

$$Ans. = \frac{8 \times 18 \times 20}{3 \times 5} = 192.$$

NOTE.—In working Examples by the Chain-Rule, the computer will do well to avail himself of every occasion that may offer to expunge factors common to numerator and denominator,—common, that is, to dividend and divisor, before he actually multiplies and divides. For instance, in the present example, he should deal with the fraction $\frac{8 \times 18 \times 20}{3 \times 5}$ in the more simple form $\frac{8 \times 6 \times 4}{1}$, or rather $8 \times 6 \times 4 = 192$. The 8 is retained, intact, in the numerator, because it has no integral factor (or divisor) common to either the 3 or the 5; but the factor (or divisor) 3 is common to the 18 and the 3; this common factor, expunged from both, reduces the 18 to 6 and the 3 to 1. Again; the factor 5, entering into both the 20 and the 5, is in like manner expunged from both numbers; the 20 being thus reduced to 4, and the 5 to 1. And these simplifications being made mentally, at the outset, the actual figure-work becomes abridged. When a factor is seen to be common to a number in the numerator and a number in the denominator, it will be convenient to draw the pen through both numbers, and to write, above the former and below the latter, only the factor of each which is retained, and then to work with these.

2. If twenty Spanish piastres are worth £3 7*s.* 6*d.*; and £1 be worth $25\frac{1}{3}$ French *francs*: how many francs may be had in exchange for $2\frac{1}{2}$ Spanish piastres?

$$20 \text{ piastres} = £3\tfrac{3}{8} \qquad = \frac{£27}{8}$$

$$£1 = 25\tfrac{1}{3} \text{ francs} \qquad = \frac{76}{3} \text{ francs}$$

$$\text{francs ?} = 2\tfrac{1}{2} \text{ piastres} \qquad = \tfrac{5}{2} \text{ piastres.}$$

The number of francs is therefore $\frac{27 \times 76 \times 5}{20 \times 8 \times 3 \times 2} = \frac{9 \times 19}{4 \times 2 \times 2}$ (See NOTE above) $= 10\frac{11}{16}$ francs.

3. If £1 = 420*d.* Flemish, and 58*d.* Flemish = 1 Venetian crown = 60 Venetian ducats; 1 ducat = 360 Spanish mervadies, and

272 mervadies = 1 Spanish piastre: how many piastres may be had in exchange for £1000 sterling?

£1	= 420*d.* Flemish	$\frac{420 \times 60 \times 360 \times 1000}{58 \times 272} = 5750\frac{?}{?}$
58*d.* F.	= 60 ducats	= 5750½ piastres, and a fractio[n]
1 duc.	= 360 merv.	less than one-half of a merva-
272 merv.	= 1 piastre	die. *Ans.*
piastres?	= £1000	

[On the general subject of *Exchanges*, consult Kelly's Universal Cambist.]

4. If 3 lbs. of pepper be worth 4 lbs. of mustard, and 5 lbs. of mustard be worth 12 lbs. of candles: how many lbs. of candles should be given for 20 lbs. of pepper? *Ans.* 64 lbs. of candles.

5. If the value of 5 lbs. of tea = 12 lbs. of coffee; 9 lbs. of coffee = 28 lbs. of sugar; and 13 lbs. of sugar = 18 lbs. of soap: how many pounds of soap may be had for 7 lbs. of tea?
Ans. 72⅜ lbs. of soap very nearly.

APPENDIX.

I.—Decimals.

In the foregoing treatise, we have taken for granted, as stated at the outset, that the reader is already familiar with the principles of common Arithmetic; and since even the most rudimentary books on this subject usually explain the notation of *Decimals*, and give specimens of the methods of computation with the numbers so called, we might be justified in excluding all explanations, touching the fundamental principles of decimal arithmetic, from this work. But in order to refresh the memory of what may perhaps have been but hastily gone through at school, and more especially to give a clear insight into the theory of this important part of arithmetic, we here devote a short supplementary article to the subject of Decimals.

1. The Theory of Decimals.

The common notation in which all *integer* (or whole) numbers is expressed, *is* the *decimal* notation; and what are more specially called *decimals* are only *fractions* expressed in *the same* notation; both whole numbers and fractions being thus comprehended under one and the same uniform system.

We know that in the notation for integers, the *value* of any figure of a number is *ten times* as great as it would be if that figure were advanced one place further to the *right*: thus, taking the number 4387, the 4, occupying the place of *thousands*, stands for 4000, and is read *four thousands*; but if it were removed a place further to the right, that is, to where the 3 now stands, its value would be only the tenth part of this,—it would then represent but 400, and would be read *four hundred*; if it were to replace the 8, it would represent 40, and if the 7, merely *four units*, or *ones*. Hence in writing the figures of a number in the usual way, that is, from left to right, every figure we put down is, in value, only the *tenth part* of what it would be if it were one place less in advance; and writing the figures one after another in this manner, the whole number becomes completed as soon as we arrive at the place of *units*. But there is no reason why the decimal notation should always terminate *here*, and not be extended *beyond* the unit's place; the first figure beyond representing so many *tenths* of a unit, the second, so many *hundredths*, the third, so many *thousandths*, and so on; each figure, as before, standing for only the *tenth part* of what it would have been if written in the immediately preceding place. It is this extension of the common notation which introduces what are more emphatically called *decimals*.

The number 4387, chosen for illustration above, is only a brief way of expressing $4000 + 300 + 80 + 7$. If, in addition to these whole numbers, there were the fractions $\frac{6}{10} + \frac{3}{100} + \frac{8}{1000}$, then the whole numbers and fractions together would be $4000 + 300 + 80 + 7 + \frac{6}{10} + \frac{3}{100} + \frac{8}{1000}$, which in the decimal notation would be briefly expressed by the figures 4387·638; the dot (called the decimal point) being interposed to separate the integral from the fractional part of the entire number. Each figure to the right of this separating point is thus the representative of a *fraction*, the denominator of which it is unnecessary to record: the ·6 is $\frac{6}{10}$, the next decimal, $\frac{3}{100}$, and the last $\frac{8}{1000}$; so that ·63 is $\frac{63}{100}$, and ·638 is $\frac{638}{1000}$.

We can as readily pronounce upon the *value* of any figure on the *right* of the decimal point, as we can pronounce upon the value of any figure on the left of that point: the place next to the unit's place, on the *left*, is the place of *tens*; the

place next to the unit's place, on the *right*, is the place of *tenths*; the place next to that of tens, on the left, is the place of *hundreds*; whilst the place next to that of tenths, on the right, is the place of *hundredths*; and so on. Take for instance the mixed number 1234·5678: then the 4 being in the place of units occupies the *first* place in the arithmetical scale; the 2, on the *left*, the third place; and the 6, on the *right*, the third place from the unit's place also: the value of the 2 is 200, or two *hundreds*; the value of the 6 is $\frac{6}{100}$, or six *hundredths*; in like manner, the value of the 3 is 30, or 3 *tens*, and the value of the 5,—equally distant from the unit's place, but in the opposite direction, is $\frac{5}{10}$, or 5 *tenths*; the value of the 1 is 1000, and that of the 7, $\frac{7}{1000}$; and lastly, the value of the 8 is $\frac{8}{10000}$.

From what has now been explained, it will be clear, that in order to express a decimal in the form of a common fraction, we have only to write the figures of the decimal for the numerator, and to put for denominator 1, followed by as many ciphers as there are figures, or places of figures, beyond the decimal point. In the particular decimal given above, for instance, it was seen that $\cdot638 = \frac{638}{1000}$: in like manner, $\cdot3456 = \frac{3456}{10000}$; as is obvious, because $\cdot3456 = \frac{3}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{6}{10000}$, the sum of which is the single fraction just written. Again, $\cdot027 = \frac{27}{1000}$; $\cdot036 = \frac{36}{1000}$; $\cdot0036 = \frac{36}{10000}$; and so on; for $\cdot027 = \frac{2}{100} + \frac{7}{1000} = \frac{27}{1000}$; $\cdot036 = \frac{3}{100} + \frac{6}{1000}$; and $\cdot0036 = \frac{3}{1000} + \frac{6}{10000} = \frac{36}{10000}$. We thus see, too, how materially *ciphers* immediately after the decimal point, that is, *before* the significant figures, affect the values of those figures; yet ciphers *after* them have no effect at all, and are quite useless: thus, though ·36 is ten times ·036, and *this*, ten times ·0036, yet ·36 and ·36000 are identical in value, the terminating ciphers being superfluous, and counting as *nothing*; since a fraction with a denominator of *some* definite value, and 0 only for numerator, has *no* value at all.

It further appears that the removal of the decimal point one place more to the right, is equivalent to *multiplying* the number by 10; its removal two places to the right, the same as multiplying by 100; and so on: thus, $2\cdot468 \times 10 = 24\cdot68$, where each figure is 10 times what it was before: $2\cdot468 \times 100 = 246\cdot8$; each figure being 100 times what it was before: the 2 is now 200; the 4, instead of $\frac{4}{10}$, is now

40; the 6, instead of $\frac{6}{100}$, is now 6; and the 8, instead of $\frac{8}{1000}$, is now $\frac{8}{10}$. The removal of the point in the *other* direction is the same as *dividing* by 10, 100, &c.: thus 24·68 is a tenth part of 246·8; 2·468, a hundredth part of 246·8; and ·2468, a thousandth part: and all this is obvious from the very notation of decimals.

The following examples will sufficiently show how to convert decimals into common fractions.

1. $13·5 = 13\frac{5}{10} = 13\frac{1}{2}$; 2. $13·25 = 13\frac{25}{100} = 13\frac{1}{4}$; 3. $13·75 = 13\frac{75}{100} = 13\frac{3}{4}$; from which we see that the fractions $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{3}{4}$, expressed in decimals, are respectively ·5, ·25, and ·75.

---

2. The Practice of Decimals.

Addition and Subtraction.

Rule.—As in integers, units are to be placed under units, tens under tens, and so on,—so in decimals we must place tenths under tenths, hundredths under hundredths, and so on; that is, the decimal points must all range in the same vertical line or column, as in the following examples.

Addition.

32·863	·0074
5·27	3·6208
43·814	·1526
61·503	5·4903
72·006	6·021
·234	12·56
·901	·0038
216·591	27·8559

Subtraction.

473·2538	52·0472
264·3297	17·65836
208·9241	34·38884
2478·36	74·
35·2096	·0062
2443·1504	73·9938

In a similar manner the sum of 23·462 + 7·38 + 26·151 + 53·84 + 78·3584 is found to be 189·1914; and the sum of 4·70234 + ·3258 + 15·1602 + ·0043 + 37·10021 + 18 is found to be 75·29285; also 683·2031 − 479·8627 = 203·3404; and 1·4310063 − 1·3648163 = ·06619.

Multiplication and Division.

Rule.—In each of these operations, proceed exactly as you would do if dealing with integers only, disregarding the decimal point altogether. But in *Division*, remember that you are at liberty to annex to the decimals, in the dividend, as many ciphers as you please (p. 208); so that although

(disregarding the decimal point) the dividend may be *less* than the divisor, yet, by annexing ciphers, it may always be made *greater*; and thus the operation may be carried on. How to ascertain the correct number of decimal places to be pointed off, in the result of either operation, will be shown presently.

EXAMPLES.

1. Multiply 325·201 by 2·43. Disregarding the decimal point, the work is as here annexed; and the question now is—whereabouts in this product must the decimal point be introduced? In order to ascertain this, we are to observe that the factors multiplied together are $325\frac{201}{1000}$ and $2\frac{43}{100}$; that is, they are $\frac{325201}{1000}$ and $\frac{243}{100}$; the product of which, as we have just seen, is $\frac{79023843}{100000}$; and the *five* ciphers in the denominator imply that *five* places of decimals must be pointed off (p. 208); so that the correct product is 790·23843;—the number of decimal places being the *sum* of the number in *both* factors: and it is pretty obvious that such must be always the case; because, as in the present instance, each of the two factors may be replaced by a *fraction*, the numerator of which is a whole number consisting of the very same figures, and of which the denominator is 1 followed by as many ciphers as there are figures in the decimal portion of that factor; so that in multiplying these fractions one by the other, the denominator of the product will be 1 followed by as many ciphers as there are figures in the decimals of *both factors together*; and therefore as many places must be pointed off in the product as this sum amounts to. We add an example or two for exercise, in the working of which the computer is to remember, whenever the decimals in the product terminate with ciphers, that although these ciphers may be suppressed, as non-significant, yet that they must be included in counting the number of places of decimals to be pointed off. Should the product fall short of the requisite number of figures, the deficiency must be supplied by *prefixing* as many ciphers as will make up that number; the decimal point being placed before them.

$$\begin{array}{r} 325{\cdot}201 \\ 2{\cdot}43 \\ \hline 975603 \\ 1300804 \\ 650402 \\ \hline 790{\cdot}23843 \\ \hline \end{array}$$

2. 32·605 × 6·417 = 209·226285; 3. ·038 × ·072 = ·002736;
4. ·0431 × ·217 = ·0093527; 5. 43·92 × 2600 = 114192;
6. 24000 × ·0016 × ·35 = 13·44; 7. 1·4 × ·04 × ·4 × ·004 = ·0000896.

Multiplication being clearly understood, there can be but little difficulty about *Division:* instead of any formal Rule, we shall therefore proceed at once to examples.

Divide 2·5 by ·32; also 5·714 by 827·5; and ·07908 by ·83497.

Looking at these instances, we see that, paying no regard

to decimal points, and viewing the numbers as *whole* numbers, each of the divisors is greater than the corresponding dividend: we therefore annex ciphers to the latter, or conceive them to be annexed, as follows:—

```
·32) 2·50 (78125
     224
     ---
      26
      256
      ---
        4
       32
       --
        8
       64
       --
       16
       160
       ---
```

```
827·5) 5·7140 (6905
       49650
       -----
        7490
        74475
        -----
          425
         41375
         -----
          1125
```

```
·83497) ·0790800 (947
         751473
         ------
          39327
          333988
          ------
           59282
           584479
           ------
             8341
```

[Ciphers are here *supposed* to be annexed to the remainders: the actual insertion of them is not necessary.]

In the second and third of these cases, the division may be still carried on; but arresting the work at the present stage, let us see how many figures of the respective quotients are to be pointed off for decimals. In the first case, besides the 2·5 in the dividend *five* annexed ciphers have been used: hence, in the dividend, *six* decimals have been used altogether, and in the divisor *two;* and since (by multiplication) the number of decimal places, in divisor and quotient together, must make up the number in the dividend, the number in the quotient alone must be $6-2$, that is, *four* places; therefore the correct quotient is 7·8125. In the second case above, seven places of decimals have been used in the dividend, and only one in the divisor: therefore there must be *six* in the quotient; so that the correct quotient is ·006905. In the third case, there have been used nine decimals in the dividend, and five in the divisor: therefore the correct quotient is ·0947. The following examples are left for the learner to work out.

1. $721·17562 \div 2·257432 = 319·4672$; 2. $7·66858 \div ·0325 = 235·956$;
3. $47·298 \div 6·029 = 7·845$; 4. $103·936 \div 1059·108 = ·0981355$.

In the first of these examples, the quotient is carried to four places of decimals; in the second and third, to three;

and in the fourth, to seven places. In general, the decimals may be extended to an indefinite number; but of course only a limited number can be retained, those which would follow, upon prolonging the operation, being disregarded. This, it is true, causes the quotient, thus curtailed, to be short of strict accuracy; but since we may carry on the division to as many quotient-figures as we please, the deficiency is of no practical consequence. A figure in the fourth place of decimals has for value a fraction of which that figure, taken as a whole number, is the numerator, and 10000 the denominator; and a figure in the fifth place is a fraction of which the denominator is 100000. From the nature of the inquiry in hand, we can always tell whether such small fractions of unity are worth recognition or not: if they are, we may extend the work to six, seven, eight, &c., places of decimals in the quotient, whenever the operation does not spontaneously terminate, as it does in the first example at p. 211.

Application of Decimals to Concrete Quantities.

Examples.

1. What is the value of £·842; as also of £·375? The work here annexed shows that £·842 = 16*s*. 10*d*. + $\frac{32}{100}$*f*.; and £·375 = 7*s*. 6*d*.

1st.	2nd.
·842£.	·375£.
20	20
16·840*s*.	7·500*s*.
12	12
10·08*d*.	6·0 *d*.
4	
·32*f*.	

2. Express $\frac{5}{8}$*s*. in decimals of a £; and reduce 2*s*. 9¾*d*. to the decimal of 7*s*. 9¾*d*.

1st. 8) 5·000

20) ·625*s*.

= £·03125 = $\frac{5}{8}$*s*.

$$\text{2nd.}\quad \frac{2s.\ 9\tfrac{3}{4}d.}{7s.\ 9\tfrac{3}{4}d.} = \frac{11\tfrac{1}{4}d.}{2s.\ 7\tfrac{1}{4}d.} = \frac{45f.}{125f.} = \frac{9}{25} = \cdot 36.$$

Hence 2*s*. 9¾*d*. is the 36 *hundredth* part of 7*s*. 9¾*d*.

3. Express £7 14*s*. 9*d*. in decimals. Here we commence with the 9*d*., and convert it into decimals of 1*s*.; then prefixing the 14*s*. to the result, we divide by 20, thus converting 14*s*. 9*d*., or 14·75*s*., into decimals of £1; to which the £7 being prefixed, the result is the equivalent of £7 14*s*. 9*d*. expressed in decimals. (See Table I. at the end, by aid of which the proper decimals are found by inspection.)

12) 9*d*.

20) 14·75*s*.

£7·7375 = £7 14*s*. 9*d*.

4. How many square feet are there in the surface of a slab 4 ft. 6 in. by 3 ft. 9 in.? $4\frac{1}{2} = 4·5$, and $3\frac{3}{4} = 3·75$; so that the work is that here annexed.

$$\begin{array}{r} 3·75 \\ 4·5 \\ \hline 1875 \\ 1500 \\ \hline 16·875\text{Ft.} \end{array}$$ *Ans.*

5. Reduce $7\frac{1}{2}d.$ to the decimal of a £.
$7·5d. \div 12 = ·625s.$, and $·625s. \div 20 = ·03125$ £. *Ans.*
6. What decimal of £1 is 1*s.* $6\frac{1}{2}d.$ *Ans.* £·07708333...
7. Find the value of £·07708333... *Ans.* 1*s.* $6\frac{1}{2}d.$

[In this decimal, the figures after the 8 are an interminable series of *threes*; and since the fraction $\frac{1}{3}$ is convertible into such a series, that is, since $\frac{1}{3} =$ ·333..., the decimal of a £, in this example, may be otherwise expressed, thus: £·07708$\frac{1}{3}$. Now disregarding, for the moment, the supplementary fraction, the work for the pure decimal will be that here annexed. Then ·0000$\frac{1}{3}$* × 20 × 12 = ·00001 × 80 = ·0008: therefore, adding this to the result in the margin, we find the value of the proposed decimal to be 1*s.* $6\frac{1}{2}d.$ *exactly*. In like manner the *repeating decimal* ·999... may always be replaced by $\frac{9}{9}$ or 1; ·111... by $\frac{1}{9}$; ·666... by $\frac{6}{9}$; ·222...by $\frac{2}{9}$; &c.]

$$\begin{array}{r} ·07708£ \\ 20 \\ \hline 1·5416s. \\ 12 \\ \hline 6·4992d. \\ ·0008d. \\ \hline ·5000d. \\ \hline \end{array}$$

8. Reduce 13*s.* $4\frac{3}{4}d.$ to the decimal of £1. *Ans.* ·669791£.
9. Reduce 2 qrs. 14 lbs. to the decimal of 1 cwt. *Ans.* ·625 cwt.
10. How many grains are there in ·002084 lbs. troy? *Ans.* 12·00384 grs.

This brief sketch of the theory and practice of Decimals must terminate here: the use of these numbers is very extensive, especially in the more advanced parts of practical mathematics,—trigonometry, navigation, astronomy, &c.; and for their general applications in purely arithmetical inquiries, upon which we have not space here to enter, the student is referred to the treatise on Arithmetic, in Weale's *Rudimentary Series*. We shall now give a concluding article on the important subject of Decimal Coinage.

II. On the Decimal Coinage.

A change in the monetary system of this kingdom has long been under the consideration of the government, and various schemes for the purpose have been submitted to

* The 8 and the $\frac{1}{3}$ *both* occupy the *fifth* place of decimals.

examination. The object proposed to be accomplished is, to simplify the keeping of accounts, and to facilitate all calculations in which money is concerned.

There is no difference of opinion as to the *decimal* subdivisions of our coinage being the best that can be chosen for these purposes; the various denominations, from the highest to the lowest, descending uniformly by *tenths*. Taking the sovereign as the highest denomination, the next denomination (at present), shillings, are so many *twentieths;* the next, pence, so many *twelfths;* and the next, farthings, so many *fourths;* so that in casting up a money account, the *carryings* each conform to a different scale, till we arrive at the columns of *pounds*, when uniformity, in-this respect, comes at length to be observed: the decimal system is then adopted, and the carryings continue henceforward to be invariably so many *tens*. It is evidently most desirable that the same uniformity should prevail throughout the entire operation; and in order to this we venture here to submit the following suggestions.

The value of the highest money-unit should be determined on with a view, principally, to these two objects, namely: 1st. It should be chosen so as to disturb as little as possible existing notions and prejudices. 2nd. It should be chosen so as to render it easy to convert a sum of money, expressed in the old notation, into the equivalent sum expressed in the new notation. And it is moreover of the first importance that these objects should be accomplished without sacrificing even the smallest fraction of a farthing; mere approximations, however close, to the actual values of the coins to be superseded, would be unsatisfactory, and indeed unjust. Whatever changes be made, as to *value*, in the pieces coined, there should, we think, be as little change as possible in our familiar nomenclature—pounds, shillings, pence, and farthings; for any *general* change in this respect would be a commercial inconvenience, and would cause needless perplexity to the humbler classes. The value of the present gold piece—the *sovereign*—is 240 pence: we would recommend, if the decimal system be adopted, that the highest money-unit should be a gold piece of 250 pence, or 1000 farthings, and that it be called an IMPERIAL POUND; that a silver piece, value 25 pence, be issued as an IMPERIAL

SHILLING; a silver piece, value $2\frac{1}{2}$ pence, as an IMPERIAL (or silver) PENNY; and that the present copper penny, halfpenny, and farthing, remain untouched. The prefix, or the addition *Imperial*, to the old designations, would effectually preclude the confounding of the new Pound, Shilling, and Penny, with the old.

To distinguish between the Imperial and the old money, during the transition period, the symbols for the former should be £ S. D., those for the latter *l. s. d.* We should thus have the following money-units expressed in *pence* (*d.*):—

Imperial Pound.	*Imperial Shilling,*	*Imperial Penny.*	*Farthing.*
250*d.*	25*d.*	2·5*d.* = $2\frac{1}{2}$*d.*	·25*d.* = $\frac{1}{4}$*d.*

which values regularly descend according to the decimal scale, each coin, after the Imperial Pound, being, in value, the tenth part of that of the coin next higher in value.

Now although the decimal point appears in each of the last two values, yet, in keeping accounts, in Imperial Money, the decimal point need not ever be introduced, nor the word *decimal* be ever used or implied; so that an uneducated person would have no more need to know, in casting up an account, that he was using the notation of decimals, than he need know now, in proceeding from pence to shillings, that he is using duodecimals.

To convert old money into Imperial, we have only to apply the principle that 1000 farthings = 1 Imperial Pound; and to remember that to divide a number by 1000 we have merely to point off *three* figures of it—the last three—for *decimals*. Thus, suppose we have to find the values of the following sums in Imperial Money.

```
 l.  s.  d.                               l.  s.  d.
 9  13  7¼ Present currency.             12   8  5¾ Present currency.
 20                                      20
 ——                                      ——
 193                                     248
  12                                      12
 ———                                     ———
 2323                                    2981
    4                                       4
 ————    £  S.  D.                       ————    £  S.  D.
£9·293 = 9  2  9·3 Imperial.           £11·927 = 11  9  2·7 Imperial.
 ————                                    ————
```

In the first example, the amount, in Imperial Money, is 9 Pounds 2 Shillings and 9 Pence 3 farthings, *Imperial;* in the second it is 11 Pounds 9 Shillings and 2 Pence 7 farthings, *Imperial.* The point between the 9 and the 3, and between the 2 and the 7, may be regarded as used merely for the purpose of separating the farthings from the pence; the two results may also be written in this way, £9 2s. 9D. 3*f*., and £11 9s. 2D. 7*f*.

It thus appears, that for the conversion of old into Imperial Money, all we have to do is to reduce the proposed sum to *farthings;* and then to mark off *three* figures on the right for the Shillings, Pence, and farthings; the remaining figures, on the left, expressing the number of Imperial Pounds. If fractions of a farthing are included in the old notation, these same fractions (or their equivalent decimals) are to be appended in the new notation; thus, if the first of the above sums had been 9*l.* 13*s.* 7$\frac{1}{4}$*d.* + $\frac{1}{4}$*f*., then the same sum, in the new notation, would have been £9· 2s. 9·3$\frac{1}{4}$D., or £9·293$\frac{1}{4}$, or £9·29325.

In order to convert Imperial Money into old, it is plain that we have only to unite the figures under £. s. D. into a single number, and to reduce that number of *farthings* into *l. s. d.* Thus, £9 2s. 9·3$\frac{1}{4}$D. = 9293$\frac{1}{4}$*f*. = 9*l.* 13*s.* 7$\frac{1}{4}$*d.* + $\frac{1}{4}$*f*. The following is an example of a common account in the system here proposed: we shall present the operation in two slightly-differing forms, either of which may be adopted in actual practice, the facility of performing the work being exactly the same in both. The operations on the right hand are those necessary for the conversion of the total amount into the present currency.

£	s.	D.		£	
12	7	4·2	or,	12·742	4) 32043 farthings.
8	9	2·4		8·924	12) 8010$\frac{3}{4}$*d.*
3	6	7·5		3·675	
5	3	8·7		5·387	2,0) 66,7*s.* 6*d.*
	5	0·6		·506	
	8	0·9		·809	*Present currency* 33*l.* 7*s.* 6$\frac{3}{4}$*d.*
£32	0s.	4·3D.	*Imperial.*	£32·043	☞ To be read 32 *Pounds,* nought, four, three.

Again; let it be required to find, 1st, the cost of 423

articles at £3 7s. 4·3d. each; and 2nd, the price of 1, if 423 cost £1583 2s. 8·9d.

```
       £                            £          £
1st.  3·743         2nd.  423) 1583·289 (3·743 = £3 7s. 4·3d.
       423                     1269        Or £3 7s. 4d. 3f.
      -----                    ----
     11229                      3142
      7486                      2961
    14972                       ----
    -------                      1818
 £1583·289 = £1583 2s. 8·9d.     1692
      Or £1583 2s. 8d. 9f.       ----
                                  1269
                                  1269
                                  ----
```

The prefix *Imperial*, here proposed, would, no doubt, go out of use with the disappearance of the present coins, until which time it would effectually prevent confusion, and would also distinguish the new coins of account from the old by a dignifying appellation. There is certainly an objection to there being two kinds of penny, as a *permanence*—the present copper penny and the Imperial (or silver) Penny; but this latter may *also* be called a *Decime*—the tenth part of an Imperial Shilling; and in ordinary every-day transactions over the counter may be denominated, indifferently, either a decime, or a Penny Imperial; just as the present 20*s*. piece is now called, indifferently, either a *pound* or a *sovereign*. In accounts, *decime* would most likely be the general designation so soon as the term Imperial came to be disused in reference to the other coins.

The *coinage* need not be exclusively a decimal coinage; this would be a practical inconvenience; the want of intermediate pieces of money would, in many cases, be seriously felt, especially by the poor; but our money of *accounts* should be exclusively on the decimal system, and no denomination should occur except Pounds, Shillings, Pence (or Decimes), and farthings. And for these, which, on the above plan, regularly descend in value according to the decimal scale, there should unquestionably be appropriate representative coins. The intervals between them should, however, be narrowed by the interpolation of such other coins as would be wanted in the usual pecuniary transactions of life. We would suggest the following coins, where each set of four,

viz., from ¼*d.* to 1D., inclusive; from 1D. to 1s.; and from 1s. to £1, observe this gradation, namely: Twice the first coin of each set gives the second; twice the second, the third; and 2½ times the third (or twice the third + the second) gives the fourth, throughout.

Copper.			Silver.	Silver.				Silver.		Gold.	
¼*d.*,	½*d.*,	1*d.*,	1D.	1D.,	5*d.*,	10*d.*,	1s.	1s.,	2s.,	4s.,	1£.

These are all palpably distinct in size and weight, and the coins in each set have the same relations to one another as the present 3*d.* piece, 6*d.*, 1*s.*, and half-crown. Whether or not there should also be a gold 5s. piece, or half-pound, may be matter for consideration; but we do not think that the want of such an additional coin would be felt.

It may be interesting to notice here that, in the three sets above, the highest coin in each set may be exchanged for lower pieces, all belonging exclusively to that set, in *twelve* different ways (as in the margin); and that, whatever lower pieces in one set are exchangeable for a higher piece in that set, the corresponding pieces in either of the other sets are exchangeable for the like higher piece in *that* set,—calling the like, or corresponding pieces, those which occupy like places in the different sets,—the first place, the second place, or the third place.

1s.	2s.	4s.
10	0	0
8	1	0
6	2	0
4	3	0
2	4	0
0	5	0
6	0	1
4	1	1
2	2	1
0	3	1
2	0	2
0	1	2

10s. = £1 Imp., the coin which is exchangeable for either of the annexed rows of pieces, the values of which are denoted in the *top* row. And similarly for either of the other sets of coins.

As to the coins to be gradually withdrawn, or suffered to be thrust out of circulation, if a system such as that here proposed were to gradually replace the present system, the 3*d.* piece, the 4*d.* piece, and the 6*d.* piece, should disappear first; the 2½*d.* piece (the Decime, or Imperial Penny), and the 5*d.* piece, replacing, or rather *dis*placing them: then the 10*d.* piece and the Imperial Shilling might, in like manner, displace the present shilling and florin; and lastly, the double shilling and the two gold coins might be introduced. And during the transition period, each of the now existing coins would be readily exchangeable for the Imperial coins without the smallest fraction of loss or gain—our copper or bronze pieces

remaining as they are, and retaining their present purchasing power.

As to the specific *names* for the 10*d.* piece and the 5*d.* piece, we may mention that the designations *franc* and *half-franc* have been proposed; and we think that *silver dollar*, and *gold dollar* (or dollar, and double-dollar), would do very well for the 2s. and 4s. pieces. Our circulating coins would then bear the following distinctive names, and have the following values in old money :—

New.					Old.
*Farthing. . . .	1*f.*				= ¼*d.*
*Decime	1D. =		10*f.* =		2½*d.*
Half-franc . . .	2D.				= 5*d.*
Franc (*F*) . . .	4D.				= 10*d.*
*Imperial shilling .	1s. =	10D. =	100*f.* =	2½F. =	2*s.* 1*d.*
Silver dollar. . .	2s.			= 5F. =	4*s.* 2*d.*
Gold dollar . . .	4s.			= 10F. =	8*s.* 4*d.*
*Imperial Pound. .	10s.	= 100D. =	1000*f.* =	25F. =	20*s.* 10*d.*

* The coins thus marked are the monies of Account; the others, with these, and the penny and halfpenny, supply the coins of commerce, or the circulating currency.

It may be as well to observe in conclusion, that there is no necessity to call such a money-item as 8·9D. (p. 217) 8 Pence, or 8 decimes, 9 *farthings:* it may be read 8 decimes, 9 *tenths:* the 9 tenths being 9 farthings, or 2¼*d.*

We shall now give a sketch of a different scheme for a decimal coinage; it has been called "the pound and mil" system, and has been much advocated. With slight verbal alterations, the following exposition of that system is the same as that introduced into the former editions of this work.

The Pound and Mil System.

In the foregoing article, the plan proposed for decimalizing our coins of account is, leaving the present copper coins unmeddled with, to introduce new pieces of the respective ascending values 2½*d.*, 25*d.*, and 250*d.*; and then, in addition to these, for general commercial purposes, to supply coins of the intermediate values 5*d.*, 10*d.*, 50*d.*, and 100*d.* In the plan now to be considered, the higher coins—the sovereign

and the florin—are permanently to remain, as coins of account, and the lower coins—the penny and the farthing —to be replaced by others, of the respective values $2\frac{2}{5}d.$, and $\frac{24}{25}f.$, to be called the *cent* and the *mil*. The value of the sovereign, or £1, will thus be accurately equivalent to 100 cents, or 1000 mils; and we shall have the following table of relations, namely:—

Table.

10 mils = 1 cent.	1000 mils = 100 cents = 10 florins = £1.
10 cents = 1 florin.	
10 florins = 1 pound.	$1s. = 5c. = 50m.$; $1d. = 4\frac{1}{6}m.$
	$\frac{1}{4}d. = 1\frac{1}{24}m. = \frac{25}{24}m.$

Problem 1.

To convert shillings, pence, and farthings, into their equivalents in florins, cents, and mils.

Rule.—1. If the proposed sum involve odd farthings, reduce all to farthings, annex 00 to the result, and divide by 96.

2. If the sum involve a halfpenny only, reduce all to halfpence, annex 00, and divide by 48.

3. If the sum involve no denomination except shillings and pence, reduce all to pence, annex 00, and divide by 24.

4. But if the sum consist of shillings only, annex 00, and divide by 2. In each case the quotient will be the answer: it will be expressed in florins, cents, and mils, if there be *three* places of integers; in cents and mils, if there be *two* integral places; and in mils only if there be but *one* place of integers: whatever decimals or fractions follow will denote so many parts of a mil.

Note.—We may also deduce the required equivalents from the fundamental relations already recorded (Table above), namely:—

$$\tfrac{1}{4}d. = \tfrac{25}{24}m. = 1\tfrac{1}{24}m.;\ 1d. = 4\tfrac{1}{6}m.;\ 1s. = 5c.$$

Examples.

1. How many mils are there in $2d.$?

$$\frac{200}{24}m. = 8\tfrac{8}{24}m. = 8\tfrac{1}{3}m.\ \textit{Ans.}\quad \text{Or, } 4\tfrac{1}{6}m. \times 2 = 8\tfrac{1}{3}m.\ \textit{Ans.}$$

2. How many cents and mils are there in 5*d.*?

$\frac{500}{24}m. = 20\frac{5}{6}m. = 2c.\ 0\frac{5}{6}m.$ Or, $4\frac{1}{6}m. \times 5 = 20\frac{5}{6}m. = 2c.\ 0\frac{5}{6}m.$ *Ans.*

3. How many cents and mils are there in $3\frac{3}{4}d.$?

$\frac{1500}{96}m. = 15\frac{5}{8}m. = 1c.\ 5\frac{5}{8}m.$ Or, $1\frac{1}{24}m. \times 15 = 15\frac{5}{8}m. = 1c.\ 5\frac{5}{8}m.$ *Ans.*

4. How many florins, cents, and mils are there in 3*s.* 10*d.*?

$\frac{4600}{24}m. = 191\frac{2}{3}m. = 1fl.\ 9c.\ 1\frac{2}{3}m.$ Or, since $10d. = 4\frac{1}{6}m. \times 10 = 4c.\ 1\frac{2}{3}m.$, therefore $1fl. + 1s. + 10d. = 1fl.\ 9c.\ 1\frac{2}{3}m.$ *Ans.*

5. How many florins, cents, and mils are there in $15s.\ 7\frac{1}{2}d.$?

$$
\begin{array}{r}
15s.\ 7\frac{1}{2}d. \\
12 \\
\hline
187 \\
2 \\
\hline
4)\ 37500 \\
\hline
12)\ 9375 \\
\hline
781\frac{1}{4}m. = 7fl.\ 8c.\ 1\frac{1}{4}m.\ \textit{Ans.}
\end{array}
$$

Or: $7fl. + 1s. + 6d. + 6f. = 7fl.\ 7c.\ 5m. + 6\frac{1}{4}m. = 7fl.\ 8c.\ 1\frac{1}{4}m.$ *Ans.*

The foregoing Rule is an immediate inference from the condition $1f. = \frac{1000}{960}m. = \frac{100}{96}m.$

6. How many mils are there in $\frac{3}{4}d.$? *Ans.* $3\frac{1}{8}m.$
7. How many florins, cents, and mils are there in 5*s.*? *Ans.* 2*fl.* 5*c.*
8. How many florins, cents, and mils are there in 7*s.* 10*d.*? *Ans.* $3fl.\ 9c.\ 1\frac{2}{3}m.$
9. How many florins, cents, and mils are there in $12s.\ 4\frac{3}{4}d.$? *Ans.* $2fl.\ 1c.\ 9\frac{19}{24}m.$

Problem 2. (Converse of Prob. 1.)

To convert florins, cents, and mils into their equivalents in the present currency.

Rule I.—Twice the number of florins will be the equivalent number of shillings; and 24 times the number of mils divided by 100 will be the equivalent of these mils in pence.

Examples.

1. In $6\frac{1}{4}$ mils, how many pence are there?

$6\frac{1}{4} \times 24 = 150$; and this number of pence divided by 100 is $1{\cdot}50d. = 1\frac{1}{2}d.$ *Ans.*

2. Convert $1 fl.\ 2c.\ 9\frac{1}{8}m.$ into present currency.

$29\frac{1}{8}m. \times 24 \div 100 = 7d.$; therefore $2s.\ 7d.$ is the equivalent.

3. How many shillings are there in $9 fl.\ 7c.\ 5m.$?

This sum is only $2c.\ 5m.$, that is, $25m.$, short of £1; and $25m. = 6d.$; therefore the equivalent is $19s.\ 6d.$

Instead of working as in these examples, we may proceed as follows.

Rule II.—Express the florins, cents, and mils, in mils only; and then (by dividing by 1000) write these as the decimal of £1, which convert into shillings, pence, and farthings, in the usual way. The foregoing examples will be worked by this Rule, thus:—

Ex. 1. $$\begin{array}{r} 6{\cdot}25m. = £{\cdot}00625 \\ 20 \\ \hline {\cdot}12500s. \\ 12 \\ \hline = 1{\cdot}500d. = 1\tfrac{1}{2}d. \\ \hline \end{array}$$

Ex. 2. $$\begin{array}{r} 129\frac{1}{8}m. = £{\cdot}129\frac{1}{8} \\ 20 \\ \hline 2{\cdot}583\frac{1}{3}s. \\ 12 \\ \hline = 31{\cdot}000d. = 2s.\ 7d. \\ \hline \end{array}$$

Ex. 3. $$\begin{array}{r} £{\cdot}975 \\ 20 \\ \hline 19{\cdot}500s. = 19s.\ 6d. \\ \hline \end{array}$$

4. How many shillings are there in $3 fl.\ 9c.\ 1m.$?

$$\begin{array}{r} £ \\ {\cdot}391 \\ 20 \\ \hline 7{\cdot}820s. \\ 12 \\ \hline 9{\cdot}84d. \\ 4 \\ \hline 3{\cdot}36f. \\ \hline \end{array}$$

The work of this example, by Rule II., is that here annexed; and the answer is $7s.\ 9\frac{3}{4}d. + \frac{36}{100}f. = 7s.\ 9\frac{3}{4}d. + \frac{9}{25}f.$ The *exact* equivalent, therefore, in present currency, could not be given: either $\frac{9}{25}f.$ must be withheld, or else $\frac{16}{25}f.$ added. In the former case, the exchange would be $7s.\ 9\frac{3}{4}d.$; and in the latter, $7s.\ 10d.$ Since $1f. = \frac{25}{24}m.$, therefore $\frac{9}{25}f. = \frac{9}{24}m. = \frac{3}{8}m.$; and $\frac{16}{25}f. = \frac{16}{24}m. = \frac{2}{3}m.$ And we shall accordingly find, by working as in the margin, that $3 fl.\ 9c.\ 0\frac{5}{8}m. = 7s.\ 9\frac{3}{4}d.$ *exactly*, and $3 fl.\ 9c.\ 1\frac{2}{3}m. = 7s.\ 10d.$ *exactly*.

Miscellaneous Mercantile Calculations on the Decimal System.

Whatever values and names be given to the coins of account, so long as those values, from the highest value to the lowest, descend uniformly by a decimal gradation, arithmetical operations upon them remain the same. The following examples will be worked in accordance with "the pound and mil system," as just explained.

EXAMPLES.

1. What will 25 yards of linen come to, at 9*c*. 3*m*. per yard?

```
  93m.
  25
 ----
 465
186
 ----
2,325m. = £2 3fl. 2c. 5m. Ans.
```

2. What will 37 lbs. of tea come to, at 2*fl*. 3*c*. 7*m*. per lb.?

```
 237m.
  37
 ----
 1659
 711
 ----
8,769m. = £8 7fl. 6c. 9m. Ans.
```

3. What will be the cost of 50 yards of silk, at 2*fl*. 3*c*. 9*m*. per yard?

```
  239m.
   50
 ------
11,950m. = £11 9fl. 5c. Ans.
```

4. What will be the cost of 99 gallons of brandy, at 9*fl*. 3*c*. 7*m*. per gallon?

As 99 = 100 − 1, we work thus:

```
 93700m.
   937 (Subtract)
 ------
92,763m. = £92 7fl. 6c. 3m. Ans.
```

5. What will 721 cwt. of iron cost, at 8*fl*. 9*c*. 9*m*. per cwt.?

Since this is only 1*m*. short of 9*fl*., we proceed thus:

```
 721
   900m.
 ------
648900m.
   721m. (Subtract)
 ------
648,179m. = £648 1fl. 7c. 9m. Ans.
```

[The learner, by way of exercise, may convert the several results above into the present currency: they will be found

to be: 1st, £2 6*s.* 6*d.*; 2nd, £8 15*s.* $4\frac{14}{25}$*d.*; 3rd, £11 19*s.*; 4th, £92 15*s.* $3\frac{3}{25}$*d.*; 5th, £648 3*s.* $6\frac{24}{25}$*d.*]

6. How much in the pound (£) is £4 3*fl.* 5*c.* per cent.?

The 100th part of this is 43·5*m.* = 4*c.* $3\frac{1}{2}$*m.*, the *Ans.*, which is written down at once from mere inspection: we have only to remove each of the given denominations two places to the right.

7. How much per cent. is 3*fl.* 2*c.* 6*m.* in the pound?

100 times this is 32600*m.* = £32 6*fl.*, the *Ans.*, got by mere inspection, advancing the decimal denominations two places to the left.

8. What will the carriage of 80 tons of goods come to, at 2*fl.* 3*c.* 6*m.* per ton? *Ans.* £18 8*fl.* 8*c.*
9. What will 212 yards of lace come to, at 7*fl.* 5*c.* 8*m.* per yard? *Ans.* £160 6*fl.* 9*c.* 6*m.*
10. What will 1476 tons of coal come to, at 5*fl.* 3*c.* 7*m.* per ton? *Ans.* £792 6*fl.* 1*c.* 2*m.*
11. How much per cent. is 6*fl.* 6*c.* $1\frac{1}{2}$*m.* in the pound? *Ans.* £66 1*fl.* 5*c.*
12. If £457 2*fl.* 1*c.* 5*m.* be divided among 255 persons, what will be the share of each? *Ans.* £1 7*fl.* 9*c.* 3*m.*

We have thus largely illustrated the practical facilities of decimalizing our coins of account conformably to the pound and mil system, because it is that system which is most in favour. Calculations by the former system, however, are executed with precisely the same facility; and therefore one of these systems can claim a preference to the other only on grounds altogether distinct from considerations as to *facility of Calculation.* It is on these other grounds that the balance of advantage or disadvantage is to be estimated. It is a great recommendation of the plan just discussed that the *Pound* preserves its present value, and maintains its present place in the proposed currency; but it is an objectionable necessity of the scheme that the present currency cannot be accurately translated into the new, except in the cases of such sums of money as are multiples of 6*d.* For any sum below 6*d.* there is no exact equivalent in mils, nor for any sum between 6*d.* and 1*s.*, between 1*s.* and 1*s.* 6*d.*, and so on; and we think that this circumstance would be a source of perplexity long after our present copper coins had been displaced by the mil; for although the penny, the halfpenny, and the farthing, should disappear from the eye, yet they would long continue to be present to the mind, in all small money transactions, as standards of comparison. In fact, a person would be obliged to *express* himself in cents and mils

(so to speak), when all the while he was *thinking* of pence and farthings. From this objection the former plan is wholly free; but it has the disadvantage of displacing the pound of accounts, and substituting for it what we have called the *Imperial Pound*, value 20*s*. 10*d*., or 25 francs. Yet, in this system, every existing coin would circulate at its present value, concurrently with the proposed new coins; and in giving change, as well as in paying or receiving payments—whether old or new money exclusively, or a combination of both, were employed—nothing would be lost or gained on either side; every old coin would have its exact equivalent in the new coinage, and every new coin its exact equivalent in the old. During the transition period, there would indeed be a superabundance of coins; but there would be no bewilderment as to the evaluation of the exact purchasing power of any of them; so that not even one of the existing coins need, *of necessity*, be called in: it would be sufficient that more were not sent out, and that as they *came* in they were *kept* in. And it is further deserving of notice that the coin which, in this system, we have proposed to call a *Dollar* (4*s*. 2*d*.), would circulate at that value in all the British Colonies; for, by an Order in Council, issued in 1845, it was directed that Spanish, Mexican, and South American dollars, should be legal tenders at 4*s*. 2*d*. sterling; and this is also very nearly the value, sterling, of the dollar of the United States.

For the off-hand transactions of the shop and the market our existing coinage is fully as convenient as any other that can be substituted for it, at least under our present system of weights and measures; and till this system be also decimalized, we apprehend that a decimal coinage, for shop and market purposes, would be found far *less* convenient than that in present use. But the work of the Accountant and Money-Calculator would unquestionably be facilitated if our moneys of account *only* were according to a decimal system; and *any* decimal system, for this purpose merely, would do as well as any other; but to secure such an advantage, we do not think that the currency of the humbler classes, that is, of those who have to do more with the penny and the farthing than with the pound and the florin, should be perplexed and complicated—much less deteriorated.

It should always be borne in mind that in deciding upon what decimal *coinage* it is best to adopt, the requirements of the accountant and the calculator should be left out of consideration altogether; because to *them*, as already observed, one decimal system would be just as acceptable as another: the *choice* of system should be determined solely in reference to the public convenience in general; and this is the *only* purpose to be kept in view in considering the question—Which of our existing coins should be preserved as the basis of the new system? Shall we take the *highest*, the £, and descend, by *submultiples* of it, down to the coin which should be the lowest in the decimal series; or shall we take the *lowest*, the farthing, and ascend, by *multiples* of it, up to that which should be the highest, interposing, in either case, the coins best suited for general purposes?

Some of the first scientific authorities in the kingdom are unanimous in the preference of the former basis: we think that a commission of retail dealers, small traders in general, and the habitual customers of such, would be quite as unanimous in favour of the latter basis—if any change at all were made imperative. They would say, "If Financiers, Bankers, Accountants, and Calculators, want any new coins for *their* business, let them have them; *we* shall be willing enough to receive and pay these new coins, whenever they come in our way, provided only that we be informed of their exact purchasing power. We shall soon become used to them, and perhaps even thankful for them; but don't take any of ours away—at least unless you replace them, as you have replaced the old copper money, by their exact equivalents in purchasing power." And the remonstrance would be a reasonable one.

From the best consideration he can give to the subject, the present writer is of opinion that the public in general would sooner become reconciled to the first of the changes proposed in this Appendix than to the second. It is not the poor only who would be inconvenienced and perplexed by the abolition of the penny, but the community at large. There are cab fares, omnibus fares, small railway fares, porterages, &c.; and there are besides numberless pecuniary transactions in which pence and halfpence, at the definite values with which everybody is familiar, are in daily request; and no sum of money, estimated according to our existing

system, could be liquidated with exactitude by the coins of the pound and mil system, if that sum, be it small or great, involved odd pence other than sixpence.

It may be objected to the penny system that certain of its coins, so long as the coins in present use coexisted with them, would approach so nearly in size to certain of the latter as to create perplexity, and occasion mistakes; but a disc of metal may be of any superficial extent we please, so that this merely mechanical difficulty could be easily overcome. If the old guinea were now in circulation, concurrently with the sovereign, there would be no hazard of confounding the one coin with the other. Besides, two coins, nearly of the same size, the edge of one being smooth and that of the other milled, could be readily distinguished the one from the other, even in the dark. There would, however, be additional security against mistakes of this kind if the Imperial Pound and the Imperial Shilling were each marked with the number 25, indicating twenty-five *francs*, and twenty-five *pence* respectively; the sovereign and florin being respectively twenty-four francs, and twenty-four pence. And in like manner the franc and half-franc might be marked 10*d.* and 5*d.*; and the dollar, the gold dollar, and the decime (the Imperial or Silver Penny), 5F., 10F., and 2½*d.*, respectively.

[The writer of the foregoing article on Decimal Coinage has endeavoured to represent fairly the peculiar merits and demerits of each of the two systems proposed for adoption; and although it will have appeared that he has a leaning rather to the first-mentioned of these systems than to the second, yet, so impressed is he with the conviction of the satisfactory adaptation of the existing coins to the daily-recurring pecuniary transactions of the great bulk of the community, that he does not *advocate* any alteration at all. With our present coins, a person never feels any difficulty in paying for any purchase—of whatever amount—nor in counting his change, whether he tender shillings, florins, sovereigns, or even bank-notes; and he would most likely consider any proposal to simplify and expedite these ready-money transactions as an offer to "encumber him with help," and that the proffered aid would prove to be a hindrance.]

SUPPLEMENTARY TABLES.

Table I.—This Table shows the values, expressed in decimals of £1, of all sums from $\frac{1}{4}d.$ up to 20*s.* The following instances will sufficiently exemplify the use of it.

1. Express the value of 11*s.* $5\frac{3}{4}d.$ in decimals of £1.

By turning to p. 232 of the Table, we find that 11*s.* $5\frac{3}{4}d.$ is £·57395833.

2. Required the value of £7 8*s.* $7\frac{1}{2}d.$ in decimals of £1.

By the Table, 8*s.* $7\frac{1}{2}d.$ = £·43125 (page 232), therefore, £7 8*s.* $7\frac{1}{2}d.$ =£7·43125.

3. What is the value, in £ *s. d.*, of £3·4628? Here we look in the Table for the decimal ·4628, and at page 232 we find the number the nearest to it to be ·4625, against which stands 9*s.* 3*d.*; therefore £3·4628 = £3 9*s.* 3*d.*, *to the nearest farthing.*

To find the exact value of £3·4628 by *calculation*, we should proceed as in the margin; from which we see that £3·4628 = £3 9*s.* 3*d.* + $\frac{288}{1000}f.$, this latter fraction being a little more than a quarter of a farthing. We know that $\frac{1}{4}f.$ is ·25*f.*; and ·288 is ·038 *greater* than this; that is, it exceeds $\frac{1}{4}f.$ by $\frac{38}{1000}f.$, which is less than a 25th part of a farthing.

$$\begin{array}{r} £3{\cdot}4628 \\ 20 \\ \hline 9{\cdot}256s. \\ 12 \\ \hline 3{\cdot}072d. \\ 4 \\ \hline {\cdot}288f. \end{array}$$

But the value of the small difference between any given decimal of £1, and the approximation to that decimal, in the Table, may be readily calculated, whenever it is thought necessary to do so, by simply converting the small difference into the decimal of a farthing; thus, in the case before us, the difference between the tabular number ·4625 and the given number ·4628 is ·0003; and this multiplied by 20 × 12 × 4 is ·288*f.*

Table II.—This Table expresses any number of days, from 1 day to 365 days, in decimals of a year, and will be found useful in determining the Interest of any sum of money, at a given rate per cent., for any specified number of days. In using the Table for *this* purpose, it is necessary to remember that the yearly interest of £1, at any rate per cent., is the 100th part of that rate; that is, it is the rate itself with a cipher prefixed and preceded by the decimal point: thus—The interest of £1, at 2 per cent., at $2\frac{1}{2}$, at 3, at $3\frac{1}{2}$, at 4, at $4\frac{1}{2}$, is £·02, £·025, £·03, £·035, £·04, £·045, and so on. As an instance of the application of the Table, let us take Example 1, at p. 189, the principal being £325 7*s.*, the time 89 days, and the interest $4\frac{1}{2}$ per cent.

Now 7*s.* = £·35 ; this is, of course, given in Table I.; but there is never any necessity to refer to the *Table* in the case of *shillings only,* since we have merely to take half the number of shillings, and then point off two decimal places. The given sum is therefore £325·35, and this multiplied by ·045, the rate per cent., gives 14·64075. By Table II., the decimal for 89 days is the number ·24384* (limiting the decimals to five places,—a number quite sufficient), and 14·64075 × ·24384 = 3·57. (See Note below.) Turning now to Table I., we find that the nearest decimal to ·57 is ·56979, to which corresponds 11*s.* $4\frac{3}{4}$*d.* Hence the required interest, to the nearest farthing, is £3 11*s.* $4\frac{3}{4}$*d.*, as at p. 189.

Table III., on the weights and measures of France, and on the monetary system of that nation, requires no explanation here.

Note.—Whenever numbers involving several places of decimals are to be multiplied together, or to be divided the one by the other, it is the better way to perform the operations by what are called *Contracted Multiplication* and *Contracted Division;* as taught in most books on common Arithmetic.† The multiplication of 14·64075 by ·24384, indicated above, is here executed by both the common method and the contracted method.

Common method.	*Contracted method.*
14·64075	14·64075
·24384	48342
5856300	2928150
11712600	585630
4392225	43922
5856300	11713
2928150	586
3·5700004800	3·57000

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* Whenever any of the terminating decimals are rejected, as being superfluous for the case in hand, the *last* of the figures retained should always be increased by a unit, if the first of the rejected figures be a 5 or a greater number. It is plain, in the instance in the text, that *five* decimals from the Table will be a sufficient number to employ; for the error of so much as even a unit in the fifth place would be an error only of $\frac{1}{100000}$; and it is easy to foresee that this part of £14·64075 can be but an insignificant fraction of a farthing; for this part of £100 even is less than a whole farthing.

† See the "Arithmetic" in Weale's *Rudimentary Series.*

TABLE I. Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
0	¼	·00104167	1	0¼	·05104167	2	0¼	·10104167	3	0¼	·15104167
0	½	·00208333	1	0½	·05208333	2	0½	·10208333	3	0½	·15208333
0	¾	·003125	1	0¾	·053125	2	0¾	·103125	3	0¾	·153125
0	1	·00416667	1	1	·05416667	2	1	·10416667	3	1	·15416667
0	1¼	·00520833	1	1¼	·05520833	2	1¼	·10520833	3	1¼	·15520833
0	1½	·00625	1	1½	·05625	2	1½	·10625	3	1½	·15625
0	1¾	·00729167	1	1¾	·05729167	2	1¾	·10729167	3	1¾	·15729167
0	2	·00833333	1	2	·05833333	2	2	·10833333	3	2	·15833333
0	2¼	·009375	1	2¼	·059375	2	2¼	·109375	3	2¼	·159375
0	2½	·01041667	1	2½	·06041667	2	2½	·11041667	3	2½	·16041667
0	2¾	·01145833	1	2¾	·06145833	2	2¾	·11145833	3	2¾	·16145833
0	3	·0125	1	3	·0625	2	3	·1125	3	3	·1625
0	3¼	·01354167	1	3¼	·06354167	2	3¼	·11354167	3	3¼	·16354167
0	3½	·01458333	1	3½	·06458333	2	3½	·11458333	3	3½	·16458333
0	3¾	·015625	1	3¾	·065625	2	3¾	·115625	3	3¾	·165625
0	4	·01666667	1	4	·06666667	2	4	·11666667	3	4	·16666667
0	4¼	·01770833	1	4¼	·06770833	2	4¼	·11770833	3	4¼	·16770833
0	4½	·01875	1	4½	·06875	2	4½	·11875	3	4½	·16875
0	4¾	·01979167	1	4¾	·06979167	2	4¾	·11979167	3	4¾	·16979167
0	5	·02083333	1	5	·07083333	2	5	·12083333	3	5	·17083333
0	5¼	·021875	1	5¼	·071875	2	5¼	·121875	3	5¼	·171875
0	5½	·02291667	1	5½	·07291667	2	5½	·12291667	3	5½	·17291667
0	5¾	·02395833	1	5¾	·07395833	2	5¾	·12395833	3	5¾	·17395833
0	6	·025	1	6	·075	2	6	·125	3	6	·175
0	6¼	·02604167	1	6¼	·07604167	2	6¼	·12604167	3	6¼	·17604167
0	6½	·02708333	1	6½	·07708333	2	6½	·12708333	3	6½	·17708333
0	6¾	·028125	1	6¾	·078125	2	6¾	·128125	3	6¾	·178125
0	7	·02916667	1	7	·07916667	2	7	·12916667	3	7	·17916667
0	7¼	·03020833	1	7¼	·08020833	2	7¼	·13020833	3	7¼	·18020833
0	7½	·03125	1	7½	·08125	2	7½	·13125	3	7½	·18125
0	7¾	·03229167	1	7¾	·08229167	2	7¾	·13229167	3	7¾	·18229167
0	8	·03333333	1	8	·08333333	2	8	·13333333	3	8	·18333333
0	8¼	·034375	1	8¼	·084375	2	8¼	·134375	3	8¼	·184375
0	8½	·03541667	1	8½	·08541667	2	8½	·13541667	3	8½	·18541667
0	8¾	·03645833	1	8¾	·08645833	2	8¾	·13645833	3	8¾	·18645833
0	9	·0375	1	9	·0875	2	9	·1375	3	9	·1875
0	9¼	·03854167	1	9¼	·08854167	2	9¼	·13854167	3	9¼	·18854167
0	9½	·03958333	1	9½	·08958333	2	9½	·13958333	3	9½	·18958333
0	9¾	·040625	1	9¾	·090625	2	9¾	·140625	3	9¾	·190625
0	10	·04166667	1	10	·09166667	2	10	·14166667	3	10	·19166667
0	10¼	·04270833	1	10¼	·09270833	2	10¼	·14270833	3	10¼	·19270833
0	10½	·04375	1	10½	·09375	2	10½	·14375	3	10½	·19375
0	10¾	·04479167	1	10¾	·09479167	2	10¾	·14479167	3	10¾	·19479167
0	11	·04583333	1	11	·09583333	2	11	·14583333	3	11	·19583333
0	11¼	·046875	1	11¼	·096875	2	11¼	·146875	3	11¼	·196875
0	11½	·04791667	1	11½	·09791667	2	11½	·14791667	3	11½	·19791667
0	11¾	·04895833	1	11¾	·09895833	2	11¾	·14895833	3	11¾	·19895833
1	0	·05	2	0	·1	3	0	·15	4	0	·2

Table I.—*continued.* Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
4	0¼	·20104167	5	0¼	·25104167	6	0¼	·30104167	7	0¼	·35104167
4	0½	·20208333	5	0½	·25208333	6	0½	·30208333	7	0½	·35208333
4	0¾	·203125	5	0¾	·253125	6	0¾	·303125	7	0¾	·353125
4	1	·20416667	5	1	·25416667	6	1	·30416667	7	1	·35416667
4	1¼	·20520833	5	1¼	·25520833	6	1¼	·30520833	7	1¼	·35520833
4	1½	·20625	5	1½	·25625	6	1½	·30625	7	1½	·35625
4	1¾	·20729167	5	1¾	·25729167	6	1¾	·30729167	7	1¾	·35729167
4	2	·20833333	5	2	·25833333	6	2	·30833333	7	2	·35833333
4	2¼	·209375	5	2¼	·259375	6	2¼	·309375	7	2¼	·359375
4	2½	·21041667	5	2½	·26041667	6	2½	·31041667	7	2½	·36041667
4	2¾	·21145833	5	2¾	·26145833	6	2¾	·31145833	7	2¾	·36145833
4	3	·2125	5	3	·2625	6	3	·3125	7	3	·3625
4	3¼	·21354167	5	3¼	·26354167	6	3¼	·31354167	7	3¼	·36354167
4	3½	·21458333	5	3½	·26458333	6	3½	·31458333	7	3½	·36458333
4	3¾	·215625	5	3¾	·265625	6	3¾	·315625	7	3¾	·365625
4	4	·21666667	5	4	·26666667	6	4	·31666667	7	4	·36666667
4	4¼	·21770833	5	4¼	·26770833	6	4¼	·31770833	7	4¼	·36770833
4	4½	·21875	5	4½	·26875	6	4½	·31875	7	4½	·36875
4	4¾	·21979167	5	4¾	·26979167	6	4¾	·31979167	7	4¾	·36979167
4	5	·22083333	5	5	·27083333	6	5	·32083333	7	5	·37083333
4	5¼	·221875	5	5¼	·271875	6	5¼	·321875	7	5¼	·371875
4	5½	·22291667	5	5½	·27291667	6	5½	·32291667	7	5½	·37291667
4	5¾	·22395833	5	5¾	·27395833	6	5¾	·32395833	7	5¾	·37395833
4	6	·225	5	6	·275	6	6	·325	7	6	·375
4	6¼	·22604167	5	6¼	·27604167	6	6¼	·32604167	7	6¼	·37604167
4	6½	·22708333	5	6½	·27708333	6	6½	·32708333	7	6½	·37708333
4	6¾	·2228125	5	6¾	·278125	6	6¾	·328125	7	6¾	·378125
4	7	·22916667	5	7	·27916667	6	7	·32916667	7	7	·37916667
4	7¼	·23020833	5	7¼	·28020833	6	7¼	·33020833	7	7¼	·38020833
4	7½	·23125	5	7½	·28125	6	7½	·33125	7	7½	·38125
4	7¾	·23229167	5	7¾	·28229167	6	7¾	·33229167	7	7¾	·38229167
4	8	·23333333	5	8	·28333333	6	8	·33333333	7	8	·38333333
4	8¼	·234375	5	8¼	·284375	6	8¼	·334375	7	8¼	·384375
4	8½	·23541667	5	8½	·28541667	6	8½	·33541667	7	8½	·38541667
4	8¾	·23645833	5	8¾	·28645833	6	8¾	·33645833	7	8¾	·38645833
4	9	·2375	5	9	·2875	6	9	·3375	7	9	·3875
4	9¼	·23854167	5	9¼	·28854167	6	9¼	·33854167	7	9¼	·38854167
4	9½	·23958333	5	9½	·28958333	6	9½	·33958333	7	9½	·38958333
4	9¾	·240625	5	9¾	·290625	6	9¾	·340625	7	9¾	·390625
4	10	·24166667	5	10	·29166667	6	10	·34166667	7	10	·39166667
4	10¼	·24270833	5	10¼	·29270833	6	10¼	·34270833	7	10¼	·39270833
4	10½	·24375	5	10½	·29375	6	10½	·34375	7	10½	·39375
4	10¾	·24479167	5	10¾	·29479167	6	10¾	·34479167	7	10¾	·39479167
4	11	·24583333	5	11	·29583333	6	11	·34583333	7	11	·39583333
4	11¼	·246875	5	11¼	·296875	6	11¼	·346875	7	11¼	·396875
4	11½	·24791667	5	11½	·29791667	6	11½	·34791667	7	11½	·39791667
4	11¾	·24895833	5	11¾	·29895833	6	11¾	·34895833	7	11¾	·39895833
5	0	·25	6	0	·3	7	0	·35	8	0	·4

TABLE I.—*continued.* Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
8	0¼	·40104167	9	0¼	·45104167	10	0¼	·50104167	11	0¼	·55104167
8	0½	·40208333	9	0½	·45208333	10	0½	·50208333	11	0½	·55208333
8	0¾	·403125	9	0¾	·453125	10	0¾	·503125	11	0¾	·553125
8	1	·40416667	9	1	·45416667	10	1	·50416667	11	1	·55416667
8	1¼	·40520833	9	1¼	·45520833	10	1¼	·50520833	11	1¼	·55520833
8	1½	·40625	9	1½	·45625	10	1½	·50625	11	1½	·55625
8	1¾	·40729167	9	1¾	·45729167	10	1¾	·50729167	11	1¾	·55729167
8	2	·40833333	9	2	·45833333	10	2	·50833333	11	2	·55833333
8	2¼	·409375	9	2¼	·459375	10	2¼	·509375	11	2¼	·559375
8	2½	·41041667	9	2½	·46041667	10	2½	·51041667	11	2½	·56041667
8	2¾	·41145833	9	2¾	·46145833	10	2¾	·51145833	11	2¾	·56145833
8	3	·4125	9	3	·4625	10	3	·5125	11	3	·5625
8	3¼	·41354167	9	3¼	·46354167	10	3¼	·51354167	11	3¼	·56354167
8	3½	·41458333	9	3½	·46458333	10	3½	·51458333	11	3½	·56458333
8	3¾	·415625	9	3¾	·465625	10	3¾	·515625	11	3¾	·565625
8	4	·41666667	9	4	·46666667	10	4	·51666667	11	4	·56666667
8	4¼	·41770833	9	4¼	·46770833	10	4¼	·51770833	11	4¼	·56770833
8	4½	·41875	9	4½	·46875	10	4½	·51875	11	4½	·56875
8	4¾	·41979167	9	4¾	·46979167	10	4¾	·51979167	11	4¾	·56979167
8	5	·42083333	9	5	·47083333	10	5	·52083333	11	5	·57083333
8	5¼	·421875	9	5¼	·471875	10	5¼	·521875	11	5¼	·571875
8	5½	·42291667	9	5½	·47291667	10	5½	·52291667	11	5½	·57291667
8	5¾	·42395833	9	5¾	·47395833	10	5¾	·52395833	11	5¾	·57395833
8	6	·425	9	6	·575	10	6	·525	11	6	·575
8	6¼	·42604167	9	6¼	·47604167	10	6¼	·52604167	11	6¼	·57604167
8	6½	·42708333	9	6½	·47708333	10	6½	·52708333	11	6½	·57708333
8	6¾	·428125	9	6¾	·478125	10	6¾	·528125	11	6¾	·578125
8	7	·42916667	9	7	·47916667	10	7	·52916667	11	7	·57916667
8	7¼	·43020833	9	7¼	·48020833	10	7¼	·53020833	11	7¼	·58020833
8	7½	·43125	9	7½	·48125	10	7½	·53125	11	7½	·58125
8	7¾	·43229167	9	7¾	·48229167	10	7¾	·53229167	11	7¾	·58229167
8	8	·43333333	9	8	·48333333	10	8	·53333333	11	8	·58333333
8	8¼	·434375	9	8¼	·484375	10	8¼	·534375	11	8¼	·584375
8	8½	·43541667	9	8½	·48541667	10	8½	·53541667	11	8½	·58541667
8	8¾	·43645833	9	8¾	·48645833	10	8¾	·53645833	11	8¾	·58645833
8	9	·4375	9	9	·4875	10	9	·5375	11	9	·5875
8	9¼	·43854167	9	9¼	·48854167	10	9¼	·53854167	11	9¼	·58854167
8	9½	·43958333	9	9½	·48958333	10	9½	·53958333	11	9½	·58958333
8	9¾	·440625	9	9¾	·490625	10	9¾	·540625	11	9¾	·590625
8	10	·44166667	9	10	·49166667	10	10	·54166667	11	10	·59166667
8	10¼	·44270833	9	10¼	·49270833	10	10¼	·54270833	11	10¼	·59270833
8	10½	·44375	9	10½	·49375	10	10½	·54375	11	10½	·59375
8	10¾	·44479167	9	10¾	·49479167	10	10¾	·54479167	11	10¾	·59479167
8	11	·44583333	9	11	·49583333	10	11	·54583333	11	11	·59583333
8	11¼	·446875	9	11¼	·496875	10	11¼	·546875	11	11¼	·596875
8	11½	·44791667	9	11½	·49791667	10	11½	·54791667	11	11½	·59791667
8	11¾	·44895833	9	11¾	·49895833	10	11¾	·54895833	11	11¾	·59895833
9	0	·45	10	0	·5	11	0	·55	12	0	·6

TABLE I.—*continued.* Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
12	0¼	·60104167	13	0¼	·65104167	14	0¼	·70104167	15	0¼	·75104167
12	0½	·60208333	13	0½	·65208333	14	0½	·70208333	15	0½	·75208333
12	0¾	·603125	13	0¾	·653125	14	0¾	·703125	15	0¾	·753125
12	1	·60416667	13	1	·65416667	14	1	·70416667	15	1	·75416667
12	1¼	·60520833	13	1¼	·65520833	14	1¼	·70520833	15	1¼	·75520833
12	1½	·60625	13	1½	·65625	14	1½	·70625	15	1½	·75625
12	1¾	·60729167	13	1¾	·65729167	14	1¾	·70729167	15	1¾	·75729167
12	2	·60833333	13	2	·65833333	14	2	·70833333	15	2	·75833333
12	2¼	·609375	13	2¼	·659375	14	2¼	·709375	15	2¼	·759375
12	2½	·61041667	13	2½	·66041667	14	2½	·71041667	15	2½	·76041667
12	2¾	·61145833	13	2¾	·66145833	14	2¾	·71145833	15	2¾	·76145833
12	3	·6125	13	3	·6625	14	3	·7125	15	3	·7625
12	3¼	·61354167	13	3¼	·66354167	14	3¼	·71354167	15	3¼	·76354167
12	3½	·61458333	13	3½	·66458333	14	3½	·71458333	15	3½	·76458333
12	3¾	·615625	13	3¾	·665625	14	3¾	·715625	15	3¾	·765625
12	4	·61666667	13	4	·66666667	14	4	·71666667	15	4	·76666667
12	4¼	·61770833	13	4¼	·66770833	14	4¼	·71770833	15	4¼	·76770833
12	4½	·61875	13	4½	·66875	14	4½	·71875	15	4½	·76875
12	4¾	·61979167	13	4¾	·66979167	14	4¾	·71979167	15	4¾	·76979167
12	5	·62083333	13	5	·67083333	14	5	·72083333	15	5	·77083333
12	5¼	·621875	13	5¼	·671875	14	5¼	·721875	15	5¼	·771875
12	5½	·62291667	13	5½	·67291667	14	5½	·72291667	15	5½	·77291667
12	5¾	·62395833	13	5¾	·67395833	14	5¾	·72395833	15	5¾	·77395833
12	6	·625	13	6	·675	14	6	·725	15	6	·775
12	6¼	·62604167	13	6¼	·67604167	14	6¼	·72604167	15	6¼	·77604167
12	6½	·62708333	13	6½	·67708333	14	6½	·72708333	15	6½	·77708333
12	6¾	·628125	13	6¾	·678125	14	6¾	·728125	15	6¾	·778125
12	7	·62916667	13	7	·67916667	14	7	·72916667	15	7	·77916667
12	7¼	·63020833	13	7¼	·68020833	14	7¼	·73020833	15	7¼	·78020833
12	7½	·63125	13	7½	·68125	14	7½	·73125	15	7½	·78125
12	7¾	·63229167	13	7¾	·68229167	14	7¾	·73229167	15	7¾	·78229167
12	8	·63333333	13	8	·68333333	14	8	·73333333	15	8	·78333333
12	8¼	·634375	13	8¼	·784375	14	8¼	·734375	15	8¼	·784375
12	8½	·63541667	13	8½	·68541667	14	8½	·73541667	15	8½	·78541667
12	8¾	·63645833	13	8¾	·68645833	14	8¾	·73645833	15	8¾	·78645833
12	9	·6375	13	9	·6875	14	9	·7375	15	9	·7875
12	9¼	·63854167	13	9¼	·68854167	14	9¼	·73854167	15	9¼	·78854167
12	9½	·63958333	13	9½	·68958333	14	9½	·73958333	15	9½	·78958333
12	9¾	·640625	13	9¾	·690625	14	9¾	·740625	15	9¾	·790625
12	10	·64166667	13	10	·69166667	14	10	·74166667	15	10	·79166667
12	10¼	·64270833	13	10¼	·69270833	14	10¼	·74270833	15	10¼	·79270833
12	10½	·64375	13	10½	·69375	14	10½	·74375	15	10½	·79375
12	10¾	·64479167	13	10¾	·69479167	14	10¾	·74479167	15	10¾	·79479167
12	11	·64583333	13	11	·69583333	14	11	·74583333	15	11	·79583333
12	11¼	·646875	13	11¼	·696875	14	11¼	·746875	15	11¼	·796875
12	11½	·64791667	13	11½	·69791667	14	11½	·74791667	15	11½	·79791667
12	11¾	·64895835	13	11¾	·69895833	14	11¾	·74895833	15	11¾	·79895833
13	0	·65	14	0	·7	15	0	·75	16	0	·8

TABLE I.—*continued.* Decimals of a £.

s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£
16	0¼	·80104167	17	0¼	·85104167	18	0¼	·90104167	19	0¼	·95104167
16	0½	·80208333	17	0½	·85208333	18	0½	·90208333	19	0½	·95208333
16	0¾	·803125	17	0¾	·853125	18	0¾	·903125	19	0¾	·953125
16	1	·80416667	17	1	·85416667	18	1	·90416667	19	1	·95416667
16	1¼	·80520833	17	1¼	·85520833	18	1¼	·90520833	19	1¼	·95520833
16	1½	·80625	17	1½	·85625	18	1½	·90625	19	1½	·95625
16	1¾	·80729167	17	1¾	·85729167	18	1¾	·90729167	19	1¾	·95729167
16	2	·80833333	17	2	·85833333	18	2	·90833333	19	2	·95833333
16	2¼	·809375	17	2¼	·859375	18	2¼	·909375	19	2¼	·959375
16	2½	·81041667	17	2½	·86041667	18	2½	·91041667	19	2½	·96041667
16	2¾	·81145833	17	2¾	·86145833	18	2¾	·91145833	19	2¾	·96145833
16	3	·8125	17	3	·8625	18	3	·9125	19	3	·9625
16	3¼	·81354167	17	3¼	·86354167	18	3¼	·91354167	19	3¼	·96354167
16	3½	·81458333	17	3½	·86458333	18	3½	·91458333	19	3½	·96458333
16	3¾	·815625	17	3¾	·865625	18	3¾	·915625	19	3¾	·965625
16	4	·81666667	17	4	·86666667	18	4	·91666667	19	4	·96666667
16	4¼	·81770833	17	4¼	·86770833	18	4¼	·91770833	19	4¼	·96770833
16	4½	·81875	17	4½	·86875	18	4½	·91875	19	4½	·96875
16	4¾	·81979167	17	4¾	·86979167	18	4¾	·91979167	19	4¾	·96979167
16	5	·82083333	17	5	·87083333	18	5	·92083333	19	5	·97083333
16	5¼	·821875	17	5¼	·871875	18	5¼	·921875	19	5¼	·971875
16	5½	·82291667	17	5½	·87291667	18	5½	·92291667	19	5½	·97291667
16	5¾	·82395833	17	5¾	·87395833	18	5¾	·92395833	19	5¾	·97395833
16	6	·825	17	6	·875	18	6	·925	19	6	·975
16	6¼	·82604167	17	6¼	·87604167	18	6¼	·92604167	19	6¼	·97604167
16	6½	·82708333	17	6½	·87708333	18	6½	·92708333	19	6½	·97708333
16	6¾	·828125	17	6¾	·878125	18	6¾	·928125	19	6¾	·978125
16	7	·82916667	17	7	·87916667	18	7	·92916667	19	7	·97916667
16	7¼	·83020833	17	7¼	·88020833	18	7¼	·93020833	19	7¼	·98020833
16	7½	·83125	17	7½	·88125	18	7½	·93125	19	7½	·98125
16	7¾	·83229167	17	7¾	·88229167	18	7¾	·93229167	19	7¾	·98229167
16	8	·83333333	17	8	·88333333	18	8	·93333333	19	8	·98333333
16	8¼	·834375	17	8¼	·884375	18	8¼	·934375	19	8¼	·984375
16	8½	·83541667	17	8½	·88541667	18	8½	·93541667	19	8½	·98541667
16	8¾	·83645833	17	8¾	·88645833	18	8¾	·93645833	19	8¾	·98645833
16	9	·8375	17	9	·8875	18	9	·9375	19	9	·9875
16	9¼	·83854167	17	9¼	·88854167	18	9¼	·93854167	19	9¼	·98854167
16	9½	·83958333	17	9½	·88958333	18	9½	·93958333	19	9½	·98958333
16	9¾	·840625	17	9¾	·890625	18	9¾	·940625	19	9¾	·990625
16	10	·84166667	17	10	·89166667	18	10	·94166667	19	10	·99166667
16	10¼	·84270833	17	10¼	·89270833	18	10¼	·94270833	19	10¼	·99270833
16	10½	·84375	17	10½	·89375	18	10½	·94375	19	10½	·99375
16	10¾	·84479167	17	10¾	·89479167	18	10¾	·94479167	19	10¾	·99479167
16	11	·84583333	17	11	·89583333	18	11	·94583333	19	11	·99583333
16	11¼	·846875	17	11¼	·896875	18	11¼	·946875	19	11¼	·996875
16	11½	·84791667	17	11½	·89791667	18	11½	·94791667	19	11½	·99791667
16	11¾	·84895833	17	11¾	·89895833	18	11¾	·94895833	19	11¾	·99895833
17	0	·85	18	0	·9	19	0	·95	20	0	1·

Table II. Decimals of a Year.

Days.	Years.	Days.	Years.	Days.	Years.	Days.	Years.
1	·00273973	51	·13972603	101	·27671233	151	·41369863
2	·00547945	52	·14246575	102	·27945205	152	·41643836
3	·00821918	53	·14520548	103	·28219178	153	·41917808
4	·01095890	54	·14794521	104	·28493151	154	·42191781
5	·01369863	55	·15068493	105	·28767123	155	·42465753
6	·01643836	56	·15342466	106	·29041096	156	·42739726
7	·01917808	57	·15616438	107	·29315068	157	·43013699
8	·02191781	58	·15890411	108	·29589041	158	·43287671
9	·02465753	59	·16164384	109	·29863014	159	·43561644
10	·02739726	60	·16438356	110	·30136986	160	·43835616
11	·03013699	61	·16712329	111	·30410959	161	·44109589
12	·03287671	62	·16986301	112	·30684932	162	·44383562
13	·03561644	63	·17260274	113	·30958904	163	·44657534
14	·03835616	64	·17534247	114	·31232877	164	·44931507
15	·04109589	65	·17808219	115	·31506849	165	·45205479
16	·04383562	66	·18082192	116	·31780822	166	·45479452
17	·04657534	67	·18356164	117	·32054795	167	·45753425
18	·04931507	68	·18630137	118	·32328767	168	·46027397
19	·05205479	69	·18904110	119	·32602740	169	·46301370
20	·05479452	70	·19178082	120	·32876712	170	·46575342
21	·05753425	71	·19452055	121	·33150685	171	·46849315
22	·06027397	72	·19726027	122	·33424658	172	·47123288
23	·06301370	73	·20000000	123	·33698630	173	·47397260
24	·06575342	74	·20273973	124	·33972603	174	·47671233
25	·06849315	75	·20547945	125	·34246575	175	·47945205
26	·07123288	76	·20821918	126	·34520548	176	·48219178
27	·07397260	77	·21095890	127	·34794521	177	·48493151
28	·07671233	78	·21369863	128	·35068493	178	·48767123
29	·07945205	79	·21643836	129	·35342466	179	·49041096
30	·08219178	80	·21917808	130	·35616438	180	·49315068
31	·08493151	81	·22191781	131	·35890411	181	·49589041
32	·08767123	82	·22465753	132	·36164384	182	·49863014
33	·09041096	83	·22739726	133	·36438356	183	·50136986
34	·09315068	84	·23013699	134	·36712329	184	·50410959
35	·09589041	85	·23287671	135	·36986301	185	·50684932
36	·09863014	86	·23561644	136	·37260274	186	·50958904
37	·10136986	87	·23835616	137	·37534247	187	·51232877
38	·10410959	88	·24109589	138	·37808219	188	·51506849
39	·10684932	89	·24383562	139	·38082192	189	·51780822
40	·10958904	90	·24657534	140	·38356164	190	·52054795
41	·11232877	91	·24931507	141	·38630137	191	·52328767
42	·11506849	92	·25205479	142	·38904110	192	·52602740
43	·11780822	93	·25479452	143	·39178082	193	·52876712
44	·12054795	94	·25753425	144	·39452055	194	·53150685
45	·12328767	95	·26027397	145	·39726027	195	·53424658
46	·12602740	96	·26301370	146	·40000000	196	·53698630
47	·12876712	97	·26575342	147	·40273973	197	·53972603
48	·13150685	98	·26849315	148	·40547945	198	·54246575
49	·13424658	99	·27123288	149	·40821918	199	·54520548
50	·13698630	100	·27397260	150	·41095890	200	·54794521

Table II.—*continued.* Decimals of a Year.

Days.	Years.	Days.	Years.	Days.	Years.	Days.	Years.
201	·55068493	251	·68767123	301	·82465753	351	·96164384
202	·55342466	252	·69041096	302	·82739726	352	·96438356
203	·55616438	253	·69315068	303	·83013699	353	·96712329
204	·55890411	254	·69589041	304	·83287671	354	·96986301
205	·56164384	255	·69863014	305	·83561644	355	·97260274
206	·56438356	256	·70136986	306	·83835616	356	·97534247
207	·56712329	257	·70410959	307	·84109589	357	·97808219
208	·56986301	258	·70684932	308	·84383562	358	·98082192
209	·57260274	259	·70958904	309	·84657534	359	·98356164
210	·57534247	260	·71232877	310	·84931507	360	·98630137
211	·57808219	261	·71506849	311	·85205479	361	·98904110
212	·58082192	262	·71780822	312	·85479452	362	·99178082
213	·58356164	263	·72054795	313	·85753425	263	·99452055
214	·58630137	264	·72328767	314	·86027397	364	·99726027
215	·58904110	265	·72602740	315	·86301370	365	1·
216	·59178082	266	·72876712	316	·86575342		
217	·59452055	267	·73150685	317	·86849315		
218	·59726027	268	·73424658	318	·87123288		
219	·60000000	269	·73698630	319	·87397260		
220	·60273973	270	·73972603	320	·87671233		
221	·60547945	271	·74246575	321	·87945205		
222	·60821918	272	·74520548	322	·88219178		
223	·61095890	273	·74794521	323	·88493151		
224	·61369863	274	·75068493	324	·88767123		
225	·61643836	275	·75342466	325	·89041096		
226	·61917808	276	·75616438	326	·89315068		
227	·62191781	277	·75890411	327	·89589041		
228	·62465753	278	·76164384	328	·89863014		
229	·62739726	279	·76438356	329	·90136986		
230	·63013699	280	·76712329	330	·90410959		
231	·63287671	281	·76986301	331	·90684932		
232	·63561644	282	·77260274	332	·90958904		
233	·63835616	283	·77534247	333	·91232877		
234	·64109589	284	·77808219	334	·91506849		
235	·64383562	285	·78082192	335	·91780822		
236	·64657534	286	·78356164	336	·92054795		
237	·64931507	287	·78630137	337	·92328767		
238	·65205479	288	·78904110	338	·92602740		
239	·65479452	289	·79178082	339	·92876712		
240	·65753425	290	·79452055	340	·93150685		
241	·66027397	291	·79726027	341	·93424658		
242	·66301370	292	·80000000	342	·93698630		
243	·66575342	293	·80273973	343	·93972603		
244	·66849315	294	·80547945	344	·94246575		
245	·67123288	295	·80821918	345	·94520548		
246	·67397260	296	·81095890	346	·94794521		
247	·67671233	297	·81369863	347	·95068493		
248	·67945205	298	·81643836	348	·95342466		
249	·68219178	299	·81917808	349	·95616438		
250	·68493151	300	·82191781	350	·95890411		

TABLE III.

FRENCH WEIGHTS, MEASURES, AND MONEY, WITH THEIR ENGLISH EQUIVALENTS.

1. WEIGHTS.

[The French unit of weight is the GRAMME = 15·44 grains English. It is the weight of a cubic centimètre of distilled water.]

Milligramme	= 1000th of a gramme	. . .	= ·0154	grains English.
Centigramme	= 100th „	. . .	= ·1544	„
Décigramme	= 10th „	. .	= 1·544	„
GRAMME	. . . .	. .	= 15·44	„
Décagramme	= 10 grammes	. .	= 154·4	„
Hectogramme	= 100 „	.	= 1544	„
Kilogramme	= 1000 „	.	= 32⅙ oz. Troy = 2·2057	lbs. av.
Myriagramme	= 10000 „	.	= 321⅔ oz. „ = 22·057	„

⁎ 51 Kilogrammes make 1 cwt. and very nearly ½ lb. besides.

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2. MEASURES.

Length.

[The French unit of linear measure is the MÈTRE = 39·3708 inches. It is the 10 millionth part of the arc of the meridian from the equator to the pole.]

Millimètre	= 1000th of a metre	. . .	= ·03937 inches.
Centimètre	= 100th „	. . .	= ·39371 „
Décimètre	= 10th „	. .	= 3·93708 „
MÈTRE	. . . .	. .	= 39·3708 = 3·2809 ft.
Décamètre	= 10 mètres	. .	= 32·809 ft. = 10·9363 yds.
Hectomètre	= 100 „	.	= 328·09 ft. = 109·363 yds.
Kilomètre	= 1000 „	.	= 1093·63 yds. = ·62138 miles
Myriamètre	= 10000 „	.	= 10936·33 yds. = 6·21382 miles.

NOTE 1.—Since the fraction ⅝ is equal to the decimal ·625, the French kilomètre differs but little from the ⅝ths of an English mile; the difference being ·625 — ·62138 = ·00362, which is less than the $\frac{4}{1000}$th, or the $\frac{1}{250}$th of a mile; so that by estimating a kilomètre at ⅝ths of an English mile, we make an error, in excess, of less than 1 mile in 250 miles. For the ordinary purposes of comparison therefore we may regard 8 kilomètres as equal to five miles; so that the distance between any two places, expressed in kilomètres, may be converted into English miles, near enough for general itinerary objects, by multiplying the number of kilomètres by 5, and then dividing the product by 8; as in the instance in the margin, where we see that 40 kilomètres make 25 miles.

```
   40 kilos.
    5
   ---
8) 200
   ---
   25 miles.
   ---
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2. Certain French linear measures have been abolished since the year 1840. The principal of these are the following, frequently mentioned in books anterior to that date.

The French inch	(*pouce*) = 12 lines (*lignes*)	=	1·094 inches.
„ foot	(*pied*) = 12 pouces . .	=	13·124 „
„ toise = 2 mètres =	6 pieds . .	=	6·5618 feet.

The old French itinerary league (*lieue*) was 2·4222 English miles; that is, $2\frac{3}{7}$ miles very nearly.

Surface.

[The French unit of superficial measure is the ARE = 119·603 sq. yds. It is the square of 10 mètres; that is, of a décamètre.]

Centiare	= 100th of an *are* = 1 square mètre	=	1·196 sq. yds.
ARE	. . . .	=	119·603 „
Decare	= 10 ares	=	·2471 acres.
Hectare	= 100 ares	=	2·471 „

Capacity.

[The French unit of capacity is the LITRE = 61·02705 cubic inches. It is the cube of one-tenth of a mètre, that is, of a décimètre.]

Millitre	= 1000th of a litre = 1 cubic centimètre	= ·06103 cub. in.	
Centilitre	= 100th „	= ·6103 „	
Décilitre	= 10th „	= 6·1027 „	
LITRE	. . . .	= 61·02705 „	
Décalitre	= 10 litres	= 610·2705 „ = 2·201 gallons.	
Hectolitre	= 100 litres	= 3·53166 cub. ft.	= 22·0097 gal.
Kilolitre	= 1000 „ = a cub. métre	= 35·31658 „	= 220·0967 „
Myrialitre	= 10000 „ . . .	= 353·1658 „	= 2200·9667 „

In the measurement of *solids*, a cubic mètre is called a *stère*, a 10th part of which is a *Décistère*, and 10 stères is a Décastère.

It will be seen, from the preceding Tables of Weights and Measures, that the Mètre,—the unit of *length*, is an element entering into even the system of *weights*, as well as into linear, superficial, solid, and quantitative measurements. It is on this account that the French system of weights and measures is called the METRICAL SYSTEM: it is at the same time

a *Decimal* system; because, proceeding from the fundamental unit, the ascending gradations are uniformly at a tenfold rate, and the descending gradations are uniformly by *tenths*.

The money denominations too, as will appear from the Table next following, are likewise according to the decimal system. Accounts however are usually kept, not in francs, décimes, and centimes, but in francs and centimes only.

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3. Money.

[The French unit of money is the Franc = 9·4 pence sterling.]

Centime = 100th of a franc . . . . =	0*s*.	·094*d*.
Décime = 10th „ . . . . =	0*s*.	·94*d*.
Franc . . . . =	0*s*.	9·4*d*.
5 franc piece (silver, *and* gold) . . . . =	3*s*.	11*d*.
10 franc piece (gold) . . . . =	7*s*.	11*d*.
20 franc piece or Napoleon (gold) . . . . =	15*s*.	10*d*.

[Among the money denominations now disused, were the *Sol* or *Sou*, half a decime, or the 20th part of a franc; the *Ecu* = 6 francs; and the *Louis d'Or* = 24 francs. The franc was formerly called a *livre*. It is proper to add, however, that the half-decime continues to be a current coin, and bears the inscription CINQ CENTIMES (five centimes), and that it is still commonly called a *sou*.]

Note.—In the above Table, the value of each of the several French coins, in English money, is the *intrinsic* value; that is, the value as respects the weight and fineness of the metal: but the *exchangeable* values of the coins of the one country for those of the other are regulated by additional considerations,—political and commercial. The £1 or 20*s*. sterling, exchanges, in general, for only 25 francs, which is less than would be given according to the foregoing Table. This number, 25, facilitates the conversion of francs into their exchangeable equivalent in pounds: and the converse; inasmuch as that to divide a number by 25 is the same as to multiply the number by 4 and then divide the product by 100, this latter division being effected by merely pointing off the last two figures for decimals. We thus have the following Rules.

Rule I.—*To convert francs into their equivalent in pounds sterling.* Point off the last two integral figures for decimals, and then multiply the number by 4: the product will express the equivalent in pounds.

Rule II.—*To convert pounds sterling into their equivalent in francs.* Add two ciphers to the integer number denoting the pounds, and then divide by 4 : the quotient will express the equivalent number of francs.

Note.—It is to be understood that the number, denoting the francs to be converted into pounds, does not *itself* involve decimals of a franc : if it do, the pointing off of the last two integral figures will be the advancing of the decimal point two places to the *left*.

The number, too, denoting the pounds, to be exchanged for francs, is, in like manner, considered to be an integral number ; but if it involve decimals of a pound, the decimal point is to be removed two places to the *right ;* the wanting place, if there be but one place of decimals in the pounds, is to be supplied by a cipher. We shall give an example or two of these conversions.

1. How many pounds are there in 2500 francs?
   25·00 × 4 = £100, the *Ans.*
2. How many pounds are there in 3684 francs?
   36·84 × 4 = £147·36 = £147 7*s.* 2⅖*d.*, the *Ans.*
3. How many francs are there in £120?
   12000 ÷ 4 = 3000 fr., the *Ans.*
4. How many francs are there in £147·36?
   14736 ÷ 4 = 3684 fr., the *Ans.*
5. How many francs are there in £238·648? 23864·8 ÷ 4 = 5966·2 fr. = 5966 francs, 2 décimes, or 5966 francs, 20 centimes. *Ans.*

The foregoing are the common Rules for reducing francs to pounds, and pounds to francs : we shall now give a different mode of proceeding ; one which will be found to be more especially convenient when the sums proposed are of but small amount.

Since 25 francs = 20*s.*, it follows that any number of francs are equivalent to that same number of shillings diminished by one-fifth of the number ; and that any number of shillings are equivalent to that number of francs and one-fourth of the number more. Thus :—

1. 125 fr. = 125 − 25 shillings = 100*s.* 2. 37 fr. = 37*s.* − 7⅖*s.* = 29⅗*s.* = £1 9*s.* 7⅕*d.* 3. (Ex. 2 above.) 3684 fr. = 3684*s.* − 736⅘*s.* = 2947⅕*s.* = £147 7*s.* 2⅖*d.* 4. 20 fr. = 16*s.* 5. 25 fr. = 20*s.* 6. 1 fr. = 1*s.* − ⅕*s.* = 9⅗*d.*

Again 1. 14*s.* = 14 fr. + 3½ fr. = 17 fr. 50c. 2. 17*s.* = 17 fr. + 4¼ fr. = 21 fr. 25c. 3. £5 = 100*s.* = 125 fr. 4. £19 = 380*s.* = 380 fr. + 95 fr. = 475 fr. 5. 13*s.* 6*d.* = 13½ fr. + 3⅜ fr. = 16⅞ fr. = 16 fr. 87½c. 6. 1*s.* = 1¼ fr. = 1 fr. 25c. And even from this the equivalent of any number of shillings may be readily found : thus, 14*s.* = 14 fr. + 250c. + 100c. = 17 fr. 50c.; by regarding 14 times 25c. as 10 times and 4 times.

To what has now been said respecting the comparative values of the French and English coins, we may add, for the practical purposes of visitors to France, that in all the ordinary money transactions of every-day life, *ten* centimes count as *one* penny, and *five* as *one* halfpenny: thus, $75c. = 7\frac{1}{2}d.$; $25c. = 2\frac{1}{2}d.$; $15c. = 1\frac{1}{2}d.$; and so on.

NOTE ON PROB. 6, p. 109.

This problem may be otherwise investigated as follows. Suppose q gallons to be the quantity of water, sweetened or unsweetened, to be added: then it is plain that q (cost of a gallon of spirits − cost of a gallon of water) = profit on the $1 + q$ gallons;

therefore, profit per gallon $= \frac{q}{1+q}$ (cost of a gallon of spirits − cost of a gallon of water);

therefore, $\frac{1+q}{q}$ profit per gallon = cost of a gallon of spirits − cost of a gallon of water;

therefore, $\frac{1}{q}$ profit per gallon = cost of a gallon of spirits − (cost of a gallon of water + profit per gallon).

$$\text{therefore } q = \frac{\text{Profit per gallon of mixture.}}{\text{cost of a gal. of spirits} - (\text{cost of a gal. of water} + \text{profit per gal. of mixture}).}$$

And this is the RULE at page 109.

But the Rule may be established in another, and a more easy way, by aid of the general principle at page 124, as follows.

Let A be the number of shillings, or of pence, in the cost of a gallon of the spirits; C, the number in the cost of a gallon of the water; and B, the number in the cost of a gallon of the *mixture:* also, let P denote the number of shillings, or pence, in the *profit*, per gallon, on this mixture. Then, by the principle referred to, a compound, at the stipulated price per gallon, will be obtained by mixing $B - C$ gallons of spirits with $A - B$ gallons of the water; that is, since $A - B = P$, and therefore $B = A - P$, and $B - C = A - C - P$,

$A - (C + P) =$ the number of gallons of spirits.
and P = " the water.

Hence to *one* gallon of spirits there must be added $\frac{P}{A - (C + P)}$ gallons of the water, as above.

THE END.

LONDON: PRINTED BY WILLIAM CLOWES AND SONS, STAMFORD STREET
AND CHARING CROSS.

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TESTIMONIALS ADDRESSED TO MR. D. O'GORMAN.

Royal Mint, London.

DEAR SIR,—I have read with great pleasure your last edition on calculations, the easiest and the best I have ever met with. I have shown it to my friends, engineers, and moneyers, who are all unanimous in pronouncing it the most useful book they have ever seen.

WILLIAM BUCKLE,
Royal Engineer.

Lancashire and Yorkshire Railway,
General Manager's Office, Manchester.

DEAR SIR,—I duly received a copy of your new "Intuitive Calculations," and I can assure you the methods adopted therein are far beyond what I expected. A good knowledge of the tables, I find, is all that is required to glide easily into the midst of calculations which, except by your ingenious invention, would be most tedious and cumbersome. I am very much interested in your book, and glad to find you have such extended patronage among railway managers and clerks. Your rules for calculating the carriage of goods and for the business of the clearance-house must be invaluable; the adoption of them will greatly lighten drudgery, and the sooner your system is adopted in such places the better. In fact, I find nothing omitted in general use for the business man.

Very truly yours,
M. F. FENNELLY.

"Nautical Almanac" Office, 3, Verulam Buildings,
Gray's Inn.

DEAR SIR,—Accept my thanks for the copy of the eighteenth edition of your "Intuitive Calculations" which you were kind enough to send me. It is certainly a departure, in the right direction, from everything of the kind which has preceded it. The whole book bears indelibly the stamp of originality, and its usefulness in the ordinary business matters of life is strikingly apparent. The rules are written in a clear and pleasing manner, which, with the various notes, are exceedingly well calculated to create that feeling of interest so essential to the acquisition of commercial knowledge. Large as has been the sale of the work up to the present time, it falls short of what its merits deserve.

I am, dear Sir, faithfully yours,
RICHARD FARLEY, F.R.A.S.,
Chief Calculator in the "Nautical Almanac" Office, London.

O'GORMAN'S "INTUITIVE CALCULATIONS," 24th Edition. Edited by PROFESSOR YOUNG, 1871.

Cambridge Observatory.

DEAR SIR,—I have carefully examined the eighteenth edition of your "Intuitive Calculations," and have great pleasure in bearing testimony to the ingenuity of your rules, which produce such brevity in operation. You have arranged a system that must render great and important service to the mercantile community, as well as to the rising generation, notwithstanding that many have grappled with the subject of which you treat. Some brought out works obscurely written; whilst others were found more cumbrous than even the common routine; but the ease with which you arrive at conclusions, and the concise and clearly developed rules, as well as the simplicity of your methods, belong to a different order, and must be incalculably invaluable in the various mercantile pursuits in life. No branch of mercantile calculations appears to be overlooked in your treatise, and the brevity with which you calculate the most minute fraction, at once shows that you were fully competent for the work you had undertaken. Indeed, I do not wonder at the numerous list of subscribers that fill your pages; and I have no doubt, the more your book is known, the more will the public appreciate the services you have rendered to the busy part of mankind.

I am, Sir, respectfully yours,
JAMES BREEN.

Royal Mint.

DEAR SIR,—Your welcome note of the 28th of September, and copy of your thirteenth edition of "Intuitive Calculations," came duly to hand, for which I feel much obliged. I have hastily looked over it, and cannot say too much of its usefulness. It will be my desk-companion.

Yours very truly,
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